MULTIMODAL SEMIOTICS OF MATHEMATICS TEACHING AND LEARNING

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By

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ABSTRACT

The practice of mathematics education is fundamentally multimodal. It incorporates not only talk and embodied action, but also technical notation and diagrams, brought into discourse through verbal and gestural reference. As this interplay of semiotic systems arises in interaction, it can be interpreted by analyzing sequences of talk, writing, and gesture, but a better understanding requires an ethnographic perspective to contextualize interaction with reference to its physical surroundings, institutional setting, and enduring relationships within the community. Thus, classroom interaction is best understood as a multimodal ecology in which micro-level discursive practices, the history of a community, and the biographies of its members mutually influence and determine one another.

Beginning from the ecological perspective on classroom interaction, this dissertation presents an analysis of observations and video recordings collected during a semester's multi-site ethnographic fieldwork with both a middle school math class for English learners and a quasi-remedial college calculus section. To model how any perceptible feature of the environment may be taken as meaningful, I draw on the semiotic theory of C. S. Peirce, not only in interpreting observational data, but as an organizing principle, as the analysis moves from qualities, to particular instances, to recurring patterns. First, I consider the ontological status of mathematical notation as a quality of interaction, investigating its capacity for representing continuous phenomena. Second, I take up actually occurring sequences of interaction, showing how students’ participation in conversational repair offers insights into ideologies of classroom authority and mathematical knowledge. Third, I address students’ and teachers’ identities in the classroom as social perceptions that are constructed and recognized through patterns of interaction. Each area of
inquiry is then reconsidered to identify semiotic affordances that are made available in classroom interaction, and to explicate problems of practice that prevent students from seeing themselves as successful mathematics learners. The problems I identify are similar to those addressed by mathematics education researchers in the learning sciences, so I conclude by proposing a future research trajectory combining linguistic anthropology, the learning sciences, and classroom teaching practice.
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Chapter 1
FINDING MEANING IN MATHEMATICS

1.1. MATHEMATICS AS COMMUNICATION

Mathematicians understand their discipline to be a process of logical reasoning about abstract mathematical objects such as numbers, sets, and relations between them, through which discoveries may be made with potential implications for our understanding of the physical world. Students, on the other hand, often conceive of mathematics as a collection of algorithmic techniques for manipulating formal notation, disregarding the concepts that those symbols might represent; as a result, the study of math is seen to be nothing more than a sequence of progressively more sophisticated manipulations that enable the student to answer more complex questions posed by the teacher or textbook. While this aspect of mathematics is the most readily apparent to students, practitioners view it as the least interesting part, talking derisively about “plugging and chugging,” the mechanistic computation of solutions to equations, as well as “drilling and killing,” a style of teaching that presents students with long sets of plug-and-chug problems in order to increase their fluency with the notation.

Teachers and researchers struggle to find ways of bridging the gap between their conception of mathematics and that of their students. One common way to understand the difference between the two views is to focus attention on the ontological status of the notation. Written equations and graphs present a sort of map of an abstract territory, and one that students often mistake for the territory itself, understanding the object of study to be the notation rather than the mathematical models and abstractions that it represents. This confusion has long been recognized as a problem with broader consequences for mathematics education:

confusions ... can occur when the attention of the pupil is focused more closely on the symbols themselves (i.e. the language itself), rather than on the meanings of those symbols.
This problem arises partly because of the abstractness of the referents for many of these symbols. It also comes about from an attempt to teach to pupils the practice of successful mathematicians, who act upon the symbols as if they were the mathematical objects themselves. (Pimm 1987, 19)

So, for example, if students are presented with an equation such as $2x + 3 = 11$, it is difficult for them to take it as a question meaning “What quantity, when it is doubled and then three is added, gives eleven as the result?” Instead, they understand it to mean “This is a puzzle that must be solved by using certain procedural tricks, and the solution is an equation of the form $x = (\text{something})$.” They may know how to properly derive sequences of well-formed statements using mathematical notation, but without connecting them to any sort of real or abstract referent; in other words, they have learned the syntax of the notation, but not its semantics.

Viewed in this way, a fundamental problem of mathematics education is revealed to be essentially a question about language and communication. First, the educational goal is to help students understand that mathematical language and notation are ways of representing and communicating statements about the world. Additionally, this problem concerns the process as well as the product of education; students’ growing understanding that mathematics is communication is itself reached through communication, either in a co-present setting, as in classroom discourse, or more removed in time and place, as when students read textbooks and complete problem sets. And ultimately, students’ success or failure at learning mathematical techniques and ways of thinking is judged by the way they express themselves, both in the answers they give to problems and in the way they articulate and justify those answers. Even the Common Core State Standards characterize (cognitive) understanding and (social) communication as inseparable: “One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, why a particular mathematical statement is true ... The student who can explain the rule understands the mathematics” (Common Core State Standards Initiative 2012).
This view of mathematics education has led to a push for interdisciplinary research that considers the classroom as a site for the embodied instantiation of multimodal discursive practices (Gutierrez, Sengupta-Irving, and Dieckmann 2010), and that situates its analysis at the intersection of the communicative and the cognitive (Sfard 2012):

if mathematical understanding involves multiple modalities and artifacts, including oral and written language, gesture, the body, symbols, representations, and the like, then studies that take an interdisciplinary approach would enhance the field of mathematics education research. In particular, situated cognition, and anthropological, activity theoretic, social practice, and ethnomathematical perspectives could deepen our understanding of mathematics as a multimodal practice. This view of mathematics as a multimodal and multi-semiotic activity, we believe, moves us toward a more expansive way of approaching mathematical communication and learning. (Gutierrez, Sengupta-Irving, and Dieckmann 2010, 32)

This dissertation answers this call by considering formal notation as one among many meaning-making resources in the rich semiotic environment of the mathematics classroom. Through an analysis of the complete communicative ecology of classroom interaction, including speech, written language, diagrams, and gesture as well as notation, it becomes possible to present a multimodal analysis that aims to account for the semiotic complexity of classroom interaction.

1.2. **What a Sociocultural Linguist Sees in a Mathematics Lesson**

To illustrate the perspective I bring to mathematics education research, I offer an anecdote, first of all as a slice of life from one of my ethnographic field sites, but also as a specific example to ground the following discussion of the motivations for this study. The scene that I will describe took place one Thursday morning in January in Ms. M’s middle school math class, during a lesson that focused on converting between fractional, decimal, and percentage representations. Earlier in the lesson, the teacher had described this task as an act of translating between three different
languages, a familiar metaphor for her students, who were all English learners. As the story begins, Ms. M has completed her initial explanation and is calling on students to help her model a few practice problems.

1.2.1. **Fifty out of a Hundred**

“Who wants to shade this decimal?” asked Ms. M, circling 0.5 on the white board. The usual suspects raised their hands, but she waited for a moment, hoping more students would volunteer. When no one did, she tilted her head and stared directly at Alberto in the second row, focusing the full force of her gaze on him. Receiving no response, she began to bend slightly forward at the waist, moving toward him to add that little extra bit of intensity. Alberto’s hand reluctantly went up.

“Yeah,” said Ms. M brightly, calling on Alberto as if his participation had been his own idea. She walked over to offer him the whiteboard pen as he clambered out of the too-small space between his desk and the one behind him.

Alberto was a softspoken boy, a tall, lanky seventh grader who had moved here from Bolivia within the previous year or so. Ms. M found him “smart, but lazy”—he usually got right answers on homework and tests, but didn’t like to speak up in class—and this may have been the reason she singled him out that day. He took the pen from the teacher’s hand, turned to smile over his shoulder as he passed a friend, reached the board, and began to write up his answer.

A ten-by-ten grid was displayed on the board, and the task was to color in a proportion of the grid that corresponded to the given number. Alberto began to draw a line downward through the first column of the grid. Once he had got through five squares, Ms. M said “Could you just lift your hand?” and adjusted something on the board. She stepped back to allow him to continue, but instead, he raised his hands to chin height and gave a flick of the wrist, as if to say, *Ta-dah!*

“That’s it?” asked Ms. M, and then, as he walked back to his seat, she asked the class: “Give me a thumbs up if you agree with him, thumbs down if you disagree. I want to see everyone with a thumb up or a thumb down…. Hmm, everyone agrees with him?”
“It should be fifty,” Nayan called out.

“I think Rita disagrees with him. Come up, Rita, and show me what you would do.”

As Rita walked to the board from her seat in the back row, there were a few anticipatory oohs and ahs from the class—“Oh, do you see it?” Ms. M asked Tiana—and a few more after Rita had filled in the first five columns in the grid and walked back to her seat.

Ms. M continued: “Alberto, why were you wrong?... How many squares do we have here? A hundred, right? But this,” she said, pointing to 0.5, “is five out of how many.”

“Oh,” said Alberto, “it’s fifty.”

“Ten, right?” Ms. M said. “So out of one hundred how much would that be?”

“Fifty!” Fernando called out.

“Fifty out of a hundred,” said Ms. M.

1.2.2. Observations, Reflections, and Areas of Inquiry

Ms. M’s class was one of two mathematics classes where I conducted ethnographic fieldwork and video recording during the spring semester of 2014. I wrote the above narrative by referring to the video record, as well as my own impressions and hypotheses at the time, as recorded in my field notes. In Chapter 2, I provide detailed descriptions of the two field sites and outline how I collected evidence for analysis.

As a participant observer in these two classrooms, I reflected on what I had observed, focused my attention in ever more directed ways in each following observation, and through this iterative process, gradually articulated the research questions that I address in this dissertation. First, consider the metaphor in which fractions, decimals, and percentages are seen as “three different languages.” If we believe, as Ms. M told her students, that $\frac{1}{2}$, 0.5, and 50% are in some way comparable to English water, Spanish agua, and Bengali pāni, then what corresponds to the clear liquid substance composed of H₂O molecules? Ms. M’s presentation displayed a photograph of a falling water droplet; is this image somehow comparable to Rita’s grid with 50 squares shaded? In
semiotic terms, words and numerals are *symbols*, while diagrams and photographs are *icons*; but if the photograph is an iconic representation of an actual water droplet, what is represented by the grid?

Going further, we see that mathematical communication is not limited to notation and diagrams, but also includes the words written on the board or spoken by students and teachers. In the classroom setting, we see that additional communicative work is done in embodied channels, such as Alberto’s shrug, Ms. M’s offer of the pen, or Rita’s choice to sit in the back row. To understand the nature of mathematical objects as it is made visible through classroom interaction, I can examine the notation and diagrams that are used to represent these objects, but such technical representations come into discourse as components of a complex multimodal ecology. Accordingly, I ask, *what do technical mathematical representations mean, and how are they incorporated into multimodal configurations?* This question motivates the analysis in Chapter 3.

Another observation that caught my attention is in the teacher’s wrap-up that occurred at the end of my retelling. As she is explaining the difference between the decimal 0.5 and the concept *five out of a hundred*, she points to the number and asks, “This is five out of how many?” And Alberto says, “Oh, it’s fifty.” He doesn’t answer her question directly, but he does attend to it and identify his mistake. *Fifty* is not the correct answer to Ms. M’s prompt, but it is the solution to the problem, and we see this a moment later, when Fernando repeats “fifty” and is tacitly recognized by the teacher. So what is Alberto doing, and why doesn’t Ms. M acknowledge it? Drawing from Erving Goffman’s (1981) discussion of the ways one utterance follows on another, we might say that Alberto is *responding* to Ms. M, but he isn’t *replying* to her question. In that moment, she’s anticipating “ten,” and when she doesn’t get it, she supplies it herself. She could potentially validate Alberto’s response by saying something like “Yes, you’ve got it,” but instead, she seems locked into a step-by-step explanation, offered for the sake of a student who has just shown that he doesn’t need it. It almost looks as if she’s correcting Alberto once again, but if this is the case, what is the mistake to be
corrected? *Fifty* is the correct number of squares to shade in, but Ms. M doesn’t recognize it because Alberto is providing it too early in the sequence of talk. In fact, Nayan volunteered *fifty* even earlier in the sequence, but received no response of any kind. In this sense, we might say that “the right answer” does not mean “the mathematically accurate answer,” but rather “the answer that the teacher anticipates,” or more technically, “the appropriate answer at this moment in sequence.”

It seems, then, that *being correct* is a consequence of the timing and manner of a student’s participation in interaction, just as much as it is a consequence of the mathematics. But what does this mean for our common understanding that there is always one objectively correct answer to any mathematical question? This idea is central to many people’s understanding of the discipline; I have taught students who love math because of this, and at least as many who hate it for just the same reason. But at the interactional moment we are considering now, *ten* is the one right answer to Ms. M’s intermediary question, and Alberto gets it “wrong” because he is answering the problem as a whole. Mathematically, his answer could be considered correct, but in sequence, it cannot be taken up just yet. In other words, it is the structure of the interactional sequence, not the mathematics, that demands a unique right answer. So I ask, how does this come about? *How do sequences of classroom interaction realize ideologies of mathematical knowledge?* This question will be taken up in Chapter 4.

Finally, what does Alberto’s performance signify? His reluctance to answer, his slow walk to the board, that shrug of the shoulders—at first, they seemed to me an exaggerated show of nonchalance, as if he knew he wasn’t going to have the right answer, and he meant to defend against the attendant loss of face by saying, “I don’t really care about this anyway.” He’s taller than most of his classmates, maybe old for his grade, but his English proficiency is low and his education in Bolivia might not have prepared him even for this special ESL math class; the coolness in his demeanor, I thought, might be masking a deeper shame. But as I got to know Alberto, I came to understand not only that he did care about doing well, but that he was in fact often quick to find the
right answer. He spoke English hesitantly and avoided talking in whole-class settings, but as the semester went on and newly arrived students joined the class, Alberto was often the one to sit next to them and help them get oriented to mathematics content as well as classroom procedures. What was it about his self-presentation that led me to misjudge him so completely?

The question, essentially, is What does it mean to be a “good math student”? Since my own days teaching math, I have been curious about why people tend to see this issue in such essentialist terms; you even hear educated adults say “I’m not a math person,” with no more than the barest trace of embarrassment. Someone who claims to be “not a math person” presupposes that some people are in fact “math people”—but how can those people be recognized? An ethnomethodologist might phrase the question, How does one do being-a-math-person? We can therefore consider “good math student” to be a performed identity rather than a level of academic achievement, but this raises the question, How is the “good math student” identity performed, and how does it relate to other aspects of students’ identities, as they are expressed in the classroom? These questions will be considered in Chapter 5.

In summary, this dissertation addresses three primary research questions that view mathematics classroom interaction through the lenses of technical notation, interactional sequence, and performed identity. As the “Fifty out of a Hundred” anecdote illustrates, these aspects of the communicative ecology are always present in the classroom scene, and each of them may be made visible by different analytic choices. At the same time, however, they seem to reflect diverse types of phenomena occurring at different levels of social organization. Each of them has meaning, but each belongs to a different class of signs, and I turn to C. S. Peirce’s ([1902] 1955) classification to characterize the differences between them. In Peirce’s phenomenology, potentially meaningful phenomena may be classified as Firsts, or qualities; Seconds, actually existing objects and events; or Thirds, more general patterns, laws, or regularities. In this framework, a notational form such as 0.5 is best understood as a Firstness, a quality of interaction, which exists as a potentiality that may be
instantiated in particular occurrences. A sequence such as How much would that be? Fifty! is a
Secondness, an actual instance of interaction that can be viewed as a constellation of qualities, but
whose meaning is contingent on the particularities of how it took place. Finally, a perception of
identity such as He’s a lazy student (or I’m not a math person) is a Thirdness, a more general
understanding derived from a regular pattern that occurs in a series of interactional events,
including occasions that feature the same individual—He never volunteers to speak up in class—as
well as experiences of a number of individuals—Good students are active class participants. Peirce’s
framework can thus be seen as an organizing principle for this dissertation (see Appendix A,
column 2), and I use it as well in following chapters to illustrate the semiotic foundation of
discursive phenomena such as professional vision (Goodwin 1994), social agency (Kockelman
2007), stancetaking (Du Bois 2007), and positioning (Harré and van Langenhove 1999); a reference
table is provided in Appendix A.

In following chapters, I use Peirce’s semiotic theory to pursue the areas of inquiry outlined
above, regarding notation, sequence, and identity. To understand these semiotic entities and
processes as they arise in the course of classroom interaction, I operationalize this theory through
the methodologies of linguistic ethnography and multimodal interaction analysis, which allow me
to observe as broad a range of phenomena as possible in order to describe and interpret particular
communicative practices and instances. What follows in the rest of this chapter is a summary of
classroom ethnography, semiotic theory, and multimodal interaction analysis, followed by an
overview of the chapters to come.

1.3. Theory and Method

All human interaction is to some degree multimodal. Even text-based forms of
communication, such as telegrams or Ph.D. dissertations, emerge from situated cultural practices
that involve technologically mediated interactions between social actors, and that are informed by
participants’ prior embodied experiences. The use of technology in these cases does function to
elide the multimodal character of interaction, however, while in classroom discourse, the richness of the environment is readily apparent. We see it in the simultaneous use of speech and writing, the physical and temporal coordination among participants, the gestural and prosodic techniques that teachers use to direct and maintain students’ attention. And in mathematics instruction, entirely new technical semiotic systems of notation and diagram come into play as both objects and tools of disciplinary content learning. By focusing on mathematics teaching and learning, then, we are forced to address more general questions regarding how communication includes but is not limited to language, how various social actors and semiotic channels combine to create our holistic understanding of what is happening, and how researchers can achieve an analytic perspective on all of it. To confront these challenges, I draw upon the scholarly traditions of the ethnography of communication, semiotic theory, and multimodal interaction analysis, which are described in detail in this section.

1.3.1. Ethnographic Perspectives in the Classroom

I referred above to the “complete communicative ecology of classroom interaction,” a foundational concept that deserves fuller explanation. By using this phrase, I mean to align with the theoretical stance that

syntax [for example] is but one component of a larger ecology of meaning-making resources used to build action in interaction. It is thus important not to focus our analysis exclusively on the properties of individual semiotic modalities, but instead we should explore and explicate the intricate process of ongoing synthesis and mutual calibration of disparate kinds of information provided through multimodal conduct. (Hayashi 2005, 47)

Hayashi’s concern is with syntax, but the same could be said of any single element of communication, not limited to talk but also including embodied actions, written inscriptions, and other salient aspects of material culture. In addition, this “ecology” incorporates not only the array of available communicative channels and technologies, but also, as Erickson (2015) details, the
range of participant roles, as participants mutually influence one another through looking and listening behaviors, and reciprocally coordinate their timing as interaction proceeds. Calling the semiotized social world an “ecology” means that we cannot think of entities without thinking of relationships, not only the connections between an organism and its environment that reveal cognitive processes to be situated and embodied (Bateson 1972), but also the social ecology within which behavior comes to be seen as social action and cultural practice (Lave and Wenger 1991). This theory motivates a methodology that does not privilege language a priori, but rather views it as one component integrated within a more general social and communicative whole; this is the methodological project that led Dell Hymes and John Gumperz to develop the *ethnography of communication*, calling for studies that “take as context a community, investigating its communicative habits as a whole, so that any given use of channel and code takes its place as but part of the resources upon which the members of the community draw” (Hymes 1964, 3). These resources include not only “channel and code,” but also the physical arrangement of the space where interaction occurs (Wolfgram 2014), the selection and identification of participants (Goffman 1964), and the ways in which these people share physical space and temporally coordinate action (Erickson 2010).

For researchers interested in how learning happens in formal educational environments, it is clear that the classroom can be viewed as fundamentally linguistic as well as fundamentally multimodal; it is the case not only that “reading and writing float on a sea of talk” (Britton 1970, 164), but also, that *talk* is too narrow a focus; “the study of behavior while speaking and the study of behavior of those who are present to each other but not engaged in talk cannot be analytically separated” (Goffman 1964, 134), in the classroom as much as anywhere else. Responding to this semiotic complexity, the Ethnography of Communication methodology was quickly adopted for use in classroom settings (Cazden, John, and Hymes 1972; Gilmore and Glatthorn 1982; Green and Wallat 1981). Heath (1982) provides an overview, beginning with the methods and theories of
ethnography as originally developed by sociocultural anthropologists working in relatively isolated tribal communities, and going on to critique the adaptations and applications of ethnographic methods to study formal education in industrialized societies. While she finds that ethnographies of speaking often promise more than they can deliver in terms of providing a neutral, holistic view of a cultural group, she nevertheless sees them as essential to education research, since “large-scale surveys, correlational studies, and exclusively quantitative studies do not provide actual data about events either in the classroom or the communities of students and teachers” (Heath 1982, 43). By focusing on actual events, ethnographers are able to address questions of “what people customarily do [and] how the doings get done” (Erickson and Mohatt 1982, 138); Patricia Duff (2002) identifies ethnography as a methodology that can combine not only macro and micro analyses, but also researchers’ etic and participants’ emic perspectives.

As David Bloome (2012) points out, to articulate a clear vision of what ethnography is, it is important to distinguish “between an ethnographic perspective and ethnographic tools” (Bloome 2012, 11). The familiar techniques of ethnographic research—participant observation, collection of artifacts, open-ended interviewing—may be used in qualitative studies of any philosophical and theoretical orientation. What distinguishes a study as “ethnographic” is not the choice of tools, but the methodological emphasis on producing a “thick description” (Geertz 1973) of a cultural setting, which allows the researcher to reach conclusions in a principled way about participants’ emic understanding of their own cultural practices. Ethnography is not necessarily distinct from quantitative research, as counting and calculating can be used as a means to answer ethnographic questions (e.g. Hammersley 1990, chap. 4); in fact, ethnographic claims about what typically happens in a social setting are implicitly quantitative, though not necessarily quantifiable. Instead, classroom ethnography is positioned in contrast with a process-product methodology that treats students as cognitive-psychological units and seeks to measure the impact on their learning of some external factor such as socioeconomic status or experimental teaching methods. In addition, while
discourse analysis, framed as *microethnography*, is a typical component of linguistic ethnography, these are not equivalent terms. Rather, it is possible to conduct analyses of interaction without taking up the more holistic orientation that distinguishes ethnography, and current overviews of methodologies in classroom discourse analysis tend to list ethnography as one of the main approaches, either as an analytic methodology in its own right or to add depth to a microanalysis such as those attained through Conversation Analysis or Systemic Functional Linguistics (Duff 2002; Zuengler and Miller 2008).

Writing for an audience of mathematics education researchers, Margaret Eisenhart (1988) advocates for the value of ethnographic methodology in addressing the questions raised through constructivist and sociocultural perspectives on education. In her account, other qualitative education researchers tend to conceive of their role in normative terms, seeking to *improve* teaching and learning, while ethnographers take a more descriptive approach designed to *interpret* teaching-learning behavior as it occurs in practice. In this sense, even a study such as Castanheira et al. (2001), which presents an analysis of videotape data provided by outside researchers, can be considered “ethnographic” in that it seeks to describe the emergence of situated cultural practices across diverse interactional contexts.

Among educational psychologists and other learning scientists who work on mathematics teaching and learning, there is a style of research termed “communicative,” “sociocultural,” “activity theoretic,” or “social constructivist” that fits the above definition of ethnographic research, even though the researchers’ involvement in the classroom community may not resemble what sociocultural anthropologists would traditionally call ethnography. In this tradition, video is often recorded to evaluate the effect of an experimental intervention, and used to study topics such as the emergence of normative conventions for expressing mathematical ideas (Forman, McCormick, and Donato 1997; Yackel and Cobb 1996) or the effect of inscriptions and other technological tools on students’ mathematical thinking (Sfard and McClain 2002). While they do not use participant
observation as such, and arguably record interactions that are in some sense "inauthentic" because of the presence of an experimental intervention, these studies provide a valuable insider-outsider perspective as a result of the researchers’ being mathematics educators themselves, and therefore asking research questions that reflect emic concerns regarding mathematical thought, symbolic representation, and linguistic expression. In contrast, linguistic ethnographers working in mathematics classrooms tend to focus on more traditional anthropological concerns such as the growth of communities of practice among students and teachers (Brilliant-Mills 1994; Hansen-Thomas 2009).

Between linguistic ethnography of education on the one hand, and learning sciences research on the other, there is room for a productive interdisciplinary dialogue that brings a detailed analysis of social interaction into the improvement of educational practice, potentially leading to an understanding of classroom social processes that neither discipline can achieve alone. One researcher who combines the strengths of both learning sciences research and classroom ethnography is Judit Moschkovich, who uses discourse analysis of video-recorded classroom interaction to argue that “the language of mathematics” is best understood as primarily a discourse-level rather than a lexical phenomenon (e.g. Moschkovich 2002; Moschkovich 2007), but in the learning sciences in general, these linguistic and semiotic issues tend to be undertheorized. In addition to the three research questions articulated above, then, this can be considered a sort of meta-question or methodological problem to be addressed in this dissertation: to demonstrate the relevance of semiotic theory and ethnographic methodology to the learning sciences, and at the same time, the value of learning sciences research in directing the ethnographer’s analytic attention in useful directions. I will return to this point in Chapter 6.

1.3.2. Semiotics and Multimodality

The “linguistic anthropology of education” is well established as a research program (summarized in Wortham 2008), but the present research on the richly semiotized environment of
the mathematics classroom is perhaps best framed more broadly as “semiotic anthropology of education.” A semiotic theory of interaction allows the analytic focus to be expanded beyond the narrowly linguistic to address all potentially meaningful aspects of the environment. By appealing to a theory of how anything perceptible may be taken as meaningful, we can observe how speech, prosody, written language, formal notation, visual images, gesture, body positioning, eye gaze, choice of technology, and classroom layout all contribute to the communication of meaning in various ways, and all are situated in an interactional setting and sequence. In fact, I argue that classroom discourse is not really comprehensible as such if discourse is defined in narrowly linguistic terms; rather, in practice, utterances can only be understood with reference to gestures and inscriptions, and an analysis of social action in interaction must take all of these modalities into account.

In Elizabeth Mertz’s (2007) summary, semiotic anthropology is characterized as a synthesis of sociocultural and linguistic anthropology that draws on the work of C. S. Peirce (e.g. Peirce [1902] 1955) to model linguistic signs and the social context of their use as mutually constitutive and analytically inseparable. In contrast to Saussure’s conceptualization of the sign as a static, arbitrarily defined relation of signifier to signified, Peirce considers the semiotic process as more than just a sign’s indicating its object, proposing a third piece, called the interpretant, that indicates the result of the sign’s being taken to stand for its object. To use an example taken from Enfield (2013, chap. 4), dark clouds can be viewed as a sign whose object is the impending rain, and whose interpretant might be an individual’s deciding to carry an umbrella that day. In this framework, a possible “representamen,” or meaningful feature of the environment, is seen to extend far beyond the linguistic; as Enfield points out elsewhere (Enfield 2011), observed phenomena can be taken as meaningful even in the absence of an agentive “speaker.” This decoupling of meaning from speaker’s intent leads to an analysis of semiosis as a process of perception and interpretation, in which a sign is defined as any perceptible quality, object, event, or pattern that is treated as
meaningful by an interpreter. Accordingly, “the object of a sign is that to which all (appropriate and effective) interpretants (of that sign) correspondingly relate” (Kockelman 2005, 242), which is to say that the meaning of a word (or an equation, or a cloud) only becomes visible in the way that interpreters respond to it.

Much of Peirce’s semiotic framework is grounded in one fundamental phenomenological point: for Peirce, our experience of the world consists of qualities, actual objects and events, and more general laws, which he terms Firsts, Seconds, and Thirds. These distinctions have implications throughout the theory; for example, picking up an umbrella is considered to be an energetic interpretant (action) of the dark clouds, which is a Second, distinct from an affective interpretant (change in bodily state, a First) such as a feeling of sadness brought on by the cloudy weather, or a representational interpretant (proposition, a Third) such as the utterance “Tut tut, it looks like rain.”

More famously, Peirce also writes about three different mechanisms through which a sign can be linked to its object. An icon, such as a portrait of me, indicates its object through shared qualities; an index, such as an arrow pointing to me, indicates its object through actual co-occurrence; and a symbol, such as my name, indicates its object through convention. This is an oversimplification, of course, and signs can blur the boundaries between these categories; a photograph is an index (created through the action of reflected light on a photosensitive medium) as well as an icon (it looks like its object). As for words, the prototypical symbol, they may be learned as indexes through repetitive use in the presence of their object, with each successive repetition iconically referring to previous ones; words also indexically suggest lexical collocates and other words of the same syntactic class, semantic field, or enregistered voice. This typology demonstrates that, far from being arbitrary, the meaning of a sign is grounded in social actors’ direct experience of the world through relations of iconicity and indexicality. Viewed diachronically as they develop in the individual and in the cultural system, even symbolic signs can be seen to have some iconic or indexical character, and as Terrence Deacon (2003) argues, a symbol such as a common noun can
only refer to a specific object or process in the world by being paired with an index such as a demonstrative determiner or pointing gesture.

Having seen that meaning may reside in any aspect of the social environment, we are faced with a methodological problem: how can we describe the "semiotic carrying capacity" of such "multi-modal interactive environments" (Sicoli 2013; Sicoli 2015; Sicoli forthcoming) in which the production of any given social action may be distributed across diverse modalities and participants? In a methodological overview, Carey Jewitt (2014) identifies three main approaches to multimodality, two of which developed from Michael Halliday's functional grammar, while the third is based on interactional sociolinguistics. First, she describes social semiotic multimodality, an expansion of Halliday's (1978) social-semiotic theory of language to account for multimodal texts, incorporating such features as color, layout, images, and multimedia (Kress 2010; Kress and Van Leeuwen 2001), as well as situated discourse practices, including gesture and visual aids (Kress et al. [2001] 2014). Next, Jewitt summarizes multimodal discourse analysis, which adapts not only Halliday's social semiotic theory, but also his methods for grammatical analysis (Halliday and Matthiessen 2004), to work with non-linguistic and multimodal texts. This approach was initiated by O’Toole (1994), who identified grammar-like constituents within visual images, and it has been extended to a number of social domains, notably mathematical communication and mathematics classroom discourse (O’Halloran 2000; O’Halloran 2004). Using formal constructs that were developed for the analysis of English grammar, O’Halloran constructs system networks for a typology of mathematical visual images, and proposes new process types that only exist in mathematical equations, not in natural-language sentences. While the use of essentially grammatical categories to understand multimodality is not well suited to my research questions, some of O’Halloran's insights are useful to the present study. In particular, she makes it clear that writing per se is not a single modality, but rather, graphs and equations are best viewed as separate
semiotic systems, each with its own “grammatical” structure, communicative affordances, and typical function in discourse.

Ultimately, the Hallidayan approach is limited by its focus on the text as a unit of analysis, and by its privileging of the role of the speaker over that of the listener; this is a social semiotic theory in which “sign-makers and their agency as social actors are in the foreground” (Kress 2010, 34), concerned less with the way interpreters perceive meaning in their environment, and more with the agentive design of particular signs by social actors. This perspective privileges the agency of the sign-maker over the perception of the interpreter in a way that both overestimates and undertheorizes the semiotic agency of participants. That is, not only do a student’s capacity to participate in class, to answer a question, and to answer a question correctly represent three distinct forms of agency, but they are each visible in the teacher’s reaction to the student rather than the student’s utterance per se. (A fuller treatment of semiotic agency is provided in Chapter 4.)

Finally, Jewitt (2014) outlines the primary approach used in this dissertation, which, following Sigrid Norris (2004), she terms multimodal interactional analysis. Jewitt traces this approach back to Ronald and Suzanne Scollon’s (2003; 2004) work on mediated discourse analysis, which brings a multimodal dimension into interactional sociolinguistics, although Erickson (2004a; 2011) and Kendon (1990) identify antecedents back into the 1950s, including Trager and Hall’s (e.g. 1954) studies of proxemics and paralanguage, McQuown and Bateson’s unpublished Natural History of an Interview project of 1955–56, and Scheflen’s (e.g. 1966) context analysis. In addition, more recent work presented by Streeck, Goodwin, and LeBaron (2011) is more ethnomethodological in character, but shares a similar interest in the production of social action through the coordination of diverse semiotic systems. What all of these scholars share in common, and what sets them apart from other approaches to multimodality, is their focus on social action. Multimodality is not the subject of analysis for its own sake, but rather, in the course of studying how people engage in coordinated action, they come to recognize the importance of a variety of
communicative channels beyond language. By focusing on instances of interaction rather than idealized multimodal communicative systems, this approach considers multimodal configurations to be newly co-constructed in each interaction, and their meaning to reside in the way participants orient to them; while not explicitly Peircean, a focus on social action is most compatible with Peirce’s consideration of semiosis as centrally involving the perception of meaning, rather than merely its agentive production.

Multimodal interactional analysis extends not only to language and gesture, but also incorporates participants’ interaction with physical objects and technologies as phenomena that are taken as meaningful and therefore subject to analysis. In addition, the notion of the “social actor” as identifiable unit is problematized; not only do groups of participants often collaborate in the production of single actions, but also, as Sicoli (2015) discusses in detail, individuals often simultaneously engage in multiple and even contradictory actions (for example, being “of two minds” about something). For this reason, a focus on embodied action is supported with a theory of embodied cognition according to which action, cognition, and the social environment can only be understood through their involvement with one another, existing within a communicative ecology as described above. Applications of this methodology to classroom settings have focused on semiotic topics such as the coordination of speech, writing, and gesture (Ford 1999), ethnomethodological topics such as the production of recipient-designed utterances (Koschmann and LeBaron 2002), and anthropological topics such as the emergence of communities of practice (Evnitskaya and Morton 2011). By combining multimodal interactional analysis with Peircean semiotic theory, Chapters 3–5 present a similarly broad range of analytic focus, addressing mathematics classroom interaction on the levels of notation, sequence, and identity.

1.4. Outline of Chapters

Following this introductory chapter, Chapter 2 provides an ethnographic sketch of the two field sites where I conducted fieldwork and recorded video of classroom interaction, and outlines
the contents of the corpus that I collected. Next, Chapters 3–5 provide the core of the dissertation, in which I present example instances of classroom interaction and use them as evidence for particular analytical claims. These three chapters respectively address my three research questions, corresponding to Peirce’s First, Second, and Third. Finally, Chapter 6 reviews the preceding analytic findings and proposes implications both for classroom interactional practice and for interdisciplinary ethnographic research in education.

The methodological description in Chapter 2 provides two essential perspectives that would otherwise be elided in the following analytic section. First, it gives a sense of what the two classrooms were like in general, beyond the particular interactional episodes that I have transcribed for analysis; to understand the moment-by-moment give and take of situated action, we begin with an idea of who the players were, why they found themselves together at this time and place, and what sort of work they were engaged in. At the same time, I account for my own peculiar forms of participation by outlining the methodological choices that I made in the collection of the corpus. As Chapter 2 shows, these choices were grounded in the theoretical frameworks just outlined, motivated by my research questions, and designed to facilitate certain types of analysis.

Analysis begins in Chapter 3 with an analysis of technical notational forms in use in the classroom, considering the practical capabilities of the semiotic system to be a quality of mathematical communication, and therefore a First. To address the research question What do technical mathematical representations mean, and how are they incorporated into multimodal configurations? Chapter 3 considers whether mathematical notation is able to provide symbolic representations of continuous phenomena. The common understanding is that it does, but there is a paradox here: if mathematical notation is a symbolic system, and symbols are perceived as tokens of normatively defined types, then a given symbol must be either this or that. How then can continuity be represented? Looking at teachers’ explanations of relevant mathematical concepts, I use microanalysis of talk, inscription, and gesture to show that the notation does not actually
represent continuity, but rather approximates it through the semiotic process of \textit{fractal recursivity}, defined by Irvine and Gal (2000, 38) as a “dichotomizing and partitioning process” through which a single unit may also be construed as a set of smaller units, or as a component of a larger unit, at other levels of analysis.

Next, Chapter 4 addresses the question, \textit{How do sequences of classroom interaction realize ideologies of mathematical knowledge?} The analysis considers the Secondness of actual instances of classroom interaction, specifically instances in which students participate in conversational repair. By interpreting observed patterns of repair as expressions of semiotic agency, I can make claims about the degree and kind of agency that students are granted in classroom discourse, and connect these observations to ideologies of mathematical knowledge and classroom authority. The ideology that I discuss is related to particular types of classroom interaction that are familiar from the learning sciences literature, so the chapter concludes by considering learning scientists’ calls for instructional reform from the perspective of ideology rather than practice.

Chapter 5 completes the analysis by considering the question, \textit{How is the “good math student” identity performed, and how does it relate to other aspects of students’ identities, as they are expressed in the classroom?} Identity is theorized as a structure that simultaneously emerges from social actors’ prior experience and influences their future course of action, and is therefore a Third that consists of overarching characteristics abstracted from particular instances. To analyze the way identity is constructed through cases of action in interaction, I appeal to theories of stance and positioning, which I synthesize within a Peircean semiotic frame. I then apply this framework to understand the social and interactional uses of two different types of identity: first, the locally emergent category of the “good student,” and then, the demographic category of ethnicity. Having seen that identity emerges from interaction, I consider it to exist at an intermediate level between interaction and social structure, and propose that it may be used to reveal the occurrence of social change.
To conclude, Chapter 6 looks back on the findings of the preceding three chapters to incorporate them into a discussion of the *affordances* of semiotic practices in school mathematics, considering the functions to which formal notation, interactional sequence, and stancetaking are put in classroom interaction. Each phase of the analysis is shown to be relevant to a particular problem of practice identified by mathematics educators and learning scientists; as a result, this dissertation becomes framed as an exploration of why so many students have such trouble seeing themselves as successful mathematics learners, viewed in its Firstness, Secondness, and Thirdness. In light of this observation, I go on to consider linguistic anthropology and the learning sciences as complementary perspectives in the study of interactional phenomena in the classroom, and propose an interdisciplinary approach to education research that brings the analytic tools of semiotics and ethnography to bear in the search for usable answers to practitioners’ questions.
Chapter 2

ETHNOGRAPHIC FIELDWORK AND THE VIDEO CORPUS

2.1. FIELDWORK AS CONTEXTUALIZATION

Before presenting microethnographic analyses that address particular areas of inquiry, it will be helpful to offer an ethnographic sketch of the classroom environments where fieldwork took place, as well as a detailed description of the corpus of videos that I collected. My goal is to provide some background for the reader so that when they read video transcripts in subsequent chapters, they will have an understanding of where the transcripts come from, referring not only to the classroom environments in which the videos were collected, but also considering what I did to collect and transcribe them, and how these methodological choices were motivated by theoretical concerns. If I am to give a sense of the total multimodal semiotic ecology of the classroom, as discussed in Chapter 1—the complex of participants and their environment in which meaning-making takes place—then this must be understood to comprise not only excerpted sequences of talk, writing, gesture, and coordinated action, but also the physical surroundings, institutional setting, and enduring social relationships among participants. “General ethnography provides summary accounts of what people customarily do; microethnography investigates how the doings get done. Each approach needs the other” (Erickson and Mohatt 1982, 138). For this reason, I offer this descriptive chapter as a necessary complement to the more analytic chapters that follow.

In Chapter 1, I considered Dell Hymes’s introduction of the ethnography of communication as a way of conducting studies that “take as context a community, investigating its communicative habits as a whole, so that any given use of channel and code takes its place as but part of the resources upon which the members of the community draw” (Hymes 1964, 3). To allow for the sort of investigation that Hymes envisioned, my fieldwork included the collection not only of videos, which reflect the “communicative habits” themselves, but also of interviews, field notes, and sample
student work, offering a broader range of evidence to better accomplish what Clifford Geertz called “thick description.” The primary record upon which this dissertation rests is a corpus of 33 video recordings of complete class sessions, totaling approximately 30 hours of video, collected between January and June of 2014 from two ethnographic field sites, which I refer to as the middle school class and the calculus class. I recorded all the videos personally so that I could conduct participant observation at the same time. Following this phase of fieldwork, I also conducted one-on-one interviews with both of the instructors and two of the calculus students, as well as six small-group interviews with three middle school students each. While details of the analysis will mainly address the classroom interactional record, interviews and field notes were also used to provide alternate perspectives and to allow for a more complete contextualization of findings. Contextualization here should not be taken to mean “providing ‘context’ in the form of background information.” Instead, as John Gumperz did, “I use the term ... to refer to speakers’ and listeners’ use of verbal and nonverbal signs to relate what is said at any one time and in any one place to knowledge acquired through past experience” (Gumperz 1992, 230). While Gumperz used the concept to explain how his participants contextualized specific utterances as occurring within particular types of interactions, my concern in this chapter is to relate the excerpts transcribed and analyzed in Chapters 3–5 to the knowledge I acquired through my experience as a participant observer. Thus, field notes and interviews do not constitute a secondary record, but rather a parallel record that may be contextualized—etymologically, woven together—with particular interactional episodes.

To facilitate this process of contextualization, I begin by describing the two field sites in general terms, including how I came to be present in each of them, who the instructors were, and how students were assigned to this class rather than others. I also describe the physical layout of the classrooms themselves, as well as the sequence of interactional phases that tended to occur in a typical lesson. After this, I describe the methodological choices that I made regarding classroom observation and video recording, interviewing, and transcription.
2.2. THE FIELD SITES

2.2.1. Recruiting Participants

My fieldwork consisted of weekly visits to two math classes throughout the spring 2014 semester, one in a public middle school and the other at a private university. To find potential participants, I officially began recruiting in late May of 2013, immediately after receiving IRB clearance. Traditionally, ethnography aims to describe a single social group completely on its own terms, so its findings can only be generalized through a comparative perspective; for this reason, my goal was to include two classes from different schools at the grade 6–12 level. "Ideally, microethnographic work [in education] can contribute to comparative analyses of classrooms of the same or different types, to studies of schools of varying kinds, so that some reasonable sense of wholeness or comparison may emerge" (Heath 1982, 38–39). I wanted to be able to make claims about mathematics education per se rather than limiting the scope to the practices of a single teacher and group of students, so I decided to recruit two field sites, allowing me to conduct comparative microethnography within the scope of this single study. Ultimately, ethnographic generalization is not only an interpretive process, but also "an empirical matter—it lies in the eye, mind, and experience of the reader" (Erickson 2004b, 104). That is, while I present evidence from interaction in two classrooms, things may look quite different in a third site, so any reader will judge for themself the extent to which my findings seem applicable to their own experience. Nevertheless, looking at two field sites is like looking through two eyes; while I am still limited to my own perspective, the additional input makes an additional dimension visible through comparison and contrast.

To begin recruiting, I approached middle and high schools and school districts located within a reasonable distance from where I live, making contact through official institutional channels, and also through personal contacts where possible. Inquiries were made to three public school districts, two public charter schools, and one private independent school, eventually leading
to three official rejections, two bureaucratic impasses, and an offer to work with one public middle school teacher, whom I call “Ms. M.” I visited Ms. M’s school and introduced myself to two of her classes, but in one of them, three of the twelve students did not consent to participate, so I decided it would be impractical to continue with that class. Ms. M’s other section, where I did conduct fieldwork, was a mathematics class designed for English learners, the “middle school class” that I describe below.

Since I was unable to find a second field site at the secondary level, I then used my personal network to identify potential participants at the university level, and was put in touch with the mathematics professor I call “Dr. C.” Dr. C’s teaching load in Spring 2014 consisted of several sections of introductory calculus, including “the calculus class” in which I conducted fieldwork at the postsecondary level. This class was a quasi-remedial section with a modified syllabus, which will be described in more detail below. Despite the stark differences in institutional setting and material of study, the two sections were comparable in that they were both designed for students who were viewed as having some sort of special needs, they shared a common orientation to mathematics learning as a social or communicative process, and the instructors both expressed an explicit awareness that their task was to draw students into the relevant community of practice. These were not selection criteria for participation—the vagaries of educational bureaucracy did not allow me to be selective, so I took the participants I could get—but I do believe that the kind of mathematics instructors who welcome in an outside researcher for a study on classroom communication will tend to be those who share this kind of professional orientation. At the same time, the difference between groups allows me to ask which observed aspects of mathematical semiosis and classroom communicative practice were specific to one setting, which persisted across field sites, and which showed evidence of increasing abstraction or complexity at different levels of schooling.
2.2.2. **THE MIDDLE SCHOOL CLASS**

The public middle school class was made up of students in grades 6–8, which generally include students of age 11–14. To enroll in this class, students were supposed to be English learners with low to intermediate English language proficiency, and to be below grade level in mathematics content knowledge as well; Ms. M once told me that a student newly arrived in the U.S. was to be placed in the mainstream, despite his low English proficiency, because his math skills were too advanced for her class. The students in the middle school class had mostly arrived in the United States within the preceding two years, and were therefore enrolled in a “high intensity” program for English learners, which included mathematics, social studies, and science classes specially designed for English learners, in addition to their English language development class. As a result, the students I observed were likely to attend class together for a significant portion of the day, every day.

Ms. M’s class met for two non-consecutive 45-minute periods every day, and I was typically present for the Thursday morning session. Over the course of the semester, the size of the class grew from mid-teens to low twenties as newcomers arrived and enrolled in school, although a smaller number of students left the class as well, in some cases to return to their home countries. The class was roughly balanced between genders and across grade levels, and all but two of the students were Spanish speakers from Central and South America. The teacher herself is originally from India; she is a veteran teacher with extensive training in communicative teaching methods, content-language integrated instruction, and arts and technology integration. She used the SIOP method of content-language integration (Echevarria, Vogt, and Short 2013) not only with this class, but also with the native English speakers she taught, reflecting a belief that all students benefit from explicit instruction in the language of the discipline.
As is common at the secondary level, Ms. M was assigned to a single classroom, which doubled as her office, and she was given a fair amount of freedom to choose how the room would be laid out and decorated; a snapshot of Ms. M’s classroom is provided in Figure 1. Students would circulate into her room for mathematics instruction during 45-minute blocks that were largely consistent from day to day, except when the schedule was modified for reasons such as standardized testing. In addition to seeing the ESL math section twice a day, Ms. M also taught multiple sections of a class called Math 7 Strategies, which was a second daily period of mathematics instruction for seventh graders who were not English learners, but had been deemed to need extra support in mathematics for other reasons.

Students’ desks were arranged in pairs, so that each student would have a partner sitting immediately beside them. Pairs of desks were arranged in three columns from the front to the back of the classroom, each of which was four pairs deep; this provided seating for 24 students, which was more than sufficient, and created two aisles through which students could come and go, and Ms. M could circulate to check in with students during individual or pair work phases of the lesson. Seating partners were sometimes chosen by the students and sometimes assigned by the teacher, and seating was largely consistent from week to week, although I noticed significant changes.
perhaps two or three times over the course of the semester. Students were typically, but not universally, seated in same-gender pairs. If a student was absent during a pair work activity, Ms. M directed that student’s partner to move or reorient themself so that they would have someone to work with. On top of each pair of desks was a clear plastic bucket, about the size of a shoebox, which contained materials that could be used by that pair of students; the most commonly used objects from the box were a small dry-erase whiteboard and markers, which were mainly used for practicing with newly introduced mathematical procedures, and manipulatives and other lesson-specific materials were sometimes distributed to the boxes before the beginning of class.

Sitting in one of the student desks, an observer could see on the left-hand wall a long dry-erase whiteboard, with a number line running along the top of it, and an area blocked off on the right-hand side to record the homework assignment; next to this whiteboard was a bulletin board displaying key vocabulary. Directly ahead, from left to right, were the exit door, another whiteboard, a low table holding instructional materials, and the teacher’s desk with a desktop computer. To the right and behind were painted cinderblock walls. In the middle of the whiteboard directly ahead, a “Smart Board” brand electronic interactive whiteboard had been hung, which was connected to a projector suspended from the ceiling, and could be controlled either by touching it directly or from the teacher’s desktop PC. To the left and right of the Smart Board were two visible sections of dry-erase whiteboard, which showed lesson objectives for the two different math subjects Ms. M taught, each of which included both language and content objectives, as the SIOP Model prescribes. On the cinderblock walls to the right and behind, and above the two whiteboards as well, there were colorful handmade posters that displayed summaries of key mathematics content, which seemed to appear without overt acknowledgement once the relevant content had been covered, but which Ms. M sometimes referred to in the course of subsequent lessons. These signs might include mathematical definitions, such as the names of different quadrilaterals; algorithmic procedures, such as the rules for integer addition and subtraction; or second-language
vocabulary, such as the ordinal numbers in English and Spanish. A clearer view is provided in Figure 2.

**Figure 2: Ms. M’s posters (detail of Figure 1)**

When the class was operating in a whole-class instructional grouping, the Smart Board was typically the focus of attention. The school had formerly been designated an official Smart Board Demonstration School, and Ms. M was clearly well trained in the use of this piece of instructional technology. Student handouts and worksheets were often projected on the Smart Board, orienting the class to precisely which point or problem was under discussion at any time. The interactive functionality of the board could be used to obscure portions of the screen, masking elements that might be distracting, or to add content by writing, annotating, or solving problems in the margins of a projected image or worksheet. New content was typically presented through PowerPoint-style slide shows designed and created by Ms. M, in which bright pictures and photographs were often used to illustrate real-world examples or vocabulary words. The Smart Board was also used for games such as *flyswatters* (Ms. M asked a question, a selection of candidate answers was shown on the board, and the first student to touch the correct answer with a flyswatter would score a point for their team) and *Jeopardy* (based on the popular television quiz show, the categories and point values were displayed on the Smart Board, and touching a category would cause the question to be displayed).
Like many teachers at the K–12 level, Ms. M followed a consistent sequence of activity types and procedural routines in her lesson planning, which provided a sense of familiarity and supported the maintenance of classroom discipline. Class typically began with a warmup phase in which students entered the room, immediately picked up a worksheet on the way to their desks, and spent the first five to ten minutes completing the worksheet. During this time, Ms. M would circulate in the aisles, offering assistance and answering students’ questions, and when she observed that a sufficient number of students had completed the activity, she would call their attention together and review the warmup answers in a whole-class grouping. This was typically accomplished by projecting the warmup sheet on the Smart Board, going through it problem by problem, and eliciting answers from individual students. At the end of the warmup review, Ms. M gauged the level of student performance by show of hands ("Raise your hand if you got everything right," etc.), collected the worksheets, and moved on to homework review.

Homework was also typically given as a worksheet, and Ms. M led homework review by projecting on the Smart Board an answer key, which was a version of the same worksheet with the answers written in by hand. She drew the students’ attention to one problem at a time, inviting them to ask questions about problems they might have answered incorrectly. Next, she often collected the homework as well, and moved on to the presentation of new content relevant to the day’s lesson focus; she often asked a student, usually Ricardo, to read the objectives aloud as part of the transition. The actual presentation of content could take a number of different forms, including Smart Board-supported conceptual discussion and vocabulary building, as described above; working problems in pairs, often using the small dry-erase boards from the plastic bins; or “taking notes,” in which the students were given prepared note-taking sheets with blank spaces, Ms. M defined terms or explained procedures, the students filled in the blanks based on the presentation, and the completed sheets were clipped into ring binders or pasted into composition books for future reference. Sometimes, in the midst of this phase, a bell would ring to signal the end of class,
and students would pack up and leave; at other times, Ms. M more precisely timed the lesson
delivery to fit within the allotted time, and so would conclude the class with a second reading of the
objectives or a preview of what was planned for the afternoon session.

2.2.3. **The Calculus Class**

The calculus class was designed for students with a relatively weak background in math,
compared to their peers who may have already taken calculus in high school. The department has
historically found it difficult to meet the needs of this kind of student, and had tried a number of
different approaches over the years; with this group, the standard syllabus was slowed down to
allow an entire academic year for content that is typically completed in a single semester. To
accomplish this within the structure provided by the university registrar, these students had taken
a course in the fall semester called “calculus with review,” which covered the first two chapters in
the calculus textbook. Inclusion in “calculus with review” was meant to be assigned based on
placement test results, but in fact most students did not take the placement test, and chose between
the mainstream and quasi-remedial sections by either self-selection or professor recommendation.
“Calculus with review” did not count toward fulfilling students’ mathematics requirement, but it did
grant them admission to a special spring-semester section of Calculus I, which covered textbook
chapters three through five. There were two such special sections, corresponding to the two
sections of “calculus with review” that had been offered in the fall, and taught by the same two
professors; for this reason, most of the students in the calculus class I observed had been in class
with Dr. C since the start of the preceding fall semester. By May, they were considered to have
“caught up” with their peers, who used the traditional syllabus that covered all five chapters in a
single semester; they earned the same credit, and took the same final exam, as all Calculus I
students.

The calculus class met Monday and Wednesday afternoons for an hour and fifteen minutes,
and Thursday afternoons for 50 minutes; I was typically present for the Wednesday afternoon
session. There were about 30 students in this class, a few of whom realized that they could fulfill coursework requirements in other ways, and so dropped the class mid-semester. The students were roughly balanced across genders and represented a mix of African Americans and European Americans, as well as one or two Latino students, and international students from Italy, China, and Turkey. Many were undergraduate business majors, who were required to complete a calculus course; there were also humanities and social science majors fulfilling general education requirements in mathematics; one prospective math major who, in the professor’s opinion, was looking for an easy A; and one post-baccalaureate student who had earned a bachelor’s degree outside the U.S. but needed a U.S. math credit to apply to medical schools. The professor himself was in his second year at this university. He is African American, and often used African American language and cultural references in his teaching. Many students in this class had a history of bad experiences with mathematics, and on several occasions I heard students telling the professor that this was the first time they hadn’t hated their math class. In order to help students overcome their prior bad experiences, the professor attempted to create a feeling of “family” within the group, taking advantage of the fact that they were spending an entire academic year together, and in fact I was told by students as well as the professor that I was “part of the family now.”

Unlike the middle school class, but like most classes at university level, the calculus class was not held in a space that “belonged to” the instructor, but in shared spaces that might be used at other times by any group of roughly the same size. The Monday and Wednesday class sessions took place in a classroom that had, from the student’s perspective, a blackboard in front and an instructor’s podium on the right; a photo is provided in Figure 3. There was a ceiling-mounted projector that could be operated from a computer built into the instructor’s podium, and to use it, a screen had to be lowered from the ceiling that obscured a large portion of the blackboard space. Most of the room was filled with about 40 chairs for students, which featured a small writing surface that swung upward to make it easier for students to sit down and stand up. The calculus
students chose seats in a way that self-segregated according to observable types: the back left corner was occupied by a multi-racial group of large men who were varsity athletes, possibly from the football team; in front of them, along the left-hand wall, sat a group of international students; the front of the room, from the center toward the left-hand side, was mainly occupied by Black students who were not athletes; and the rest of the white American students, including women athletes, sat along the back wall and on the right-hand side. A few individual students moved around over the course of the semester, but these groupings remained largely consistent. Thursday classes were designated as a “lab” session and took place in a room where each student had access to their own desktop computer, but the computers were not actually used; while I never conducted an observation on a Thursday, Dr. C told me that he taught in largely the same way during every class session.

![Figure 3: The calculus class in session (19 March 2014)](image)

The sequence of a calculus class meeting was not as consistently structured as the middle school class, but there were a few identifiable activity types that might take place in the course of a class session. The class typically began with an administrative phase in which homework was collected and returned, and Dr. C pointed out upcoming exams and other important deadlines on
the syllabus. Homework deadlines were often postponed at this time in order to allow students additional time to complete their work. Following this phase, most of the class consisted of lecture / demonstration, in which Dr. C defined terms and demonstrated problem-solving procedures, which he illustrated and annotated copiously on the blackboard. These lectures were interspersed with Q&A phases in which students asked for clarification or confirmed their understanding of the material just presented. These phases were sometimes initiated by Dr. C checking for understanding, but at other times a student might ask a question in the middle of a lecture which Dr. C would answer, and then go on to elicit additional questions from the rest of the class. On occasion, Dr. C assigned practice problems that students would complete during an individual work phase, and less commonly, more involved problem sets would be given for group work, which on one occasion lasted nearly 30 minutes. During these activities, Dr. C walked around the room between the desks to offer assistance and answer student questions. Both individual and group work phases were followed by debrief phases in which Dr. C went over the assigned problems and provided authoritative answers; there were also debrief phases to review homework that had been completed outside of class. Unlike in the middle school, there were no bells to signal the beginning and end of class, although the students were aware of the scheduled ending time and often began to shift in their chairs and pack up their materials as the time approached; it was up to Dr. C to signal the end of class, which he typically did with another administrative phase.

Overall, the tone of the class was much more informal than what I have typically observed at university level. Dr. C often used gentle humor to draw attention to lapses in classroom discipline, such as by saying “Class doesn’t begin until you get here, now we can get started” to a student who arrived ten minutes late, or “Cool dinosaur” to a student in the front row who was doodling in her notebook. He also tolerated fairly loud talk among students while he was lecturing, and only rarely made explicit attempts to get the students to quiet down. When I asked him about this, Dr. C said that he had made a conscious choice to be more “relaxed” in order to present a welcoming
environment for students who had become used to thinking of mathematics as something not only difficult, but inflexible. While he did admit the potential downside of his approach, he was also able to foster personal relationships with many students; for example, after he told the class that his wife was expecting, a group of students gave him an infant-size hat with the text *Cutie π*.

### 2.3. **Collection of the Corpus**

#### 2.3.1. **Classroom Observation and Video Recording**

As Erickson (2004a; 2011) discusses in a historical overview of the field, the ability of linguists and anthropologists to study language use in interactional settings has depended on the development of recording technology that allows documentation of social behavior and repeated playback for analysis. This allows the “microethnography” of discourse analysis to complement the “general ethnography” of participant observation, as outlined in foundational works on sociolinguistic ethnographic methods (Erickson and Mohatt 1982; Heath 1982). Working in this tradition, Mondada (2006) points out that technical decisions about video production reflect theoretical and methodological orientations to research. For example, recordings are generally collected in the “field” rather than in a laboratory setting where greater phonetic and gestural detail could be captured, operationalizing a belief in the situated nature of social interaction. Using a wide-angle shot to record all participants from beginning to end of an activity, rather than focusing narrowly on certain participants or phases in interaction, reflects an understanding that social actions are positioned in time and space in complex ways, and that an action or utterance of interest will need to be considered along with its temporal and physical surroundings in order to make sense of it. Given the ubiquity of video recording in sociolinguistic ethnography, I consider these concerns to be a part of the methodology, and one that the present study seeks to acknowledge in a principled, reflective way.
In both field sites, a typical lesson interspersed periods of large-group instruction and demonstration with periods of small-group cooperative problem solving. During large-group phases, the class was recorded using a tripod-mounted digital camcorder placed in a back corner of the classroom, in order to capture the instructor and as many students as possible; Figure 1 and Figure 3 above are minimally cropped to fit better on the printed page, but they give a sense of the vantage point the camera provided on the middle school and calculus classrooms, respectively. High-definition video was recorded so that the details of inscriptions written on a blackboard or projected on a screen would be available for later analysis. Sometimes a supplementary audio recorder was used as well, for instance, to record conversation among students who chatted with one another during the lecture. Field notes were used to record events that stood out to me in the moment, as well as connections and hypotheses regarding those events. Notes were color-coded, using one color to represent speech and another to represent written language and diagrams, and I sometimes used a third color to distinguish between teacher and student talk. The field notes served as an index of the video corpus in that when an event seemed notable to me as it occurred, I captured it in my notebook, and I could then locate the relevant clip for microanalysis without having to scan through dozens of hours of video. I did watch the entire corpus and produce a more comprehensive index, but it was useful to supplement this with an additional record that better captured the flow of my attention in the moment. Additionally, my field notes included more than just descriptions of events and snippets of talk, but also hypotheses, half-formed interpretations, and questions for further study, which were often instrumental in identifying units of analysis to address my research questions. In this sense, they served as a record not only of what happened while I was in the field, but also of my own developing understanding.

To record small-group phases, I used a variety of techniques. I sometimes removed the camcorder from the tripod and chose one student group to record at close range, attempting to capture gesture, eye gaze, and body position as well as writing on worksheets or notebooks. If the
membership of small groups was difficult for me to discern, I moved fluidly from group to group, or followed the instructor as they engaged with different groups in sequence. At other times, I participated in the class, either as a group member in the undergraduate section, or as a classroom assistant in the middle school. Whichever approach I took, I wrote up field notes after the small-group phase had concluded. This variety of approaches primarily reflects my own struggle to capture a kind of instructional grouping that is less centralized and more distributed than teacher-fronted lecture, presenting a challenge to the single lens and microphone of the video camera. A better record of these interactional phases might have provided valuable material for analysis; for example, in a study that was foundational for me in articulating my third research question regarding language and identity, Bucholtz et al. (2011) show how students in a small-group configuration use “math talk” to construct different kinds of scientist identities. As a relatively inexperienced fieldworker operating without the assistance of a research team, I collected little usable material from small-group interactions, so my own treatment of language and identity in Chapter 5 relies mainly on episodes from whole-class groupings, supplemented by interview data.

2.3.2. Interviews

To gain a sense of the participants’ own perspective on their mathematics teaching and learning experiences, I conducted interviews with both instructors and a subset of students from both field sites. In parallel with the general ethnographic and microethnographic orientations taken by this study overall, most interviews were divided into two sections: a set of questions designed to elicit the respondent’s reflections on their mathematics biography and global perspectives on the focal class, followed by an open-ended video playback session in which I asked for description and analysis of pre-selected clips. I waited to schedule the interviews after the classes had concluded, so that participants would be able to reflect upon the entire semester’s experience, and also because I developed the interview protocols based on questions that became salient to me through the course of participant observation. While the two interviews with instructors followed different paths
because of the specifics of the different field sites, I used a consistent interview protocol with all student interviews, and this protocol is provided in Appendix B.

As Charles Briggs (1986) cautions us, the responses of participants in an interview do not reflect some sort of unvarnished “social fact;” rather, the interview itself is a social encounter, and respondents contextualize their responses according to their own communicative norms and metapragmatic understanding in the moment. To account for this, he proposes a set of methodological adaptations for the collection and interpretation of interviews. To begin with, field observations should include “such simple facts as who talks to whom, who listens to whom, when people talk and when they remain silent, what entities are referred to directly and which are referred to indirectly or signaled nonverbally, and the like” (Briggs 1986, 94), and these observations should then “inform an in-depth investigation of the points of compatibility and incompatibility between interview techniques and the local metacommunicative repertoire” (Briggs 1986, 98). In my dealings with students, these suggestions lead me to consider what it means for me, a college-educated native English speaker in his 30s, to ask questions about mathematics and school. There seems to be an iconic resemblance between my appearance and that of a teacher—in fact, to move unobtrusively through the middle school, I dressed as I had done years before when I was working as a public school teacher, and in my dealings with the instructors, I attempted to position myself as a sympathetic colleague—but this would tend to lead students to frame my questions as attempts to elicit a “right answer,” and as opportunities for them to position themselves as “good students.” The particulars of this relationship will vary from one student to another; Briggs also notes that “in the course of conducting informal interviews with a number of members of the community, ethnographers generally form close working and often personal relationships with a few consultants” (1986, 8), and in my case, those individuals unsurprisingly turned out to be students who viewed themselves as “good at math” and generally
demonstrated a positive orientation to school. While their perspective did prove valuable, it is essential to consider this skewed participation when I present interview transcripts in Chapter 5.

Briggs’s recommendations extend beyond the collection of interviews to address the interpretation of responses as well. Not only does he caution us against interpreting interview data as “a direct outpouring of the interviewees’ thoughts or attitudes” (1986, 102), but there are also aspects of participants’ social knowledge and understanding that are not accessible to interview methods:

interview techniques rely primarily on the referential or descriptive function of language and on knowledge that lies within, in Silverstein’s (1981a) terms, the limits of awareness of speakers. This means that interviews will be totally ineffectual in dealing with some topics, and they certainly will exclude important facets of those subjects that can be treated in interviews. (Briggs 1986, 98)

For this reason, I will only consider interviews in parallel with an analysis of classroom interaction, as an additional, artificially elicited perspective to be contextualized with the record of participants’ situated social action. That is, while interview responses themselves are also social actions, the interview itself is not the setting or genre that primarily concerns us here; rather, interviews are only of interest insofar as they can help us to reach a useful understanding of what happens in the classroom. The most direct way to accomplish this is through video playback in which participants offer their own interpretation of classroom interactional episodes. As Deborah Tannen writes, “playback is the litmus test of interpretation” ([1984] 2005, 49), offering a check on the validity of the ethnographer’s analysis. At the same time, the domain of playback is the domain of microethnography, and I included more general questions in my interviews for the same reasons that I offered general ethnographic descriptions of the field sites earlier in the chapter. Responses to this type of question are not considered as “factual” accounts; rather, I subject them to discourse analytic methods similar to what I use with the classroom videos, interpreting them as participants’
means of presenting themselves to me in a certain way, at a certain time and place, for a certain reason.

The first interview I conducted was with Dr. C, on 8 May 2014, lasting about an hour. This was the only interview not to include a video playback phase, as well as the only one to be audio rather than video recorded. I had intended to follow up with a second session with Dr. C, to include video playback, but this turned out not to be practically feasible because he moved to a different university in the 2014–15 year, and we lost contact. In the middle school, student interviews took place from 27 May to 12 June 2014, two interviews a week for three weeks. These interviews were conducted with small, self-selected groups of three students each, so 18 middle school students were interviewed in total. To avoid any disruption to their class attendance, interviews took place in school during the lunch period, so they are only 25–35 minutes long. Students were offered pizza as an incentive to participate. Interviews were conducted in English, Spanish, or a combination of the two, according to the participants’ preference. The following week, I returned on 18 June to interview Ms. M; this interview took place in her classroom and lasted 47 minutes.

At the end of the fieldwork period, I began trying to schedule interviews with calculus students, but it was difficult for them to participate at the same time that they were studying for finals, so I decided to postpone these interviews until the following year. As it happened, I waited until the middle of the spring semester to contact these students, which I did via email, and I was ultimately able to schedule only two interviews. These took place on 20 March 2015, when I spoke to a student I call “Mickey” for about 45 minutes, and on 8 April, an interview of around an hour with “Sara.”
2.3.3. **Transcription**

The video corpus was annotated, and selections transcribed in detail, using the ELAN transcription software. When particular examples of classroom interaction are selected for in-depth analysis, playscript-style transcripts are provided using a set of conventions based loosely on the work of Gail Jefferson (Appendix C; cf. Jefferson 2004). To capture embodied communicative channels in addition to talk, the transcripts are enhanced to show when participants write on the blackboard or some other visible surface, and when they point to visible writing or produce other gestures. Illustrations are often provided as well to show the details of how inscriptions are laid out or gestures produced. To protect participants’ confidentiality, photographs that show faces have been reproduced as line drawings using the Edge Detect functionality of the Windows Movie Maker software, and pseudonyms are used throughout.

Any transcript of talk reflects certain theoretical presumptions on the part of the researcher (Bezemer and Mavers 2011; Erickson 2010; Ochs 1979), and I would like to be explicit as I can about the reasons for choosing this style of transcript rather than, for example, the horizontal or “quasi-musical” approaches Erickson has used to avoid “logocentrism” and highlight the multimodal character of interaction. It is true that the semiotic environment of the mathematics classroom comprises not only talk, writing, and gesture, but also the simultaneous use of these communicative modalities by multiple participants as they demonstrate attention, respond to one another, and synchronize their actions in a co-constructed rhythm; moreover, this rhythm tends to be obscured in playscript-style transcripts, which make it difficult to display the precise timing of the subtle behaviors that show listenership, coordinate attention, and maintain intersubjectivity. In his own analysis of classroom discourse at the primary level, Erickson (2004b, chap. 3) provides a compelling demonstration of the salience of this rhythmicity for the accomplishment of social roles such as attentive student. In the secondary and postsecondary classrooms I observed, however, the

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1 ELAN was developed by the Language Archive at the Max Planck Institute for Psycholinguistics, Nijmegen, The Netherlands (http://tla.mpi.nl/tools/tla-tools/elan/ and cf. Wittenburg et al. 2006).
bulk of students have been socialized into more passive ways of showing involvement. More to the point, while older students do provide indications of attention such as visibly writing in their notebooks at key moments (Kaplan 1992), the research questions I articulated lead to a focus on the coordination of talk, gesture, and writing, often by the instructor or another single participant, and this analytic orientation tends to deemphasize the coordination of social action among participants. My goal in the transcripts, then, was to highlight ways in which a single participant—usually, the instructor—produced talk, gesture, and writing in coordination, and to characterize other participants' next actions as a response to this. For this reason, I decided to consider an intonation unit in talk (Chafe 1994), along with co-occurring gesture and writing, to be a single unit for the purpose of transcription, and I transcribed each such unit as a single line of transcript. I consider the relationship between this format and Erickson's quasi-musical approach to be like the relationship between topographical and geopolitical maps of the same territory, representing the same referent, but highlighting different features of it, reflecting the different uses for which they were designed.

To illustrate the necessity of multimodal transcription, consider Example 1, in which I have transcribed only the spoken-language channel from a strip of interaction in the middle school class.

In this excerpt, Ms. M is calling on Alberto to report his answer to a worksheet problem.

*Example 1: Transcript of talk*

1  Ms. M  um
2  Alberto  give me the next one.
3  Alberto  One and one fourth.
4  Ms. M  Give me a thumbs up if you agree with Alberto
5    thumbs ↓ if you don't.
6    (1.9)
7  Really?
8  I thought it was this.
9  (2.3)
10  I thought it was this°<
11  Tiana  L((inaudible))
12  Nayan  ↑The same
13  Fernando  L((inaudible))
Viewing this strip of talk separated from its co-occurring non-linguistic modalities, it is impossible to say what “this” (lines 8, 10, 19, 21) and “this answer” (line 24) refer to, but these ambiguities are easily resolved by looking at the inscriptions on the Smart Board and the teacher’s indexical gestures. Similarly, in lines 4–5, Ms. M can be heard to call for a gestural response from the students, but the response itself is completely erased in a transcript of talk alone, so it is not clear what Ms. M is responding to beginning in line 7. This coordination between modalities demonstrates that, to make sense of language, we must look beyond language; in fact, much of the talk in this example seems designed specifically to draw the listener’s attention to the Smart Board text, which carries the actual propositional content.

More subtly, it is difficult to recover from talk alone what speech acts Ms. M is accomplishing in this excerpt, and whether she is positioning Alberto’s response in line 3 as correct or incorrect. In lines 7–10 (“Really? I thought it was this”), she proposes an alternative to Alberto’s response, which, given her institutional positioning as the local mathematical authority, is readily interpreted as a correction of Alberto. Next, when Nayan and Ricardo (and perhaps Tiana and Fernando as well) respond that Alberto’s first answer and Ms. M’s repair candidate were “the same” or “both right,” she insists that one was “more right” than the other (line 20). Overall, Ms. M’s talk is consistent with a reading in which Alberto’s answer is, if not incorrect, at least dispreferred. If we look at the video record, though, we can see that the problem that Alberto is solving is to write the
number 1.25 as a fraction, so his response in line 3 was correct. The dispreferred answer is not
Alberto’s response, but Ms. M’s candidate correction, which is $1\frac{25}{100}$, his response, not hers, is the one
that she indicates in lines 19–21 as being “more right.” To capture the communicative channels of
writing and gesture in the transcript, I developed a multimodal transcription style, demonstrated in
Example 2. In this transcript, bold text represents writing, italic text is gesture, and vertical lines
indicate synchronization across modalities by a single participant.

Example 2: Multimodal transcript (see also Example 16, p. 111)

1 Ms. M um
   | point at Alberto
2 Alberto One and one fourth.
3 Ms. M | Give me a thumbs up if you agree with Alberto
4 thumbs | $1\frac{1}{4}$
5 thumbs down if you don’t.
6 | (1.9)
7 class hold thumbs down gesture, survey student responses
8 thumbs up held high by Santiago, Nayan, Alberto, Sandra, Ricardo, Fernando; held in front of body by Elena, Rita, Jamie
9 Ms. M Really?
10 I thought it was this. | (2.3)
11 Tiana still writing $1\frac{25}{100}$ arrow $-$ $1\frac{25}{100}$
12 Nayan |((inaudible)) BH raise and lower
13 Fernando |((inaudible)) The same
14 Ms. M O:::::okay.
15 Ricardo But if we-
16 both of-
17 RH point at board
18 Fernando |((inaudible)) LH point at board
19 Ms. M This is –
20 more right
21 circle $1\frac{1}{4}$
22 than this, | circle $1\frac{25}{100}$
if you're i- when we're testing you and stuff
\textit{turn to Ricardo/Fernando, LH palm up rotating wrist}
we look for this answer, okay?
pen point at 1\textsubscript{1/4} and tap pen

In this transcript, the students' gestural responses are recorded in line 7, and the referent of Ms. M's "this" is made clear in lines 9–10, 19–21, and 24. Beyond correcting these apparent deficiencies of Example 1, additional nuances are made clear in the multimodal transcript. For example, we can see that certain speech acts are accomplished multimodally: Ms. M's nominating Alberto in line 2 is preceded by a gesture, her call for “thumbs up” or “thumbs down” in 4–5 is accompanied by a gesture, and Alberto's talk in 3 is intersemiotically translated to smart board text in 4. The two transcribed pauses are also explained with reference to writing and gesture: the first, in line 6, occurs as Ms. M is gauging the class's gestural response, and the second, in line 9, occurs while she is writing on the board. We can even begin to consider the way multimodal utterances are produced and processed, as the frequent intonation unit breaks in 19–21 ("this is—more right—than this") co-occur with changes in deictic referent, which are physically marked with writing on the board.

Transcribing other communicative channels in addition to talk was an important component of this project, not only for presentation, but also for analysis. In a recent review of automated speech recognition software and its potential for use in transcription, Bolden (2015) argues that the process of manually transcribing audio recordings requires the researcher to focus their attention on the data in productive ways, and therefore represents a first pass at analysis. The point holds even more for the transcription of gesture. Not only did I have to decide how to describe particular gestures, as movements of the hands and body do not have a standard orthography; because speakers' and listeners' bodies are in constant motion as they interact, I also had to decide which movements deserved to be represented in the transcript. In many cases, this led me to pay closer attention to gesture than to talk. I decided that fine phonetic details were
generally not relevant to my analysis, and so modified Jefferson’s conventions to use more standard spelling and punctuation. To transcribe gesture, however, no such option was available. For example, the gesture illustrated in Figure 4 was transcribed in Example 3 as “LH boundary, RH beat, moving away,” with each successive beat annotated at the point where it co-occurred with talk.

*Figure 4: Illustration of gesture (see also Figure 8, p. 66)*

*Example 3: Coordination of transcript and image (see also Example 5, p. 64)*

<table>
<thead>
<tr>
<th>1</th>
<th>Ms. M</th>
<th>Would it be closer to zero point six, zero-</th>
<th>LH boundary, RH fingertip touch, RH beat, moving away</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>point</td>
<td>seven, zero point beat</td>
<td>eight, zero point beat</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>nine, beat</td>
</tr>
</tbody>
</table>

Although I had to pay close attention to the video to produce this representation, the transcription of gesture still feels more imprecise than the transcription of talk; I feel instinctually that “RH beat, moving away” is a mere *description* of what Ms. M did, while “would it be closer to zero point six” is *what she actually said*. Reflecting on this, however, I believe my first instinct to be an illusion caused by literacy, a consequence of the logocentric perspective that Erickson warned us about. The transcription of talk is also a description, and by using Jefferson’s orthographic conventions or the International Phonetic Alphabet, I could render this description both more precise and less transparent in ways that might have been beneficial for answering research questions other than my own. Going further, even the high-definition digital video recordings that I collected in the field are just another kind of description; after all, if we view communication as an ecology, as outlined in Chapter 1, then the pieces all depend on one another. The practice of
recording and transcribing—collecting a sample and removing it to the laboratory, if you will—cannot possibly provide a complete representation of what was said, because what was said depends intimately on what had just been said, what had been said on prior occasions, what was written on the board, who was present, whether they thought of themselves as “math people,” whether they had eaten breakfast that morning, and so on. Keeping this in mind, I intend my transcripts and ethnographic sketches, whatever semiotic modalities they may reflect, to provide an indication of what happened, including just enough detail for us to conduct our laboratory analyses, and considering them not as “data” per se but as a fieldworker’s description of how the ecology appeared to him at the time and place that he was present, offered here as evidence for the analytic claims that I will make.
Chapter 3

THE PROBLEM OF CONTINUITY IN MATHEMATICAL REPRESENTATION

In Chapter 1, I considered a claim made by the middle school teacher, Ms. M: that fractions, decimals, and percentages are like three languages that express the same thought. If 0.5 and 50% are like the words water and agua, I asked, then what corresponds to the water itself? What do technical mathematical representations mean, and how are they incorporated into multimodal configurations? Motivated by that question, this chapter presents a multimodal interaction analysis of particular episodes from both the middle school class and the calculus class, considering the way notation, diagram, talk, and gesture are coordinated for the description of mathematical objects. To identify interactional episodes that offer insight into this question, I focus on lessons that address a kind of mathematical object that natural language is particularly poorly suited to describe: instances of continuous gradient change.

3.1. SEMIOTICS AND CONTINUITY

As a semiotic system grounded in symbolic reference, language provides a finite number of signs with which to divide up an infinite space of potential objects, thereby allowing language users to categorize and label aspects of the semiotized world. As a result, language most naturally expresses categorical distinctions among types of phenomena rather than gradual variation along a continuum; in Jay Lemke’s terminology, it functions in a “typological” fashion, in concert with “topological” modalities such as prosody, gesture, and visual images that more easily draw our attention to continuous phenomena (Lemke 1999; Lemke 2003). In the process of articulating this distinction, Lemke argues that the typological functioning of language is linked to typologies in its structure: a verb may be marked as either past or present tense, a noun phrase is either definite or indefinite, and while some languages may provide “in-between” categories, the point holds that these are realized as categories and not as gradient continua. It seems that this apparent universal
feature of language must be due to neither cultural convention nor neurobiological “universal grammar,” but rather to the semiotic processes underlying symbolic reference itself, as Terrence Deacon (2003) has argued in the case of other universals such as unbounded recursivity and predicate-argument structure.

This semiotic argument suggests that we should expect to find typological structure as a feature not only of all languages, but of all possible symbolic systems. However, Lemke does identify one symbolic system as an extension of linguistic semantics into the realm of continuity: the domain of mathematics. “As we developed technologies and economies in which more and more careful quantitative distinctions were necessary, natural languages became extended semantically by the concepts and symbol systems of mathematics in order to more efficiently represent topological meaning” (Lemke 1999, 175), and so we are now able to express topological meaning through “gestural and visual semiotic resources, as extended by mathematics and quantitative reasoning” (Lemke 1999, 183). For example, if an object is launched into the air, lexicogrammar can only encode discrete phases in its trajectory such as “it is rising, it is falling, it has landed,” but its movement can be reflected in a graph, gesture, or intonation contour in speech, and as O’Halloran (2000) points out, the equation \( s(t) = -16t^2 + v_0t \) also provides its exact height at every moment.

In this way, typological and topological semiotic systems complement one another: “Each of these basic modes of meaning making foregrounds particular kinds of potentially meaningful relations” (Lemke 1999, 176).

So mathematical communication is multimodal communication; as Lemke points out, “The normal mode of technical communication ... integrates verbal, visual-graphical, and mathematical semiotic resources at every turn” (Lemke 2003, 229). O’Halloran (2000) offers further specificity, providing a semiotic typology that identifies language, formal notation, and visual diagrams as three distinct resources that are brought together in mathematics classroom discourse. Considering the different uses of these meaning-making resources, she concludes that they function as
complementary components of the semiotic ecology of school mathematics, with each taking on a distinct role: “mathematical symbolism contains a complete description of the pattern of the relationship between entities, the visual display connects our physiological perceptions to this reality, and the linguistic discourse functions to provide contextual information for the situation described symbolically and visually” (O'Halloran 2000, 363). To this inventory of semiotic channels, Lemke adds embodied action, and connects it specifically to topological meaning: “Gestures, and more generally actional movements in space and time, are a primary meaning-making resource for imitative, or iconic, representation of meaning-by-degree as opposed to meaning-by-kind” (Lemke 2003, 221). In fact, as we will see, gesture provides essential evidence to help us understand how symbolism, visual images, and language are coordinated in the flow of talk.

It seems fitting that visual images, gestures, and intonation should be able to model gradient phenomena. These representations are grounded in iconic reference, so the continuous nature of the object is both encoded and recognized in continuously changing aspects of the sign: the shape of the curve, the movement of the hand, the rise and fall in pitch. In the case of the equation, however, we see a symbol—a sign that, by necessity, is realized as a string of discrete elements chosen from a finite repertoire—that nevertheless is taken to encompass the infinitesimal changes in position that occur as the object moves through space. Lemke introduces the typological / topological distinction in order to understand discourse processes in medical education, and he recognizes “the tension between the typological-categorial norms in medical diagnostic discourse and the topological-quantitative nature of the biological phenomena being discussed” in this setting (1999, 178). Rather than considering mathematics to be a linguistic expression of continuity as Lemke does, I argue that the same tension exists within the mathematical notation itself: the position function as a mathematical object is taken to be continuous, providing an infinitesimally different value at each instant in time, but when the object is incorporated into mathematical discourse—particularly in its instantiation as a symbolic equation, but also, in a way, in its graphical version—the typological
aspects of its meaning are foregrounded. Specifically, I will show that when such mathematical objects are referenced in actual classroom practice, they are understood by being broken down into discrete components or subsections as the task requires, and then recombined to present a single solution to a given problem.

Among all the speech situations where mathematical formalism is used, the classroom stands out in that the primary purpose of interaction is not to represent natural phenomena or advance mathematical theory, but to socialize students to the practices of the mathematics epistemic culture, which I understand in Karin Knorr Cetina’s sense to be “those amalgams of arrangements and mechanisms ... which, in a given field, make up how we know what we know” (Knorr Cetina 1999, 1; emphasis in original). The instructor is the locally recognized representative of this culture, the expert who is responsible for bringing newcomers into the community, and thus the instructor’s participation in discourse is often used as a means for demonstrating how mathematicians recognize meaning in the world—as Charles Goodwin (1994) puts it, the mathematician’s professional vision. In this chapter, as I analyze instances of classroom discourse that focus on issues of discreteness and continuity, I will consider what these episodes show us about professional vision, not only with reference to mathematics instruction, but as a set of semiotic processes more generally.

3.2. Continuity and Iconicity in Language and Mathematics

Considering the problem of continuity from a semiotic standpoint, it becomes apparent that icons are the only signs capable of representing continuous phenomena. As noted above, symbols are realized as paradigmatic choices among discrete and mutually exclusive options from a lexicon of types; when edge cases exist that do not fit neatly into existing categories, new ad-hoc categories are invented such as “blue-green,” but this sort of usage still presupposes a bounded range of hues that could all be considered “blue-green.” As for indexes, they encode no representational meaning, but only direct the interpreter’s attention to their object. To represent differences of degree rather
than differences of kind, the only option is to use iconically grounded signs that are themselves continuous, reflecting the continuous variation in the object to which they refer. To provide a theoretical background in the semiotics of continuity, this section will outline cases where both continuity and iconicity have been identified in language in general, and in mathematical communication specifically.

3.2.1. CONTINUITY AND ICONICITY IN LANGUAGE

Few semioticians have dealt specifically with the representation of continuity. Lemke's (1999; 2003) treatment, outlined above, identifies continuity as a particular challenge for linguistic representation: “natural language is very bad at giving precise and useful descriptions of natural phenomena in which matters of degree, or quantitative variation, are important” (Lemke 2003, 220), and mathematics has been developed to represent phenomena in ways that are not accessible to language.

Ma (2009) offers another approach unrelated to mathematics, dealing more directly with the “fuzziness” of the categories indicated by linguistic signs, and ultimately uses the sense / reference dichotomy to account for this indeterminacy: while a word must have a discrete sense, its reference may vary along continuously defined parameters. This model envisions the world as phenomenologically continuous, with words taken to indicate points or ranges along the continua we perceive. If particular words are imposed as labels upon continuous referents, however, some meaning is lost; language has difficulty representing differences within the range of possible referents of a given word (a sudden handclap in a quiet room is “loud,” but so is a TNT detonation), as well as phenomena that fall between ranges (blue-green, etc.). As a result, formal semantic theories of gradable adjectives and quantifiers (L. R. Horn 1984; L. R. Horn 1989), as well as functionalist treatments of gradient expressions of evaluation (J. R. Martin and White 2005, chap. 3), tend to account for such words as being meaningful mainly in comparison to one another; warm essentially means “not as hot as hot.” Other domains of continuous experience also seem to be
represented through language in ways that are semantically discrete or discretizing, such as progressive verbal aspect (e.g., “it is rising”), which has been analyzed using modal semantics (Portner 1998).

These approaches to continuity most commonly take up the abstracted meaning (sense, intension) of individual words, thereby highlighting the symbolic nature of language, and we began with the observation that symbolism will generally be unable to reflect continuous aspects of meaning. At the same time, however, language is by no means exclusively symbolic; on the contrary, iconicity in language is also well documented. The debate between theories of meaning-by-nature and meaning-by-convention is generally traced back to Plato’s *Cratylus*; the modern study of linguistic iconicity (see De Cuypere and Willems 2008 for a comprehensive overview) begins with Jakobson (1965), who introduced Peirce’s concept of the icon into linguistic theory in response to Saussure’s principle of the arbitrariness of the sign. Jakobson finds iconicity in the ordering of syntactic constituents, which often reflects nonlinguistic sequences such as the temporal order of events (*veni, vidi, vici*), the social hierarchy of participants (“the president and the secretary of state”), or the perception of causality (in an overwhelming majority of languages, the subject of a sentence precedes the object). He also cites the increasing phonological length of positive, comparative, and superlative adjectives, as well as the phonological correspondences that can be found within lexical sets (e.g., *father, mother, brother*), portmanteaus (words created by blending other words, e.g., *smog = smoke + fog*), and ideophones (“marked words that depict sensory imagery” (Dingemanse 2012, 655) such as reduplication or onomatopoeia). Givón (1995) generalizes Jakobson’s examples to postulate a “semantic principle of linear order,” “pragmatic principle of linear order,” “quantity principle,” and “proximity principle,” as well as a “meta-iconic markedness principle” according to which “categories that are cognitively marked—i.e. complex—tend to be structurally marked” (Givón 1995, 68; cf. Sicoli et al. 2015). Subsequent research has found additional examples of iconicity in morphosyntax; for instance, grammatically inflected forms
If icons can represent continuity, and language has iconic as well as symbolic features, then why does language have such difficulty representing continuity? To understand how iconicity creates a potential for the representation of continuity, it may be helpful at this point to go back and consider precisely how Peirce defined icons and iconic signs. According to Peirce, not only is it the case that “Anything whatever … is an Icon of anything, in so far as it is like that thing and used as a sign of it” (Peirce [1902] 1955, 102); more specifically,

A sign by Firstness is an image of its object and, more strictly speaking, can only be an idea. For it must produce an Interpretant idea; and an external object excites an idea by a reaction upon the brain…. A possibility alone is an Icon purely by virtue of its quality; and its object can only be a Firstness. But a sign may be iconic, that is, may represent its object mainly by its similarity, no matter what its mode of being. If a substantive be wanted, an iconic representamen may be termed a hypoicon. (Peirce [1902] 1955, 105)

As far as iconic representations of continuity, the only true icon, in these terms, is our idea or sense that qualities or phenomena change in continuous fashion, such as, perhaps, the feeling of falling through space, or the mental image of the spectrum of colors in a rainbow; any other representation will not be an icon, strictly speaking, but a hypoicon.

Given a hypoicon, then, the problem presents itself of how it is to be recognized as a sign. For if “the dual relation between the [iconic] sign and its object is degenerate and consists in a mere resemblance between them” (Peirce 1885, 181), then it is up to the interpreter, on perceiving the sign, to determine which of its qualities should be taken as iconic resemblance of the object—that is, to tell signal from noise. How do we know that the increasing length of the words large, larger,
largest reflects the increasing size of the referent? Why do onomatopoeia in unfamiliar languages need to be translated? Such signs seem to be iconic, but only when their object is already known, a sort of relation that Sonesson calls “secondary iconicity”: “Convention is thus needed, not only to establish the sign character, but also the very iconicity of these icons” (Sonesson 2008, 51). As Sicoli (2014) points out, an allegedly iconic linguistic sign such as an ideophone—or, by extension, any of Sonesson’s “secondary icons”—does not relate to its object in a truly iconic way, but it does give rise to an iconic interpretant. That is, the link between sign and object is conventional, but interpreters who know the convention will then recognize aspects of the sign as resembling the object in some way. Therefore, such a sign is best understood not as an icon but as a rheme, according to Peirce’s third trichotomy, rheme—dicent—argument. For example, the expression woof woof is essentially symbolic, but it is a rhematic symbol; it is recognized by convention, but the affective interpretant that it gives rise to is the idea of the sound made by a barking dog.

Applying this observation to our discussion of continuity, we see that a rhematic symbol can refer to an instance of continuous variability, but cannot model it directly. Returning to the earlier example It is rising, the progressive verb aspect has a rhematic quality that evokes a sensation or idea of continuous movement. As a symbol, however, it is applied as a label to an entire stretch of time with clearly defined boundaries; its semantics does not encode a continuous change in position, but rather a state of affairs (upward movement) that holds during a fixed temporal interval. To precisely model the projectile’s velocity, acceleration, and change in acceleration caused by variable application of force, as these quantities change over time, is clearly beyond the capability of language, and in fact this is just where we often see mathematical representation in use.

3.2.2. Iconicity and Continuity in Peirce’s Mathematics

As a logician and philosopher of science, Peirce dealt specifically with the semiotics of mathematical drawings and notation, and he often used these representations as illustrative
examples of iconicity. Specifically, they are often considered to be diagrams, or hypoicons of Secondness, “defined as hypoicons whose similarity with their objects is mostly based on shared structural or relational qualities” (Farias and Queiroz 2006, 294). Through this iconic resemblance, Peirce accounts for the ontological collapse that often happens to mathematics learners as the tools of reasoning are taken to be the actual objects of study, the map confused for the territory:

Icons are so completely substituted for their objects as hardly to be distinguished from them. Such are the diagrams of geometry. A diagram, indeed, so far as it has a general signification, is not a pure icon; but in the middle part of our reasonings we forget that abstractness in great measure, and the diagram is for us the very thing. (Peirce 1885, 181)

This description also illuminates the reason why only iconic representations are able to support mathematical reasoning. To a great extent, a symbol’s grounding in convention limits the user to predetermined conventional meanings, but an icon can encode aspects of its object that were previously unknown. Thus, knowledge about the object can be derived by carrying out operations on the iconic sign. The “capacity of revealing unexpected truth is precisely that wherein the utility of algebraical formulae consists” (CP 2.279, cit. in Nöth 2008, 97).

Peirce refers here to “algebraical formulae” rather than geometric diagrams, and in fact he did consider algebraic notation to be another sort of diagram: “every algebraical equation is an icon, in so far as it exhibits, by means of the algebraical signs (which are not themselves icons), the relations of the quantities concerned” (CP 2.282, cit. in Jakobson 1965, 27, inter alia). An equation, like a syntactic structure, can be understood as a diagram of relations among its constituents. Considering the laws by which algebraic symbols are manipulated, he describes them as “previous discoveries which are embodied in general formulae. These are patterns which we have the right to imitate in our procedure, and are the icons par excellence of algebra” (Peirce 1885, 182). So, for Peirce, both the formal notation and the schematic diagrams of mathematics are essentially iconic. Despite not being true icons, narrowly defined, their status as diagrammatic hypoicons allows them
to resemble their object closely enough to be used as tools for mathematical reasoning, which suggests that they are in a sense more iconic than language.

Despite the availability of an iconic representational system, Peirce himself struggled to articulate a theoretical definition of continuity. A contemporary of Frege, Cantor, and Dedekind, Peirce was working at a time when issues of countability, continuity, and infinity were focal to much work in mathematical and metamathematical theory, and his own thinking on these topics evolved over the course of his lifetime (Havenel 2008; Potter and Shields 1977):

Peirce’s mathematical definitions of continuity are not just the struggle to translate into mathematical language a stable intuition; but his intuition itself evolves with its mathematical conceptualizations.... However ... if one excludes his Cantorian period (1884–1892), Peirce maintains in his four other periods the “Aristotelian” idea that “there are no ultimate parts to a true continuum”. (Havenel 2008, 88)

For the purposes of the present study, the important point here is that we have an intuition of continuity—a true icon, perhaps—but we find it difficult to define continuity, or to describe phenomena in continuous terms. As a result, mathematics discourse tends to blur the lines between continuity and discreteness: “In one sense, then, continuity is totally different from any collection of discrete elements, but in another sense the larger such a collection becomes the more it resembles a continuum” (Potter and Shields 1977, 27), and conversely, we can approximate a continuum with a sufficiently large collection, designating ad-hoc “ultimate parts” as tools to reason with. From an ethnographic perspective, then, the question is not about the nature of continuity or the classification of mathematical objects as continuous or discrete; rather, considering examples of classroom interaction, I will consider the ways in which representations of discreteness and continuity are leveraged by teachers in the course of mathematical explanations.
3.3. **Discreteness and Continuity in Teacher Talk**

To address this question, I will consider one video-recorded extract of interaction from the middle school class and two from the calculus class. Episodes were chosen for microanalysis in which issues of discreteness and continuity were particularly salient. I begin with a middle school episode concerning the use of fractional and decimal representations of rational numbers, which have been considered by educational psychologists such as Rapp et al. (2014) to reflect mental models of discreteness and continuity, respectively. Next, the first calculus episode features two transcript examples from a lecture on Riemann sums, a mathematical technique that analyzes a continuous function by breaking it down into discrete chunks and then recombining them. In contrast to Lemke’s claims about mathematics as a way of representing topological meaning, multimodal analysis will show that symbolic formal notation offers a discretizing, typological view of the mathematical objects under discussion, and must be supplemented by the iconic representations of visual image and gesture in order to foreground topological meaning. Finally, the second calculus episode features a demonstration of how first and second derivatives of a function are used to translate from formal notation to a visual representation, in which I will show that even the graphical representation can be seen as typological.

In each episode, we will see that participants variably construe mathematical objects as discrete or continuous, making this distinction visible through their gestures. Psychological studies of gesture in mathematics instruction have shown that gestures, particularly those used by teachers, function both to represent mathematical concepts and to manage intersubjectivity among participants (Alibali et al. 1999; Alibali et al. 2013; Alibali et al. 2014; Goldin-Meadow, Kim, and Singer 1999; Goldin-Meadow et al. 2001). To explain these gestures in the course of microanalysis, I will consider them as *metaphors of participation*, defined as “an iconic ideological projection based upon the pattern of bodily participation in the context of social action” (Wolfram 2014, 221) in which gesture and body positioning provide a physical instantiation of a way of understanding
abstract mathematical objects. For example, in Wolfgram’s analysis, a teacher’s pointing to a triangle drawn on a whiteboard indicates that this specific triangle is being considered and analyzed as a specific token, but when the teacher turns away from the board, faces the students, and forms a triangle with her hands, it shows that she is considering the class of triangles as an idealized Platonic type. Using his terminology, gestures near the blackboard are considered to be “contextualizing gestures” that “are oriented to mathematical inscriptions on the whiteboard and they emphasize the particularities of the representation.” These gestures are contrasted with “textualizing gestures” that “occur in the space located between the board and the students” and “represent the mathematical knowledge as a thing in itself, detached from the instance of mathematical knowledge represented on the board” (Wolfgram 2014, 225–226).

Wolfgram identifies metaphors of participation that metapragmatically indicate whether the mathematical object under discussion is to be considered as a type or a token. In the present analysis, we will also see metaphors of participation that construe an object as discrete or continuous. Alibali et al. (1999, 328) considered “smooth, continuous motions” to reflect a mental representation of continuity, while “gestures that incorporated a set of discrete movements ... [or] zigzagging, circling, or spiraling motions” were taken to indicate discreteness, and I consider this distinction to be an additional metaphor of participation that sometimes co-occurs with Wolfgram’s textualizing / contextualizing dimension. We will see that metaphors of participation provide a crucial resource for tying together the diverse semiotic channels of speech, written notation, graphs, and gesture, and for bridging between typological and topological understandings of mathematical objects.

As the analysis proceeds, metaphors of participation are not only considered for their micro-level function in construing a particular mathematical representation as type or token, continuous or discrete. In addition, they reveal otherwise invisible aspects of the mathematician’s professional vision. Goodwin (1994) identifies three discursive processes that make up
professional vision: *highlighting, coding, and material representation*. As he defines them, highlighting “makes specific phenomena in a complex perceptual field salient by marking them in some fashion,” coding “transforms [these] phenomena … into the objects of knowledge that animate the discourse of a profession,” and the material representation is that multimodal discourse as it occurs (Goodwin 1994, 606). Putting this framework into semiotic terms (see Appendix A, column 3), we can describe professional vision as a process that is made explicit through the discursive functions Goodwin observes. The sign of this process is a particular phenomenon that professionals are trained to pick out from within a complex perceptual field; the object is a technical description or category that corresponds to this phenomenon; and the interpretant is the expert understanding of the phenomenon that is made possible through recognition and categorization. Each stage in this process can be realized in discourse: highlighting is a Firstness that publicly identifies the sign; coding is a Secondness that makes the object explicit; and material representation is a Thirdness in which the interpretant is physically inscribed. The following analysis will show that as instructors demonstrate how meaning is constructed with mathematical representations, they use metaphors of participation to accomplish all three of these functions.

3.3.1. **Discreteness as Pedagogical Simplification**

The first episode we will consider occurred during the “warm-up” part of a middle school class period, which always took up the first five to ten minutes of the period. In this phase of the lesson, students worked independently on worksheets, solving problems that reviewed previously covered material, and often provided a segue into that day’s lesson focus as well. On this day, the worksheet required students to convert numbers from decimal to fractional representations and vice versa; in the episode we will consider, Elena has just asked the teacher for help writing $\frac{5}{8}$ as a decimal. The desks are arranged with students sitting in pairs, and Elena’s partner, sitting directly to her left, is Nayan.
Ms. M arrives and begins walking Elena through the process of solving the problem. Rather than calculating an exact answer, she encourages Elena to consider the problem conceptually, thinking about a range of possible answers and then narrowing that range by more precise estimation. She begins by eliciting from Elena the observation that \( \frac{5}{8} \) is greater than one half and less than one; then, as shown in Example 4, she transitions from the fractional to the decimal representation.

**Example 4: ‘What decimal is equal to half’ (9 January 2014)**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ms. M</td>
<td>If it's greater than half, right hand boundary (vertical, palm open)</td>
</tr>
<tr>
<td>2</td>
<td>it's going to be greater than what decimal. RH bounce RH small-object shape</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Elena</td>
<td>five (ehm eight) point five Ms. M RH hold</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>(zero) point four? LH point and swipe</td>
</tr>
<tr>
<td>5</td>
<td>Ms. M</td>
<td>What decimal is equal to half. bounce RH one time fold hands at chin</td>
</tr>
<tr>
<td>6</td>
<td>Elena</td>
<td>Four.</td>
</tr>
<tr>
<td>7</td>
<td>Nayan</td>
<td>Like-</td>
</tr>
<tr>
<td>8</td>
<td>Ms. M</td>
<td>What decimal is equal to half. lean head forward, raise brow</td>
</tr>
<tr>
<td>9</td>
<td>Elena</td>
<td>Five. Zero point, Ms. M nod twice</td>
</tr>
<tr>
<td>10</td>
<td>Nayan</td>
<td>Loh yah.5 RH point and retract</td>
</tr>
<tr>
<td>11</td>
<td>Elena</td>
<td>Lfive.</td>
</tr>
</tbody>
</table>

In lines 1–2, Ms. M is essentially asking Elena to convert the fraction *one half* into a decimal, but this question is embedded within the frame of the problem at hand, in which the value *half* / *what decimal* is relevant because it (i.e. \( \frac{5}{8} \)) is greater than half. Elena must parse this complicated parallel syntactic structure: line 1 expresses a premise in present tense, and 2 uses the future to signal that an inference is being made based on this premise, and asks the student to supply the piece of the inference referenced in the wh- phrase “what decimal.” In 3, Elena seems to have understood both the question and the mathematics, but as she continues in 4, some level of
confusion becomes apparent, leading Ms. M to rephrase her initial question more directly in 5. Elena persists in her wrong answer in 6, but after Ms. M, in 8, stresses that she is looking for a decimal, Elena arrives at the correct answer; perhaps she was confused about whether the intended response was in fact 0.5 rather than $\frac{4}{8}$, that is, one half represented with a common denominator.

Figure 5: It's going to be greater than what decimal (Example 4, line 2)

Throughout lines 1–5, Ms. M creates with her hands an icon of the number line, which she references in tandem with her verbal description. At the beginning of the transcript, her hands are raised to her shoulders, and then in line 1, she uses an open-palm chopping gesture to represent the lower bound of the interval. By using her right hand for the lower bound, she represents the number line from the students' perspective rather than her own. Next, in 2, she bounces her right hand, again focusing attention on the lower bound, and then as she says “what decimal,” she changes her handshape to one that suggests she is holding a small object between her thumb and forefinger (Figure 5). Metaphorically, this reifies the answer to her question as an entity to be identified, or a label that corresponds to a boundary point on the number line. She holds this gesture fixed during Elena’s turn in 3–4, but in 5, when she steps out of the frame of Elena’s question to elicit information in more general terms, she also collapses the gestural number-line icon. At this point, the number line is no longer relevant, and so neither is its gestural image.

The gestural icon of the number line provides Ms. M with a phenomenon on which she can focus her professional vision. To begin walking Elena through the solution process, she highlights
the lower bound of the interval and codes it as “half,” orienting to the fractional notation that is used in the problem. She then asks Elena to recode that same point using decimal notation, bringing the student into the solution process through a task that shows a minimal level of expertise. In order to make this possible, we can infer that Ms. M has already brought her professional vision to bear on the problem in a way that is left implicit in discourse: the gestural number line is itself a material representation of a space in which the solution is to be found, which Ms. M has constructed to best support Elena’s understanding through tacit use of her teacher’s professional vision, which Shulman (1986) famously termed *pedagogical content knowledge*.

Following this excerpt, seven lines of transcript are omitted in which 1.0 is established as the decimal representation of the upper bound of the interval. Similar to Example 4, Elena remains confused between eighths and decimals—she initially proposes 0.8 as the upper bound before arriving at the correct answer—and Ms. M continues to ground the talk in a gestural representation of the number line. Having determined that $\frac{5}{8}$ is somewhere between 0.5 and 1.0, Ms. M begins to walk away, but then turns back to Elena to outline an approach to completing the problem; her instructions are transcribed in Example 5.

*Example 5: ‘Where would it fall’ (Continues from Example 4; 7 lines omitted)*

19  Ms. M | So now think about where it would be. turn out into aisle LH boundary, RH palm down interval
20     You might not get the exact decimal, point to Elena’s paper and hold
21     but, try to figure out where it would be, BH tiny objects, RH bounce
22     from zero point five to one. RH arc over interval
23     Where would it fall. boundary hands - RH moving away, slight bouncing
24     Would it be closer to zero point six, zero- LH boundary, RH fingertip touch RH beat, moving away
25     point seven, zero point eight, zero point nine, beat
26     zero point ten. beat
27  Nayan  LA: hh. Ms. M eyebrow flash, nod
In this example, Ms. M directs Elena to estimate the decimal equivalent of \( \frac{5}{8} \) as a location in abstract space between 0.5 and 1.0. She repeats this instruction four times, modifying the phrasing each time, in lines 19, 21–22, 23, and 24–26; first, she asks where it would be in general terms, then focuses attention on the range from zero point five to one, then asks where would it fall within that range, and finally subdivides the range into increments of 0.1. Metaphorically, this progressively construes the answer to the problem as a location, a location within a defined space, and an approximate location within a subdivided space. This sequence of representations serves the pedagogical purpose of simplifying the question to scaffold the student’s task. The instruction on the warmup sheet, Write the fractions as decimals, is expressed entirely in terms of formal notation, but Ms. M’s talk constructs a geometric, diagrammatic representation that foregrounds a conceptual understanding of the rational numbers as a way of identifying intermediate values between the integers.

Figure 6: Think about where it would be (Example 5, line 19)
Along with the verbal channel, Ms. M represents the interval from 0.5 to 1 using a sequence of four distinct gestures, which are instantiated over the gestural number line that was previously established in Example 4. The first two (Figure 6, Figure 7) are metaphors of participation that reflect an understanding of the interval as a continuous space, but with the second iteration emphasizing the endpoints of the range. As her words become more precise, her gestures become more discrete; in line 23, the slight bouncing of her right hand as it moves across the range reflects both discreteness and continuity, but the staccato movement in 24–26 (Figure 8), which punctuates a list of points within the interval, provides a clear metaphor of discreteness. As the question is phrased in more and more scaffolded formulations, the indication of discreteness becomes progressively stronger.
Textbook analysis and experimental research have shown that fractions are most commonly associated with portions of countable sets, while decimals are used to subdivide continuous ranges or mass quantities (Rapp et al. 2014). In this case, the assigned task requires the use of both fractions and decimals, and in completing the task, a continuous range is treated first as a continuum, and then as a discrete sequence of intermediate estimates. While it may be the case, as Peirce believed, that “there are no ultimate parts to a true continuum” (Havenel 2008, 88), we have observed that Ms. M construes the continuum as a collection, in order to make the task accessible to Elena. She points out that you might not get the exact decimal in this way, but she leaves open the possibility (and, later in the class, demonstrates the technique) for a more precise calculation at a more advanced stage of mathematical proficiency.

By presenting the gestural number line as a more and more finely subdivided space, Ms. M demonstrates a sequence of ways that it may be highlighted and coded. Understanding the interval between 0.5 and 1.0 to be what Goodwin (1994) terms a “complex perceptual field,” the solution to the problem is highlighted as falling somewhere within that interval, and is then coded variably as a certain distance from 0.5, a certain location along the interval, and an estimated value (0.6, etc.) within the interval. These alternative coding schemes may be compared with a tool used by novice archeologists at the field site Goodwin describes: the Munsell chart, a glossy sheet of paper displaying standardized color swatches, which the archeologists use to identify the color of soil. By using the chart, investigators reduce the limitless variety of colors to a finite number of standardized codes; the chart breaks up the continuous color space into a discrete typology. Similarly, metaphors of participation such as Figure 6, Figure 7, and Figure 8 identify particular locations within the undifferentiated continuum of the real number line and make them relevant, thus offering novice practitioners of mathematics a way of understanding the object of study.

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2 Ms. M’s pedagogical content knowledge happens to be in line with experimental findings, which indicate that students find it easier to understand a problem framed in discrete terms (Schwartz, Martin, and Pfaffman 2005).
3.3.2. **Equations and Diagrams as Metaphors of Participation**

The next episode, the first of two taken from the calculus class, occurred during a lecture on the Riemann sum, a computational technique for estimating the area under a curve. To calculate a Riemann sum, a designated interval of the domain of the function is partitioned into subintervals of equal length, and the value of the function is calculated at a point within each subinterval. Multiplying the value of the function by the width of the subinterval provides an estimate of the area under the curve over the subinterval, and adding up the estimates for all subintervals provides an estimate of the area over the whole interval. By increasing the number of subintervals and decreasing their width, a more accurate estimate can be obtained, and the precise area is calculated as the limit of the Riemann sum as the number of subintervals approaches infinity. A visual presentation of this technique is provided in Figure 9, and the formalism, as recorded on the blackboard at the beginning of Example 6, is provided in Figure 10. Before this technique is presented, the students have no way of calculating the area of smooth shapes other than circles, but the Riemann sum procedure sidesteps this difficulty by construing the area as a series of infinitely many rectangles of infinitesimal width. For this reason, Riemann sums offer a natural site of investigation for issues of continuity and discreteness.
As Example 6 begins, Dr. C is beginning a demonstration in which he estimates the area under the graph of the function $f(x) = \frac{1}{x}$ on the interval from 1 to 6, using Riemann sums with five rectangles. In the transcript, he begins the process by partitioning the interval.
Figure 10: Riemann sum formalism

Example 6: ‘Let’s think through it’ (16 April 2014; blackboard as in Figure 10)

1  Dr. C  And so uh the first thing that we need to do is we need to palms up
2  to come up with that little subdivision, right, BH palms facing, repeated chopping gesture (Figure 11)
3  and break the interval from one to six LH repeated chopping gesture
4  um into LH horizontal sweeping motion
intervals of equal length
LH chopping gesture twice
and those are gonna be the basis for our rectangles.
So here's one way to do it, um,
we could think about the formula and what it tells us,
so uh delta x
\( \Delta x = \) (to the right of what is shown in Figure 10)
well, first of all, let's think through it
how far is it from one to six
LH palm down sweep 3x under x axis (Figure 12) & point
class five
Dr. C five, and if I wanna break that into five intervals of equal length? how long would it be, one.
class one
Dr. C so we kinda think through it,
and figure out that this will be one,
<the next> endpoint will be two,
and then three, and then four, and then five and->
3 4 5 6 ((exact timing obscured))>
turn to class
then six<.
So we can kinda think through it.
And come up with a,
and enumerate the,
RH pointing back and forth 3x toward students (Figure 13)
endpoints
of our subdivision that way.

In lines 1–6, Dr. C provides an abstract of what is to follow. By using modal need in line 1, future time gonna in line 6, and plural pronouns we and our throughout, he represents the task as one that faces the class as a whole, and positions himself as speaking on behalf of the group. Line 7 transitions to the actual working of the problem, and in 8–9 he starts using notation to calculate the endpoints of the subintervals, going so far as to write the beginning of the equation \( \Delta x = \) before deciding in line 10 to change his approach. In contrast to his initial aborted attempt, which began to solve the problem by algorithmically working through the formula, this second attempt promises to think through it by considering the problem geometrically. This transition is marked by a physical movement from a blank area of the blackboard to an area where a graph of the function has been
drawn, and by linguistic changes at both the grammatical level, in the professor's use of I in line 13, and the discursive level, as 11–13 and 13–15 elicit student responses through two initiation – response – evaluation (IRE) sequences. In 16–20, the professor switches back to we and completes the task, considering the interval holistically and applying the partition to the graph, an iconic representation of the problem. Finally, lines 21–25 sum up the approach just demonstrated.

\[\text{Figure 11. That little subdivision (discrete, textualizing) (Example 6, line 2)}\]

\[\text{Figure 12. How far is it from one to six (continuous, contextualizing) (Example 6, line 11)}\]
The key moment in Example 6 is the shift that occurs in Line 10, where Dr. C sets aside the formalistic calculation of the partition in order to visualize it geometrically. This transition is accompanied by a change in the type and quality of gesture that he uses. In lines 2 and 3, he uses staccato vertical movements of his hands (Figure 11), suggesting an act of chopping the interval into subsections. The interval itself, which he names in line 3, is represented in line 4 by a smooth horizontal movement of the hand, but in line 5 he returns to talking about intervals (here meaning “subintervals”) and using sharp vertical movements. After the shift in line 10, however, he is no longer talking about breaking the interval up, but instead construes it as a distance (how far is it) while using the smooth horizontal gesture (Figure 12), both of which reflect an understanding of the interval as a continuous stretch. In lines 16–20, having arrived at the key idea that the subintervals will be one unit in length, he applies the partition to the graphical representation; then, when he turns back to the class to sum up, he uses the continuous gesture again in line 23 (Figure 13), but this time it is oriented not along the x axis of the graph, but along a different spatial dimension at right angles to the blackboard, between himself and the students.

In this example, we can identify three different metaphors of participation. The first is the presentation of the interval as a series of subdivisions, which is visible in the first part of the transcript, preceding line 10, and is instantiated through the word break, staccato vertical gestures, and reference to mathematical formalism. In line 10, along with the shift from symbolic to iconic...
representation of the problem, there occurs a parallel shift from typological to topological thinking. The following two metaphors of participation represent the interval as a continuous domain, but they accomplish this representation in distinct ways. In line 11, the professor gesturally highlights the $x$ axis of the inscription on the board, a gesture that occurs midway through his explanation and serves to foreground the topological understanding of the interval. In lines 4 and 23, however, we see the explanation bracketed by two continuous gestures that are similar to that in line 11, but are realized in the space between the professor and the students, rather than the space close to the blackboard. In Wolfgram’s (2014) terms, we see a textualizing gesture in lines 2–5 that represents the Riemann sum technique in the abstract as one that breaks down a continuous interval and analyzes it typologically; a contextualizing gesture in line 11 that foregrounds a topological understanding of this specific problem; and a concluding textualizing gesture in 23 that characterizes the partition as essentially continuous, a way of understanding an interval in space.

In contrast, consider Example 7, which occurs a few minutes later. In response to a student question about the use of mathematical formalism to calculate the endpoints of the subintervals, the professor summarizes the procedure in this way:

*Example 7: ‘That’s the same thing’ (16 April 2014; blackboard as in Figure 10)*

```
1  Dr. C  so I'm trying to demonstrate,  open palm point
2       uh how,  palm down point back and forth between $x_1$ and $x_2$ equations
3       how to use,  palm down sweep $2x$ over lower left sequence of equations
4       this notation.
5  To write down the endpoints of our intervals.
6  So I know $|x_{sub a}$ is gonna be one, $x_2=a$
7        $|x_{sub one}$ is gonna be $|one| +$
8        $|a| +$ $|\Delta x|$
9        which gives us back two.  hold point
10       $|x_{sub three}$ is gonna be, $x_2$
```
Like Example 6, this explanation begins in lines 1–5 with an abstract of the procedure to be outlined, accompanied in this case by contextualizing gestures that characterize the sequence of equations as both a discrete list (line 2) and an undifferentiated mass (line 3). Once the explanation begins in earnest, however, the only gestures we see are typological in character. Repeating an earlier sequence of calculations, he focuses attention on individual terms in individual equations one by one in sequence, naming them aloud at the same time as he refers to them indexically through contextualizing pointing gestures. (It doesn’t seem to matter that the words he speaks, three delta x, do not correspond precisely to the notation 2Δx on the board; rather, the point is the sequence itself, which is the spirit of the computational technique.) In this way, the process of calculating endpoints through formalism, unlike the iconic representation in the previous example, is constructed as a sequence of discrete algorithmic steps that provide a list of endpoint values. Finally, in lines 12–13, he compares this result to the geometric procedure previously described, and while he characterizes the two results as being the same thing, we see the contextualizing gesture of continuity return with his reference to the graphical representation.

Comparing these two examples, we see both the graph and the notation being viewed as icons of the same mathematical object, with the graph depicting it topologically while the notation foregrounds its typological aspects. The interval is presented as a complete whole that is then subdivided into smaller partitions, and it is easier to view it holistically in the graph, and analytically in the notation. The first indication of this semiotic division of labor is in the gestural channel, where as we have seen, icons of continuity co-occur with talk about the graph, and icons of discreteness co-occur with notation; if we interpret these gestures as metaphors of participation,
then their correlation with different forms of mathematical representation establishes an indexical relation between the graph and topology, and between the equations and typology. Going further, however, I argue that the relation is iconic rather than indexical; the professor’s contextualizing gestures draw attention to continuous aspects of the graph and to discrete aspects of the notation, such that the smooth lines of the graph are icons of the topological, and the separate terms of the equation are icons of the typological. If a metaphor of participation is defined as one that “establishes an iconic correspondence from the pattern of embodied action to some other sociocultural domain” (Wolfgram 2014, 221–222), then we can observe the blackboard writing as instantiating metaphors of participation beyond the gestural channel; the graph and the formalism are themselves metaphors of participation that respectively establish iconic correspondences to the topological and typological aspects of the mathematical objects themselves.

These iconic relations, through which the graph comes to represent continuity and the equations, discreteness, are elements of the mathematician’s professional vision. Recall that in Example 6, Dr. C’s contextualizing gesture highlighted the x axis of the graph as the focal object of attention, while in Example 7, it was the sequence of equations, indicated one by one. Moving from highlighting to coding, though, we see that the discursive act itself has a discretizing effect, even—or perhaps especially—when applied to continuous fields. Example 7 shows Dr. C highlighting the terms of the equations one by one in sequence, and recoding each expression as a simplified numerical value; each highlighted entity receives a single code; for example, in line 8 of that transcript, \( \delta x \) is recoded as one, and then in line 9, \( \text{one plus } \delta x \) is recoded as two. This piecemeal sequential coding is congruent with the discretized, modular way in which the notation had been highlighted. In Example 6, however, Dr. C applies codes to the x axis in lines 17–20, writing the numbers one through six along it. While the x axis had been highlighted as a continuous stretch, it is coded as a sequence of discrete points.
3.3.3. A Discrete Coding Scheme for a Continuous Material Representation

In the case of the Riemann sums, I presented excerpts from a lecture in which symbolic formal notation was used to make a continuous mathematical object amenable to being broken down into discrete components, while iconic images were presented in parallel with the notation to depict a continuous view of the object. In the next episode, I will consider a case in which even the graphical representation is seen as an encoding of typological meaning, representing the function as a sequence of discrete chunks.

This transcript is taken from a lecture in which Dr. C demonstrates how to graph a function by calculating its first and second derivative. In this exercise, a function was provided to the students in the form of a mathematical equation in which the variable $y$ or the expression $f(x)$ was set equal to some expression involving the variable $x$. Through the techniques of differential calculus, students calculated the first and second derivatives of the function, which were expressed as new equations that could be used to provide information about the shape of the graph.

![Typology of curve shapes](image)

*Figure 14. Typology of curve shapes*

Earlier in the lecture, a typology had been drawn on the board indicating the different curve shapes that are associated with either positive or negative first and second derivatives; this typology is reproduced in Figure 14. In this image, each of the four curve shapes is labeled with the corresponding values for the first ($f'$) and second ($f''$) derivative; for example, the shape on the upper right, which is associated with a positive first derivative and negative second derivative, is labeled.
\[ f'(x) > 0 \]
\[ f''(x) < 0 \]

The graph on the upper left is different from the others in that it shows a line beneath the curve, with the letters \( a \) and \( b \) labeling points on the line that correspond to the endpoints of the curve. This representation of the \( x \) axis shows that the relevant characteristics of the function illustrated here—specifically, that it has positive first and second derivatives, and so is shown to be increasing and concave upward—apply specifically on the interval from \( a \) to \( b \); the implication is that the other three types in the typology are also defined over specific intervals.

Example 8 occurs near the end of a demonstration in which Dr. C used this technique to graph the function \( y = 4x^5 - 5x^4 \). At this point, the first and second derivatives of the function have been determined, and the values have been calculated at which the curve changes shape; for example, the first such point is at \((0, 0)\), which is called a critical point because the first derivative changes sign here from positive to negative. The final step of the exercise is to draw the graph, an act that the professor characterizes as a process of “put[ting] this information [about the first and second derivatives] together.” Example 8 shows how this process begins.

*Example 8: ‘Can you sketch in the air’ (26 March 2014)*

1. Dr. C: But - but let’s, so let’s put this information together
2. \[ \text{from uh negative infinity} \]
   \[ \text{LH point at the left end of x axis} \]
3. \[ \text{to zero,} \]
   \[ \text{darken the negative x axis with colored chalk} \]
4. \[ \text{what happens to the first derivative positive or negative.} \]
5. students: Positive
6. Dr. C: \[ f' > 0 \]
7. student: Negative
8. Dr. C: \[ f'' < 0 \]
9. So we’re increasing and concave down, which \( \uparrow \) picture,
10. one, two, three, or four,
    \[ \text{index-finger point at each curve shape (Figure 14)} \]
11. or can you sketch in the \( \uparrow \) air what this looks like, \[ f'' < 0 \] (Figure 15)
increasing and concave down.
turn to face students

student One
student Two
NRP ((omitted))
Dr. C point and hold, upper-right shape (Figure 14)
Increasing and concave down, this is zero zero,
point to origin of the graph
Mickey Oh. I didn’t see it there
Dr. C So we increase,
curve from lower-left corner of graph to the origin
concave down.

Figure 15 shows the relevant section of the blackboard at the end of the transcript.

Figure 15. Drawing the graph

In this example, Dr. C carries out the three discursive practices of highlighting, coding, and material representation. Having identified the first critical point at (0, 0), he restricts his attention to the interval leading up to that point, which he does in lines 2–3. This act of highlighting is accomplished multimodally, as he darkens the negative $x$ axis in colored chalk at the same time that he verbally names the interval “from negative infinity to zero.” Next, in lines 4–8, the highlighted interval of the function is coded according to the sign of the first and second derivatives. This act of coding is co-constructed through the use of IRE sequences, which construe the students as active participants rather than passive listeners. As mentioned above, one of the functions of professional vision is to socialize novices into professional ways of seeing, and so students are given
opportunities to participate in professional practices, peripherally at first, but taking on more and more central roles over time. Their increasing responsibility allows novices to understand new social practices, to participate in them successfully, and to demonstrate to their teachers the extent of their capability in the moment. This discursive function is also accomplished multimodally; the “evaluation” move in the IRE sequence is provided not through speech, but tacitly, as the professor validates the students’ responses by transcribing them into mathematical notation. The expressions that he writes, $f'>0, f''<0$ (see Figure 15), identify the relevant category to which the highlighted portion of the function belongs.

It then becomes possible in lines 9–14 to identify the corresponding curve shape on the typology reproduced here as Figure 14. Lines 9–12 provide the initiation move of a second IRE sequence, eliciting an incorrect answer in 13 and a correct answer in 14. A third student does not speak, but provides an answer gesturally, responding to the professor’s invitation to “sketch in the air;” his response is shown in Figure 16, and occurs simultaneously with line 15 of the transcript. The professor’s evaluation move is accomplished by pointing to the correct shape in line 16, and he applies this result in 19–20 to produce the material representation shown in Figure 15.

![Figure 16. Tracing the curve shape](image)

In Goodwin's (1994) description, professional vision comprises the ability to section off the world, categorize discrete sections according to an institutionally validated typology, and use the typology to produce schematic representations of phenomena. In his example of an archeological dig site, an area of the site is first highlighted as being worthy of attention, and then a Munsell chart.
showing standardized color swatches is used to code the area according to the color of the soil. Similarly, the calculus lecture uses the critical points and inflection points as boundaries within the domain of the function, highlights the segments sectioned off by these points, and then codes each segment according to the sign of the first and second derivative. The curve shapes in Figure 14 perform the same function as the Munsell chart: both are iconic representations that correspond to our bodily experience, also made visible through the hand gesture depicted in Figure 16, and both are linked to a technical symbolic system through specialized semiotic practices. In each case, fitting the phenomena to the formalism requires the erasure of continuity. Similar to what we observed in Example 6, this example shows that continuous objects must be considered in a discretized way in order for codes to be applied; the difference between actual mathematical curves, like the difference between actual colors in a dig site, is infinitely gradient, but professionals can categorize a given token according to a specialized typology. The topological is made typological through the effect of professional vision.

In this case, the domain of the function is the entire set of real numbers, but the coding scheme follows the signs of the first and second derivative, and for these codes to be applied, the domain must be split up into subintervals on which the signs of the first and second derivative are uniform. The interval from negative infinity to zero, considered in this example, is one such interval. Following the act of coding, though, this example also shows Dr. C beginning to construct a material representation in the form of a mathematical diagram (Figure 15). While each coded segment of the function will receive its own material representation, we can begin to see how these representations will be combined to blend seamlessly from one into the next. In this way, the choice of an iconic modality allows the process of material representation to erase the discretizing effect of the act of coding. While the mathematician’s professional vision can recover the process by recognizing the different curve shapes in the completed graph, the representation itself serves to recombine discrete elements into a continuous whole.
3.4. A Semiotic Theory of Typology and Topology in Mathematics

To understand the relative functions of mathematical notation and visual images, we began with O’Halloran’s (2000) analysis, which states that notation provides a complete abstract description of relations among mathematical objects, while images connect this description to our bodily experience of the world. Building upon this understanding, the middle school episode showed that mathematical objects may be understood as either discrete or continuous, and that in the course of classroom discourse, teachers may choose to highlight their discreteness or continuity for reasons that align with their pedagogical goals. Next, the Riemann sum episode showed that algebraic notation provides a symbolic representation, which highlights analytic-typological aspects of the mathematical object and facilitates precise algorithmic calculation. This semiotic resource is complemented by visual images, which provide an iconic representation privileging a holistic-topological understanding, and facilitating students’ conceptual understanding of classroom discourse. As observed in the graphing episode, however, visual images can be viewed typologically as well, so there is no one-to-one correspondence between semiotic channel and typological/topological meaning; rather, mathematical representations are rhemes that may be interpreted in either a topological or a typological frame. O’Halloran also discusses the role of language; she writes that it “provide[s] contextual information” (O’Halloran 2000, 363) such as, presumably, the idea that the function represented by the equation $s(t) = -16t^2 + v_0t$, or by its graphical representation as a sketch of a parabola, may be used to model the height of a projectile in flight. Adding the gestural channel to this description, I have shown that it provides a different sort of contextual information: viewed as metaphors of participation, these gestures provide metapragmatic contextualization cues that demonstrate iconically whether a given mathematical representation is to be considered as typology or topology.

We have seen that each mathematical task is accomplished through the discursive practices of highlighting, coding, and material representation described by Goodwin (1994) as components of
professional vision. In Goodwin’s analysis, professionals bring these practices to bear in their experience of external phenomena—an archeological dig site; a piece of videotape evidence in court—and thereby construct an understanding of these phenomena that is supported and legitimated by their professional authority. By highlighting and coding relevant chunks according to professional typologies, a phenomenologically continuous experience of the world is semiotized and made available as a referent in discourse. In the case of the mathematics classroom, however, professional vision is used not to bring the natural world into professionalized discourse, but to interact with mathematical objects, which, as Sfard (2008) argues, are not natural phenomena but discursive objects created in and through mathematical discourse. Specifically, the interaction in this instance is one of what O’Halloran (2004) calls intersemiotic translation, in which a single mathematical object—the fraction to be written as a decimal; the set of endpoints of the Riemann subintervals; the function to be graphed—is translated between symbolic and iconic representations. In order to accomplish this task, relevant features of the object must be inferred from the notation, and professional vision is required to successfully identify and interpret those features.

Sfard (2008) also describes a mathematical discursive practice of *saming* in which two mathematical representations are identified as being “the same,” or a new representation is constructed that is considered “the same” as a given representation; in semiotic terms, these representations may be considered signs that share an object, and in sequence, one may arise as the interpretant of the other. In the three episodes considered here, we see the construction of graphical and notational representations that are taken to be “the same.” In the middle school episode, the written fraction \( \frac{5}{8} \) is equated to a point on the number line between 0.6 and 0.7. In the Riemann sum episode, the phrase “on the interval from 1 to 6 ... with five rectangles” is translated first into the graphical partition of the \( x \) axis shown in Figure 10, and then into the sequence of endpoints \( x_0 = 1, x_1 = 2, \ldots, x_5 = 5 \). Finally, in the graphing episode, we see in Figure 15 the
beginning of a translation of \( f(x) = 4x^5 - 5x^4 \) from a notational into a graphical representation. If these representations are all equivalent, and therefore all potentially signs or interpretants of one another, then what is their object? One way to understand the nature of the object in the Peircean framework is that “the object (of a sign) is that to which all (appropriate and effective) interpretants (of that sign) correspondingly relate. In this way, it is best to think of the object as a correspondence-preserving projection from all interpretants of a sign” (Kockelman 2005, 242; original emphasis). In the mathematical case, where the object does not exist in the natural world but only in discourse, we can think of the object as a sort of second-order abstraction that is only knowable through its semiotic representations. The function depicted in the graphing episode, for instance, can be defined as essentially that to which the equation \( f(x) = 4x^5 - 5x^4 \) and its graphical representation correspondingly relate, which is itself a relation in which each real number \( x \) is systematically paired with a single real number \( f(x) \). This object is then subject to the semiotic process of rhematization, in which a relation comes to be construed as an entity that has properties of its own (e.g. it is continuous everywhere) and that can be put into relations with other objects (e.g. it is one of the set of antiderivatives of the function \( F(x) = 20x^4 - 20x^3 \)). Even a simpler object such as \( \frac{5}{6} \) can be seen as a rhematized process, one that begins as the partitioning of an object into eight equal subdivisions and the selection of five of them, but comes to represent the proportion of the whole that is obtained as a result. This is a conjecture to be confirmed by further research, but it is my intuition that most mathematical objects—from the basic counting numbers to the most abstract entities in abstract algebra—are similarly created through rhematization.

Where does this leave us with respect to typological and topological meaning? First, it seems reasonable to claim that the process of rhematization tends to erase the topological aspects of a phenomenon; while a relation may incorporate subtle differences at different points in its domain, reanalyzing the relation as an object tends to foreground aspects of the object that demarcate it as separate from its surroundings, a figure that stands out against a ground.
Professional vision seems to be linked to rhematization in this sense; the function \( f(x) = 4x^5 - 5x^4 \) is continuous everywhere, but in order to represent it graphically, it must be divided into relevant subintervals such as the interval \((-\infty, 0)\) in which the first derivative is positive and the second derivative is negative. Typological meaning thus results from the semiotic processes of erasure and fractal recursivity described by Irvine and Gal (2000); the function can be understood as a single chunk, or as a sequence of chunks, depending on the needs of the task at hand.

If the practice of mathematics requires the mathematician’s professional vision, and the discursive processes of professional vision depend on typological aspects of meaning, this is difficult to reconcile with Lemke’s (1999) claim that mathematics allows us to communicate topological meaning in ways that language does not. Clearly, the flight of a projectile is continuous, and if the equation \( s(t) = -16t^2 + v_{0t} \) captures that continuity, then the equation must in some sense be continuous as well. At the same time, the present analysis supports the notion that when this sort of equation is used by mathematicians, it is treated as discrete and typological. If an equation seems to be continuous, this is merely a case of “icons [being] so completely substituted for their objects as hardly to be distinguished from them” (Peirce 1885, 181); like the assertion that *dogs say woof woof*, it is a consequence of a symbolic sign being rhematized and therefore perceived as an icon in its interpretant.

On the other hand, perhaps “communicating topological meaning” is always a question of translating phenomenalological continua into semiotic sequences, in a way that is not restricted to mathematics or even to professional vision. Even in everyday life, events are not discretely subdivided one from another, but must be represented in this way in order to be represented and understood in narrative: “‘narrative is a primary cognitive instrument [for] making the flux of experience comprehensible’ as event” (Louis Mink, cit. in Bauman 1986, 5), just as numbers are a primary cognitive instrument for making the flux of experience comprehensible as quantity. Whether we can truly comprehend continuity or not is a philosophical question beyond the scope of
this analysis, but discourse seems to rely on the communication of discrete units of meaning, and
the representation of continuity, like the calculation of area as the limit of a Riemann sum, seems to
be accomplished by a sort of semiotic sleight of hand.
Chapter 4

RECEIVED KNOWLEDGE AND REPRESENTATIONAL AGENCY IN CLASSROOM DISCOURSE

At the end of the anecdote I recounted in Chapter 1, it seemed that Ms. M was asking one question but Alberto was answering another. Even though Alberto answered the assigned problem correctly and thereby demonstrated understanding of the lesson content, he did not reply to Ms. M’s most recent query, and for this reason his mathematically accurate answer was not treated as correct. A few seconds later, another student who offered the same response and was credited with having found the solution, making it clear that an answer’s being seen as correct is inextricably linked to its timing and sequential context. This led me to consider the widespread belief that any math problem has exactly one objectively correct answer; the anecdote suggested that being right can be seen as an interactional accomplishment, which motivated the research question, How do sequences of classroom interaction realize ideologies of mathematical knowledge? In this chapter, I take up that question through an analysis of students’ participation in conversational repair and the insight it provides into their semiotic agency in the classroom.

4.1. WHAT’S THE RIGHT ANSWER?

Figure 17: Which one doesn’t belong? (Danielson 2014, 1)

Frustrated with what he saw as the poor quality of the shapes books that are available for children, mathematics teacher educator Christopher Danielson created his own (Danielson 2014).
Each page of his book reproduces four different figures, and readers are asked to consider which of
the four does not belong. An example is provided in Figure 17 above. Of course, there's a trick here:
any of the shapes is a potential “correct answer,” and this is precisely the concept behind the design
of the book. As Danielson writes in an explanatory blog post, “If you are thinking, ‘It depends on
how you look at it,’ then you’ve got the idea… The only measure of being right is whether your
reason is true” (Danielson 2015a, para. 6). The goal is not, as in most children’s shape books, to
teach an authoritative taxonomy for labeling and categorizing mathematical figures, but rather to
encourage creative thinking and open-ended discussion between children and their caregivers.

Having created this book, Danielson tried using some of the pages as discussion prompts
with first and second grade classes. In a follow-up blog post (Danielson 2015b), he writes about
what he learned about students’ mathematical thinking and how it differs from orthodox processes
and typical classroom tasks. Students find some concepts more difficult to understand and acquire
than teachers might expect; for example, “The 1:1 correspondence of sides to vertices in polygons is
not at all obvious to young children” (Danielson 2015b, para. 16). At other times, however, these
discussions led into surprisingly sophisticated mathematical ideas:

“What exactly is a vertex?” is a much richer and meatier mathematical question than “How
many vertices does this shape have?”… Now kids can work on stating exactly what makes a
vertex. And what makes a vertex is going to be awfully close to what makes a point of non-
differentiability ... in twenty minutes with second graders, we can get very close to
investigating things that are challenging for calculus students to describe. My point is that
second graders are ready to do some real mathematics (Danielson 2015b, para. 7–11)

What does Danielson mean by “real mathematics”? One potential reading of this phrase is
that “real mathematics” is higher mathematics; calculus is more “real” than learning the names of
shapes. This interpretation is belied by the purpose of the shapes book, however. The book is not
meant to move children more quickly through a set curriculum of mathematics topics, but rather to
engage them in math talk; notably, one of Danielson’s blogs is entitled *Talking Math with Your Kids*. In this spirit, I understand “real mathematics” to mean mathematics that is open to discussion and debate, in which “the only measure of being right is whether your reason is true.”

But how do we know whether mathematical reasoning is true? Without getting into philosophical questions of mathematical epistemology, we can view the truth or falsity of utterances as an interactional accomplishment. That is, a true statement is simply one that is treated as true within an epistemic community (van Dijk 2004), whose justification conforms to the norms of the relevant epistemic culture (Knorr Cetina 1999). Truth “refers to the unchallengeability of one’s assertion in the context of one’s addressees ... For a speaker’s representation to be ‘true’ means that others subsequently use this representation as a reason for further representations” (Kockelman 2007, 383). Considering this definition, we see that Danielson’s project challenges common understandings of mathematical truth and knowledge in at least two respects. First, he uses children’s assertions about mathematical objects as a starting point to build conversations, which means that the children are sources of true mathematical knowledge; and second, any answer to the problem could be right, as long as it is justified by “true reasoning” that is usable by other participants.

In contrast to Danielson’s approach, Alan Schoenfeld’s (1989) survey of high school geometry students revealed a broadly held belief that mathematics is an entirely objective discipline to be studied through rote memorization. Magdalene Lampert (1990) writes that this belief in objectivity is largely a consequence of the way mathematics is taught in schools, in contrast to professional mathematicians’ practice, which places more value on creativity and intellectual courage such as what Danielson’s technique requires. As a result, much of the learning sciences research on mathematics education has attempted to make school mathematics more like professional mathematics in this way. Mathematics education researcher Jo Boaler and her colleagues identify “reform” or “discussion-based” teaching practices, which contrast with
“traditional” (Boaler 2002; Boaler 2003) or “didactic” (Boaler and Greeno 2000) mathematics classes, such as the ones in Schoenfeld’s study, as well as those I observed; this taxonomy is summarized in Table 1. Drawing on the work of Etienne Wenger (1998; Lave and Wenger 1991), Boaler accounts for this contrast by identifying distinct sets of teaching practices, which give rise to qualitatively different classroom communities of practice.

Table 1: Traditional and reform teaching practices

<table>
<thead>
<tr>
<th>Course content</th>
<th>Traditional / Didactic</th>
<th>Reform / Discussion-based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructional grouping</td>
<td>Individual book work</td>
<td>Small group collaboration</td>
</tr>
<tr>
<td>Task</td>
<td>Find the right answer</td>
<td>Open-ended problem solving</td>
</tr>
<tr>
<td>Teacher’s role</td>
<td>Explain; provide information</td>
<td>Question; provide structure</td>
</tr>
<tr>
<td>Authority</td>
<td>Teachers and textbooks</td>
<td>The mathematics disciplinary community</td>
</tr>
<tr>
<td>Source of knowledge</td>
<td>Received knowing, provided by authoritative source</td>
<td>Connected knowing, emerging from interaction</td>
</tr>
</tbody>
</table>

(Boaler 2002; Boaler 2003; Boaler and Greeno 2000; Boaler, William, and Zevenbergen 2000)

This chapter is concerned with the discursive production and reproduction of two aspects of “traditional” mathematics teaching identified by Schoenfeld, Lampert, and Boaler: that there is always exactly one correct answer, and that the teacher and textbook are the only valid sources of knowledge. In traditional settings, truth is known objectively and revealed by an authority figure such as the teacher or textbook; as one of Boaler’s student participants put it, “The thing I love about traditional teaching is the teacher tells it to you and you get it” (Boaler 2003, 5). Not only is authority located in institutionally designated representatives of the discipline of mathematics, but mathematical knowledge itself, in this view, is considered direct and unequivocal; as another participant states, “It’s the only class, where there will be a right or wrong answer, there’s a way to get the right answer” (Boaler, William, and Zevenbergen 2000, 7). This positionality may give rise to feelings of powerlessness as easily as a sense of security: “There’s only one right answer ... [and]
it’s always in the back of the book right there. If you can’t get it you’re stuck” (Boaler and Greeno 2000, 179).

As with any cultural belief, the ideology underlying “traditional” practices is usually left implicit, but on occasion may be articulated more overtly—not only in reflective interviews such as those presented by Boaler, but also, on occasion, in classroom interaction. Consider Example 9, which occurred in the calculus class as the professor was pausing to check students’ understanding part way into a lecture.

*Example 9: ‘Does this look fishy to you?’ (15 January 2014)*

1 Dr. C Do you guys buy this?
2 \( (1.8) \)
3 Does this look fishy to you?
4 Wendell Is it right?
5 class \((\text{general laughter})\)
6 Dr. C Maybe,
7 maybe not.

The professor initiates this sequence by asking the students to evaluate a line of reasoning that he has just outlined. The pause in line 2 suggests that the students don’t know what to make of this initiation move, leading the professor to make a second attempt through paraphrase in line 3. One potential explanation for the students’ uncertainty is that the status of the prompt itself is, in principle, ambiguous. Its form resembles a simple check for understanding along the lines of “Does this make sense,” “Do you understand,” or even “OK?” Some education researchers have stressed the importance of this kind of understanding check, especially in mathematics, on the grounds that they encourage students to think about their responses as meaningful statements, not just candidate correct responses to pedagogical stimuli:

> having individuals ask such questions seems to be an important aspect of the thoughtfulness that all students should acquire through schooling. Rather than asking whether this is the right answer as decreed by the teacher or the textbook, students should be asking, “Does this make sense? Is this reasonable?” (Putnam et al. 1992, 222)
At the same time, however, “Does this make sense?” is a familiar sort of sequence initiation that preferentially anticipates a student response of assent, which would then license the professor to continue with his lecture. In fact, the preference for a positive response is so strong, and the face threat of a negative response is so great, that I have often observed this move to be met by silent nods without having clearly accomplished a check of student understanding. Functionally, it often seems not to provide a legitimate opportunity for students to voice questions, but rather, to accomplish a topical transition in the midst of an explanation or demonstration lecture sequence. That is, by saying “Does this make sense?” the lecturer signals that a chunk of lecture has just concluded that should hang together and “make sense” cohesively; students may ask themselves whether they are making sense of the lecture, but they rarely respond to this question out loud.

While this sequence initiation seems to work as a typical check for understanding, there is a visible hesitation on the part of the students, possibly caused by the unfamiliar phrasing. By asking whether the students “buy” his reasoning and characterizing it as potentially “look[ing] fishy,” Dr. C disaligns from his own prior talk. He accomplishes the action in a way that does not clearly prefer a response of assent, and in so doing introduces an element of uncertainty that renders his present utterance ambiguous. This interpretation resembles another strategy sometimes used by teachers: intentionally voicing a common or anticipated misconception of the material and looking for students to contradict it (Ginsberg 2013). This move is often voiced as a reversed polarity question (Koshik 2010) along the lines of “Doesn’t this proof show that two equals one,” but it may be familiar enough to students that Dr. C’s skepticism would activate a voicing-of-misconceptions schema for the students, in addition to the check-for-understanding script. In addition, Dr. C does not commonly use the voicing-of-misconceptions technique, which may add to the students’ uncertainty.

The combined effect of these conflicting interpretations is to recast Dr. C’s talk as legitimately uncertain. By stating this uncertainty in the form of a question, Dr. C positions the
students as the arbiters who will decide on the mathematical validity of the preceding strip of lecture. In line 4, Wendell rejects this positioning by appealing to an ideology of mathematical objectivity: the lecture was in and of itself either right or wrong, the professor knows the truth of it, and so the students’ opinion is irrelevant. I read the other students’ subsequent laughter in line 5 as marking a release of tension as Wendell speaks aloud what many of them were thinking: Dr. C’s questions in lines 1 and 3 are bizarre, and he shouldn’t waste time playing games if he knows the answer and he’s going to tell them eventually. And while Dr. C seems to hold on to his stance of uncertainty in lines 6–7, in fact there is nothing “fishy” in the mathematics. After the transcribed example, he calls on another student, she asks him to rerun the immediately preceding strip of lecture that he had characterized as potentially “seeming fishy,” he does so, and then he continues with his lecture. At this point, it has become apparent that Dr. C’s initial utterance in Example 9 was simply a highly marked way of bringing focus to a key point in his demonstration; what was overtly phrased as a solicitation of agreement functioned instead as a check for understanding, in the same way that an overt check for understanding such as “Does this make sense?” is more likely to function as a topic transition.

In this example, Wendell’s utterance is unusual in that it voices aloud what is pervasively true but generally left unsaid: that students generally do not expect to be asked for their opinion, but instead approach the practice of mathematics with extreme passivity and powerlessness. Students who view themselves as weak in mathematics often essentialize themselves in this respect: I’m not a math person. Even students who like mathematics and do well in class often report that the perceived objectivity of the subject is what draws them to it; “The students in didactic classes who liked mathematics did so because there were only right and wrong answers, and because they did not have to consider different ideas and methods” (Boaler 2002, 44). While these students often value their own control over the mathematical notation that allows them to
find correct answers, this is generally not framed as an ability to make truthful claims about mathematical objects.

To chart the boundaries of the student agency afforded by “traditional” teaching practices, this chapter will focus on interactional strategies that position students in relatively agentive roles. First, I will consider an interactional practice in the middle school class in which students use a “thumbs up” or “thumbs down” gesture to evaluate mathematical statements—typically, their classmates’ answers to problems posed during in-class activities—as correct or incorrect. Then, turning to the calculus class, I will present instances in which students attempted to correct the professor’s utterances. Both of these scenarios allow students the opportunity to initiate or complete repair of others’ utterances, a right that is traditionally reserved for the instructor (McHoul 1990). By interpreting these observations in light of a semiotically grounded theory of social agency (Kockelman 2007), however, we will see that, irrespective of the social positioning accomplished through these discursive practices, the students’ semiotic agency is always constrained, and in line with the “traditional” classroom ideology, the instructor’s unilateral authority is preserved.

4.2. SEQUENCE, KNOWLEDGE, AND AGENCY BETWEEN TEACHERS AND STUDENTS

One way of thinking about the “traditional” style of mathematics teaching outlined above is that there is nothing uniquely mathematical about it; on the contrary, it is a particular iteration of more general orthodox views of teacher authority, made visible through the styles of classroom discursive practice that produce and reproduce authority in interaction. Arguably the first major finding in classroom discourse analysis was that teachers often ask known-information questions and evaluate their students’ responses in the familiar initiation – response – feedback sequence (Sinclair and Coulthard 1975), later renamed initiation – response – evaluation, or IRE (Mehan 1979a). This structure canonically functions to position the teacher as both source of knowledge and evaluator of students’ classroom behavior. Similar discourse patterns were identified by
McHoul (1990), who introduced Conversation Analysis (CA) to the study of classroom discourse by using the CA model of conversational repair (Schegloff, Jefferson, and Sacks 1977) to understand teacher corrections of student errors, and in fact later researchers have used CA to look specifically at IRE sequences (Candela 1999; Hellerman 2005; Tainio and Laine 2015; Zemel and Koschmann 2011). To see how CA and Mehan’s IRE—alternative ethnomethodological approaches to discourse—would be applied to the same interactional sequences, consider the following example, an invented exchange offered by Mehan (1979b) as a prototypical IRE sequence.

Example 10: The prototypical IRE (Mehan 1979b, 285)

1 A  What time is it, Denise?
2 B  2:30
3 A  Very good, Denise

In conversation analytic terms, lines 1–2 constitute an adjacency pair—a question, together with the answer that it projects—and 3 is a sequence-closing third (SCT), defined as a single-turn post-expansion of an adjacency pair that “is designed not to project any further within-sequence talk beyond itself” (Schegloff 2007, 118). For Mehan, the difference between this example and a more familiar sort of SCT, such as “Thank you, Denise,” is that the third turn in an IRE sequence shows evaluation rather than acknowledgement, and yet, third-turn evaluations are attested outside of the classroom as well. Consider Example 11, in which a question is asked about the researcher’s video camera.

Example 11: A third-turn assessment that doesn’t do being-the-teacher (Schegloff 2007, 125)

1 Don  Is this aimed accurate enough?
2   (0.5)
3 Joh  Yes it’s aimed at the table.
4 Don  Great.

The key difference between these two examples is that Don’s evaluation in Example 11 is affective (“Great” = What you just said makes me happy), while Mehan’s hypothetical teacher in Example 10 gives an epistemic assessment (“Very good” = What you just said is correct), a
distinction that may be better articulated in terms of epistemic status and action ascription (Heritage 2012; Heritage 2013). This theory addresses the observation that a linguistic form may be ambiguous as to the social action that it accomplishes—in this case, whether an assessment in SCT position is an evaluation of the desirability of the state of affairs described in the preceding utterance, or of the accuracy of the utterance itself. The relevant social variable in resolving such ambiguity is the participants' understanding of their relative epistemic status, which "involves the parties' joint recognition of their comparative access, knowledgable and rights relative to some domain of knowledge as a matter of more or less established fact" (Heritage 2013, 558); a first-pair-part speaker would be able to judge their interlocutor's correctness only if they had greater epistemic rights over the topic. What the IRE sequence illustrates is that part of a teacher's institutional role is their privileged epistemic status within the classroom environment regarding academic content;

the classroom context will undoubtedly prime the students' understanding that the teacher has K+ status [i.e. higher epistemic status] in the general domains of the questions he asks....

all third turns that affirm or deny the correctness of students' contributions (Drew 1981; Sinclair and Coulthard 1975) retroactively reassert the K+ epistemic position that informed the question's production in the first place. (Heritage 2013, 563)

In the initiation move of an IRE sequence, the teacher does ask for information, but this is information that the teacher already possesses, and the evaluation move serves as a reminder of this difference in epistemic status, lest someone mistakenly believe that the teacher is asking a genuine information-seeking question.

If unequal epistemic status provides different speakers access to different social actions, then epistemic status can be theorized as a type of agency, following Laura Ahearn's definition of agency as “the socioculturally mediated capacity to act” (Ahearn 2001, 112). Often, one utterance may accomplish any of a number of different social actions, and the key contribution of Heritage's
framework is that this ambiguity is sometimes resolved through recognition of epistemic status. Restating this in Peircean terms, a given sign may point to different objects and give rise to different interpretants, depending on the relative social status of speaker and addressee:

the rights and responsibilities that make up a social status (such as priest, sheriff, mother, doctor) may often be described in terms of semiotic rights and responsibilities: what kinds of actions and utterances one may or must make as a function of the social context one is in. In this way, a key constraint on our residential agency is the kinds of social statuses we relationally inhabit to the extent that these statuses enable and constrain semiotic processes as to their when and where (control), their what and how (composition), and their why and to what effect (commitment). (Kockelman 2007, 382)

In this passage, Kockelman refers to “residential agency,” which is defined as a social actor’s capacity to use a given meaning-making resource. This type of agency is subdivided into a Peircean triad: control over the expression of the sign itself, composition of a sign-object relationship, and commitment to an interpretant (Appendix A, column 4). For example, as part of a mathematical modeling exercise, I may:

- write an equation (control)
- that models a phenomenon (compose)
- and thereby leads to the next step in the solution (commit).

In addition to residential agency over semiotic resources, Kockelman also outlines a theory of “representational agency,” which refers to the right or ability to make propositions about a given entity or process. The difference between the two is that “one has representational agency over that which is being represented (e.g., some person, instrument, action, thing, or event), not over that which is being used to represent it—over which one, rather, has residential agency” (Kockelman 2007, 383, note 8). This sort of agency has a firstness, secondness, and thirdness as well: the ability to “thematize a process, characterize a feature of this theme, and reason with this theme-character
relation” (Kockelman 2007, 383, emphasis added; see also Appendix A, column 5). In the particular case of mathematics,

the dimension of reason may involve the most stereotypic and celebrated of logical processes: the degree to which one may use one’s current representation to infer a new representation and/or the degree to which one has used an old representation to infer one’s current representation. (Kockelman 2007, 384)

What is at issue in the present study is the students’ degree of representational agency over mathematical objects. For example, if I demonstrate visually that two plus two equals four, then I:

- make an assertion about the expression two plus two (thematize),
- specifically, that it is equal to four (characterize),
- and provide evidence to support my claim (reason).

Intuitively, this typology seems to be distributed sequentially through the IRE script—in Example 10, the teacher thematizes the time, Denise characterizes it as being 2:30, and the teacher validates her reasoning—but it remains to be seen whether this pattern holds up in authentic IRE examples.

By either identifying or repairing a trouble source in preceding discourse, a speaker makes a claim not only to residential agency over the discourse itself, but also to representational agency over some aspect of the discourse topic. In this sense, by looking at cases when students either initiate or complete repair, we may come to understand the relative degree of representational agency they express with regard to mathematical objects. We saw earlier that a minimal IRE sequence can be parsed as an adjacency pair plus sequence-closing third; how then does repair fit into this structure? Outside of classroom talk, the literature indicates that other-initiated repair is typically accomplished in an expansion immediately following the trouble source. If the trouble source occurs in the first pair part of an adjacency pair, the repair is realized as an insert expansion, as in Example 12; and if the trouble source occurs in the second pair part, then the repair is realized as a post-expansion, as in Example 13.
Example 12: Repair initiation as insert expansion (Schegloff 2007, 102)

1. A Have you ever tried a clinic?
2. B What?
3. A Have you ever tried a clinic?
4. B ((sigh)) No, I don’t want to go to a clinic.

Example 13: Repair initiation as post-expansion (Schegloff 2007, 149)

1. Dee Well who’r you workin for.
2. Connie Well I’m working through:: the Amfat Corporation.
3. (0.8)
4. Dee The who?
5. Connie Amfah Corporation. (. ) ’ts a holding company.

The status difference in classroom interaction brings about repair sequences that look a bit different. While Example 12 provided a case where the second-pair-part speaker identifies a trouble source in the first pair part, Zemel and Koschmann (2011) found that students generally respond to teacher initiations without attempting repair, and when the teacher identifies a misunderstanding in an IRE sequence, it is up to that teacher to determine whether the trouble source occurred in the initiation or response move. In the interactions to be considered here, we will see additional ways that IRE and repair can coexist as classroom discourse patterns. In the middle-school class, the thumbs-up / thumbs-down technique allows the teacher to initiate repair as a post-expansion of the initiation-response adjacency pair; when the first student gives an incorrect response, the teacher can call on a second student to provide the correction. In the calculus class, when students initiate repair, we will see that they do not identify trouble sources in the initiation move of an IRE sequence, but in the blackboard text or the flow of an ongoing lecture, so repair is initiated in first pair parts of new side sequences.

4.3. Peer Evaluation in the Middle School Class

In the middle school, class typically began with a warm-up worksheet that students picked up as they walked into the room. Students spent as little as five minutes up to almost fifteen minutes working on it, and then Ms. M called the class to order by reviewing the problems from the
warmup. Quite often at this point she would call on one student to provide their answer, and then ask the rest of the class to evaluate that answer as correct or incorrect with a thumbs-up / thumbs-down gesture, as shown in Example 14. The problem on the board, which Ms. M indicates in line 3, is to rewrite the fraction $\frac{4}{10}$ as a decimal.

**Example 14: ‘Let’s begin with Josefina’ (24 January 2014)**

1. Ms. M | So let's begin with Josefina,
2. | turn to face the board (2.0)
3. | how do you | write this as a decimal
| point to problem | turn to face class
4. Josefina | zero point
5. four, |
6. Ms. M → | Give me a thumbs up if you agree with her. |
| 0.4 | raise thumb
7. class | most students give thumbs up gesture
8. Ms. M → | Thumbs down if you disagree. |
| thumbs down gesture
9. (0.7)
10. All | right. |
| release thumbs down gesture

This strip of talk begins like an IRE sequence, with Ms. M providing an orienting pre-expansion in lines 1–2, then initiating a question in line 3, and Josefina responding in lines 4–5. Line 6 seems at first to be an implicit positive evaluation, as Ms. M begins by recording Josefina’s answer on the Smart Board, but then her talk makes it apparent that she is simply reproducing Josefina’s response in writing, not endorsing its accuracy. Instead, she asks the class to provide an evaluation move gesturally; it seems that she has suspended her privilege to evaluate student responses, and instead has granted this authority to the class as an aggregate. In line 9, she pauses and checks on the students’ evaluations of Josefina’s answer, and in 10 endorses their approval only implicitly by moving on to the next problem in the warm-up. This observation raises the question: by calling for a thumbs-up / thumbs-down evaluation of student work, is Ms. M simply enforcing student compliance with her expected level of engagement in class—“checking in so that kids don’t check
out,” as mathematics educator Justin Lanier (2015) put it—or is she granting a certain level of agency to the students?

To begin answering this question, consider the interactional sequence of these thumbs-up / thumbs-down episodes, which I will call instances of the thumbs technique. The preceding description identifies an IRE sequence in lines 3–6 in which Ms. M asks Josefina a question, Josefina replies, and Ms. M signals uptake of Josefina’s answer by writing it on the board. Next, Ms. M’s initiation of the thumbs technique in lines 6 and 8 begins a second IRE sequence that calls for a gestural response from the class at large. This move is so familiar to the students that they supply the response in line 7, before the initiation has even concluded. Finally, in line 10, Ms. M both evaluates Josefina’s answer and endorses the class’s evaluation, thus bringing the entire sequence to a close.

In a sense, the thumbs technique allows the evaluation move of the first IRE sequence to play out as a second embedded IRE sequence. Ms. M’s writing in line 6 seems at first to be a sequence-closing third, but “the repeat in third position can be equivocal between use as a sequence-closing third on the one hand and its use as a form of repair initiation on the other” (Schegloff 2007, 126), and in fact Ms. M’s talk combines with the written inscription to initiate a post-expansion of the initial initiation-response adjacency pair. The sequence cannot conclude until Ms. M provides a true SCT in 10, which indicates how strong the preference is for the teacher to provide the evaluation move of the IRE. As the structure of insert expansions shows, “questions expect an answer in the next turn; where the adjacency criterion is not met, an answer is nevertheless still due” (Levinson 2013, 155), and along similar lines, when an IRE is initiated, not just an answer but also a third-turn evaluation is due, even if it is deferred beyond the actual third turn. As we look at additional instances of the thumbs technique, we will see that this preference persists, although to realize the final evaluation move, Ms. M can lead the correct answer to be
definitively identified in a variety of different ways, and put off voicing her final evaluation over even longer stretches of talk.

4.3.1. **Quantitative Overview of Thumbs Technique Patterns**

Before providing a microanalysis of the typical forms taken by this interactional practice, I will provide a quantitative overview of all 58 instances that occur in the video corpus. Counting and classifying the different patterns of interaction that proceed from Ms. M’s “thumbs-up / thumbs-down” prompt will allow us to identify not only trends and preferences in the interactional sequence, but also potential types of sequences that in fact do not occur in the corpus. To conduct this quantitative description, each instance of the thumbs technique was identified and tagged for a number of features:

- the phase of the lesson in which it occurred,
- whether the mathematical statement to be evaluated was correct or incorrect,
- the response of the class: mainly thumbs up, mainly thumbs down, mixed / uncertain response, or little / no response,
- and the teacher’s next move following the thumbs-up / thumbs-down display.

For example, Example 14 was coded as warm-up review, correct answer, thumbs up, no follow-up.

4.3.1.1. **Phase of the Lesson**

To begin, consider the phase of the lesson in which the thumbs technique occurs. Table 2 shows the counts across five phases of the lesson: review of the warmup worksheet, of the previous night’s homework, and of a question from a test that the class had taken previously; introduction of new material; and practice with new material.
Table 2: Lesson phase in which the thumbs technique occurs

<table>
<thead>
<tr>
<th>Phase</th>
<th>n</th>
<th>frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm-up review</td>
<td>35</td>
<td>60.3%</td>
</tr>
<tr>
<td>Homework review</td>
<td>8</td>
<td>13.8%</td>
</tr>
<tr>
<td>Test question review</td>
<td>1</td>
<td>1.7%</td>
</tr>
<tr>
<td>Introduce new material</td>
<td>3</td>
<td>5.2%</td>
</tr>
<tr>
<td>Practice new material</td>
<td>11</td>
<td>19.6%</td>
</tr>
</tbody>
</table>

As Table 2 shows, 46 out of 58 occurrences, nearly 80% of the total, occurred during either the warm-up or practice phase of the lesson. Both of these phases represent speech situations in which the students had been given a problem set earlier in the class period and had just finished working through the problems, in most cases working independently. When Ms. M observed that most students have completed the task, she initiated a review phase in which the problems were read aloud and discussed one by one, correct answers identified, and clarifications made.

While there seems to be a preference for the thumbs technique to occur during the warm-up and practice phases, I believe that the quantitative trend actually reflects a dispreference for this interactional practice in the homework and new material phases. To conduct the homework review, Ms. M typically displayed the answer key to the previous night’s homework on the Smart Board, which allowed students to compare the correct answers to their own solutions, and if they wished, to volunteer clarification questions about problems they got wrong. Ms. M was then able to presume that students had understood a problem if they asked no questions about it, so she did not have to use the thumbs technique to gauge the class’s overall or average level of understanding, although it was available to her as an option. During presentations of new material through teacher-fronted lecture or discussion, the thumbs technique was only observed three times, but this result is not surprising because the lecture / discussion format offered relatively few opportunities for students to solve problems. Finally, unlike the other four phases discussed here, test question review was not a typical daily activity, but rather occurred only once in the entire corpus. During
this activity, a question that had been previously asked on a test was then singled out for review during a later class session.

4.3.1.2. Gestural Response

Each time Ms. M initiated the thumbs technique, the response of the class was recorded impressionistically as thumbs up, few thumbs up, thumbs down, few thumbs down, uncertain, few uncertain, mixed, little response, or no response. These tags represent my impression of the overall aggregate class response based on a reviewing of the video record, not a unanimous consensus of the class, and with the understanding that some students’ responses may be obscured. For the purposes of quantitative analysis, the tags were collapsed into a three-way distinction between thumbs up, thumbs down (incorporating few thumbs up and few thumbs down, respectively), and other, allowing us to consider how often the aggregate class response corresponded to the correct answer; the totals are displayed in Table 3. While in most cases the class was asked to evaluate a single mathematical statement as correct or incorrect, there were three instances in which several students had solved a problem, and Ms. M asked the class to indicate with a thumbs-up gesture which of their solutions was accurate; these tokens are categorized as “Options” in the table below.

Table 3: Gestural responses to the thumbs technique

<table>
<thead>
<tr>
<th></th>
<th>Correct</th>
<th>Incorrect</th>
<th>Options</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thumbs up</td>
<td>33</td>
<td>2</td>
<td>0</td>
<td>35</td>
</tr>
<tr>
<td>Thumbs down</td>
<td>0</td>
<td>15</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>Other</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>36</td>
<td>19</td>
<td>3</td>
<td>58</td>
</tr>
</tbody>
</table>

These counts show that for the most part, the class as a whole gravitated toward the correct answer. In all fifteen cases when the students gave the thumbs-down signal, they had been asked to evaluate a mathematical assertion that was in fact incorrect. Out of 35 cases of thumbs up, only two occurred in response to an incorrect answer; moreover, one of these instances was a “few thumbs
up,” and in the other, the student had arrived at the correct result, but the teacher treated it as incorrect in order to point out an omission in the way the student had written it. In this latter case, it seems fair to say that her classmates were accurately evaluating her correct answer without taking account of her imprecise use of notation. While we have not yet looked at the specific phrasing and intonation that Ms. M used to initiate the thumbs technique, it seems likely that neither thumbs-up nor thumbs-down is a preferred response; unlike the case of “Does this make sense,” the students are not doing the gestural equivalent of smiling and nodding, but are carefully judging their classmates’ work and providing honest evaluations.

4.3.1.3. **Final Evaluation Move**

After eliciting a thumbs-up or thumbs-down gesture from the class, Ms. M could conclude the sequence in a few distinct ways. Aside from moving on without providing any explicit follow-up, as we saw in Example 14, she could call on the same student who provided the initial mathematical statement to explain or correct it; call on another student to explain or correct the first student’s answer; provide the correct answer herself; or restrict the focus of attention to drill down on a particular detail of the solution. The frequency of each of these options is recorded in Table 4, which shows a clear and statistically significant preference for no follow-up following a correct answer, and for another student to be called on to correct an incorrect answer.

<table>
<thead>
<tr>
<th>Options</th>
<th>Correct (N=36)</th>
<th>Incorrect (N=19)</th>
<th>Options (N=3)</th>
<th>Total (N=58)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No follow-up</td>
<td>17 (47.2%)</td>
<td>3 (15.8%)</td>
<td>0 (0.0%)</td>
<td>20 (34.5%)</td>
</tr>
<tr>
<td>Same student</td>
<td>1 (2.8%)</td>
<td>4 (21.1%)</td>
<td>0 (0.0%)</td>
<td>5 (8.6%)</td>
</tr>
<tr>
<td>Other student</td>
<td>9 (25.0%)</td>
<td>10 (52.6%)</td>
<td>2 (66.7%)</td>
<td>21 (36.2%)</td>
</tr>
<tr>
<td>Teacher</td>
<td>2 (5.6%)</td>
<td>0 (0.0%)</td>
<td>1 (33.3%)</td>
<td>3 (5.2%)</td>
</tr>
<tr>
<td>Detail</td>
<td>7 (19.4%)</td>
<td>2 (10.5%)</td>
<td>0 (0.0%)</td>
<td>9 (15.5%)</td>
</tr>
</tbody>
</table>

\( \chi^2 = 0.01073, \text{df} = 6, p = 7.15 \times 10^{-6}. \)
In the previous section, we saw that thumbs-up evaluations nearly always occurred in response to correct answers, and every thumbs-down evaluation was given in response to an incorrect answer. While this tendency does show that the class as a whole seemed to reach consensus around the correct answer to each problem, it should be noted as well that the response to each thumbs technique initiation was mixed, with only a subset of the class responding in any given instance. By combining the results shown in Table 3 and Table 4, we see two preferential patterns emerging: if the first student response is correct, it is followed by a thumbs-up evaluation, and the sequence is concluded without explicit follow-up, as in Example 14; and if the first student response is incorrect, it is followed by a thumbs-down response from the class, and a student is called on to supply the correction. In this second pattern, Ms. M is able to decide which student to call on by choosing from among the students who display thumbs down, which allows her to anticipate with some confidence a correct answer from that student.

Notably, Ms. M herself never supplies the correct answer, as shown by the count of zero in the Incorrect / Teacher cell. Instead, it is the students who supply the correction in response to fourteen out of nineteen, or 73.7%, of incorrect answers, and in the remaining cases, Ms. M either goes through the mistaken reasoning in detail so that its flaw will become self-evident, or proceeds without explicit follow-up. Rather than correcting students explicitly, she uses the thumbs technique to first build a public consensus around incorrectness and then elicit the correct response from a student, which seems to resist the prototypical use of “traditional” teaching practices to delegitimize students as sources of mathematical knowledge. By looking in detail at how these actions are accomplished, we will see ways in which students are provided opportunities to reason with mathematical representations, but only in a limited way.

4.3.2. Common Patterns: Corrections and Details

Having identified the most typical patterns of realization of the thumbs technique, we can consider in detail how these patterns play out in interaction. Example 14 above showed what tends
to happen when the answer is correct: most students confirm their understanding by displaying thumbs up, and the lesson moves forward without further comment. This section will provide examples of two additional common patterns: when Ms. M elicits a correction following a wrong answer, and when she focuses on a detail of a particular problem or solution.

4.3.2.1. Eliciting a Correction

Immediately following Example 14, Santiago was called on to answer the next problem on the same warm-up sheet, which was to supply the decimal equivalent of the mixed number $1\frac{2}{5}$.

Example 15 begins with his response.

Example 15: 'Why do you disagree?' (24 January 2014)

1 Ms. M Santiago, how do you write
2 |this as a decimal?
3 point to problem on the Smart Board
4 Santiago Zero point thirty (um)
5 thirty three repeating.
6 Ms. M $0.3\overline{3}$
7 Give me a thumbs up if you |agree with Santiago,
8 thumbs up gesture
9 if you disagree:. (0.8)
10 thumbs down gesture
11 class Through lines 6–7 a number of students give thumbs-up and thumbs-down gestures. Nayan is first with a thumbs-up, but changes to thumbs-down once other students begin to respond. By line 9, five hands are held high: four thumbs down including Nayan and Ricardo, and one thumb up from Alberto, who sits next to Santiago. Sandra is also visibly showing thumbs-down directly in front of her body.
12 Ms. M Lots of thumbs down and some thumbs up. (0.8)
13 U:h (1.4)
14 |Why do you disagree.
15 point to Sandra
16 Sandra Because
17 he didn't
18 put the whole number
19 Ms. M put down pen, pick up a different color pen
20 Are you allowed just drop that one?
21 circle "1" in problem
22 student O:h.
23 class no
24 Ms. M 'kay.
This excerpt comprises an initial question and response, followed by three recursively embedded post-expansions. The first adjacency pair, from lines 1–4, proceeds just as in the preceding example: Ms. M calls on a student and voices the question aloud, and the student answers verbally. Ms. M then begins the thumbs technique as a first post-expansion: she inscribes the student’s spoken answer onto the board in line 5 and calls for a gestural response in lines 6–7, prompting the response described in line 8. A second post-expansion is inserted into this post-expansion, as lines 9–11 recast the gestural response as an initiation. Like line 5, line 9 shows Ms. M translating a student response not just into a new semiotic modality—this time, from gesture to talk—but also into a new sequential context, as a second pair part becomes a first pair part.

Notably, Ms. M calls on Sandra, one of the students who had displayed thumbs down in the preceding sequence; by demonstrating disagreement with an incorrect answer, Sandra has indicated that there is a reasonable chance she found the correct answer, which she goes on to voice in lines 12–14. Finally, in line 16, Ms. M initiates a third recursively embedded post-expansion by stating Sandra’s response as a question, a revoicing move (O’Connor and Michaels 1993) that elicits a choral response in 18. In line 19, “kay” functions as a SCT that closes the most embedded post-expansion sequence, marking the consensus that you’re not “allowed to just drop that one,” and the logical consequence of this (marked by so in line 20) is the authoritative answer to the question that was initially posed in 1–2. This answer is provided through both speech and writing in 20–22, and in this way the larger sequence is closed.

Having seen that this sequence is structured as four adjacency pairs, and that a student or students supply the second pair part to each of them, we can consider the level of representational agency evidenced by these second pair parts: Santiago’s initial response in lines 3–4, the display of
thumbs in 8, Sandra’s correction in 12–14, and the choral response in 18. To begin with, Ms. M’s known-information questions are structured such that her initiation and the student response together form an assertion or logical argument: that $1\frac{2}{6} = 0.3\overline{3}$, that the preceding statement is false, that Santiago should have written the whole number 1 in his answer, and that the whole number 1 may not be omitted. This structure allows students to name properties of mathematical objects, but only if the object has previously been thematized by the teacher. This shows one way in which the IRE sequence limits students’ agency, by allowing them to characterize objects but not thematize them; an extreme case would be in the use of designedly incomplete utterances (Koshik 2002) such as saying “A polygon with three sides is called a …” to elicit “triangle,” and Ms. M does use this practice at times, although not together with the thumbs technique.

So, students do not thematize, but they do characterize; do they reason? If mathematical reasoning involves “the degree to which one may use one’s current representation to infer a new representation and/or the degree to which one has used an old representation to infer one’s current representation” (Kockelman 2007, 384), then we do see student reasoning, but only in highly restricted forms. Some reasoning is implicit in Santiago’s response, as he has evidently used the representation $1\frac{2}{6}$ to derive the representation $0.3\overline{3}$, but this reasoning is never made explicit, and the remainder of the sequence functions primarily to delegitimize it. (This happens because Santiago’s response is incorrect, but as we saw in the case of a correct answer in Example 14, any student reasoning must be validated by the teacher before it can be taken up as justified and true.) The class’s thumbs-down display is also derived from a preceding representation, as the preceding argument “$1\frac{2}{6} = 0.3\overline{3}$” is rhematized to become the theme of the following argument “the statement $1\frac{2}{6} = 0.3\overline{3}$ is false.” The status of this argument as a new representation based on reasoning is borderline, however, as it seems to reject the truth of a preceding statement rather than express a new statement. Santiago’s answer is labeled as an inappropriate basis for subsequent mathematical reasoning, although it remains as a prompt for metacognitive reasoning about why Santiago was
wrong. Some reasoning must have taken place here, as the students who show thumbs down have presumably derived a different decimal representation from the one voiced by Santiago, but this is only made explicit in Sandra’s spoken utterance in lines 12–14, which provides the justification for her thumbs-down response in 8. Notably, this is phrased as a specific description of what he [Santiago] didn’t do, and it remains for Ms. M to restate it as a general rule about what you are allowed to do; that is, Sandra has representational agency to reason about Santiago’s work, but not about algorithms as mathematical processes.

While there is some limited evidence of student mathematical reasoning in this sequence, the most salient process of reasoning is the pedagogical chain of inference carried out by Ms. M as she improvises a correction of Santiago’s wrong answer. First, she uses the thumbs technique to determine that his mistake is not broadly shared by his peers; then, she uses Sandra’s thumbs down response to identify her as a likely source of a correct answer; she uses Sandra’s correction in 12–14 as material to be revoiced in 16; and the discourse marker so in 20 marks the final answer as the outcome of a process of reasoning. The sequence emerges through Ms. M’s taking up the characterization in each student response and thematizing it in a following initiation, that is, recasting each comment as a topic and eliciting a further comment on it. In this way, she orchestrates a correction that is jointly constructed between herself, Sandra, and the class at large. In terms of Goffman’s (1974) participant roles, while the animator role is distributed within the classroom community, Ms. M is in a sense the author, as she has a good idea what Sandra will say before she says it, and she also becomes the principal through the final evaluation move in 20–22, which endorses both the correct answer and a fortiori the process of reasoning that led to it.

4.3.2.2. Detail

In the video corpus, Ms. M shifted focus to a detail of the problem following nine out of 58 instances of the thumbs technique. Out of these nine instances, three involved Ms. M talking through the process of a solution that a student had written on the board, one comprised a single
follow-up question about a homework problem, and the remaining five were designed to voice and contradict common student misconceptions about a content topic. Example 16, which occurred later in the same lesson phase as Example 14 and Example 15, is of this latter type. As the transcript begins, the next problem on the warm-up sheet is to write the decimal 1.25 as a fraction.

Example 16: ‘I thought it was this’ (24 January 2014)

1  Ms. M  um  \[\text{point at Alberto}\]
2  Alberto  give me the next one.
3  Ms. M  \[\text{Give me a thumbs up if you agree with Alberto}\]
4  thumbs up if you agree
5  thumbs down if you don’t.
6  (1.9)  \[\text{thumbs down gesture, survey student responses}\]
7  class thumbs up held high by Santiago, Nayan, Alberto, Sandra, Ricardo, Fernando; held in front of body by Elena, Rita, Jamie
8  Ms. M  Really?
9  I thought it was this.  \[\text{(2.3)}\]
10  I thought \[\text{it was this}\]
11  Tiana  \[\text{still writing} 1\frac{25}{100} \text{ arrow} - 1\frac{25}{100}\]
12  Nayan  \[\text{BH raise and lower}\]
13  Fernando  \[\text{L((inaudible))}\]
14  Ms. M  O:::kay.
15  Ricardo  But if we-
16  both of-
17  both ways are right.\]
18  Fernando  \[\text{L((inaudible))}\]
19  Ms. M  This is-
20  more right
21  \[\text{circle} 1\frac{1}{4}\]
22  \[\text{circle} 1\frac{25}{100}\]
23  if you're
24  \[\text{when we're testing you and stuff}\]
25  \[\text{turn to Ricardo/Fernando, LH palm up rotating wrist}\]
This sequence begins following the same pattern as the no follow-up sequence in Example 14: a question is posed, a student gives a correct answer, Ms. M initiates the thumbs technique, and the students display thumbs up. At this point, the habitual next move is for Ms. M to use a sequence-closing third such as “All right,” but here, beginning in line 8, she opts instead to initiate an extended post-expansion, which starts with an expression of disbelief. Next, in lines 9–10, she articulates a misconception of the problem: that the proper decimal equivalent of 1.25 is actually $1 \frac{25}{100}$ rather than its reduced form, $1 \frac{1}{4}$. There are several potential reasons for her to have raised this possibility: the solution does stop here for some other decimals such as 1.27; Ms. M occasionally insisted that numbers like 1.25 be read aloud “like a decimal” as one and twenty-five hundredths rather than one point two five; and as several students soon point out, it’s not technically incorrect so much as incomplete. Whatever the reason, Ms. M not only proposes an alternative response, but also aligns with it (*I thought it was this*) in a way that positions her in opposition to the class consensus around Alberto’s answer.

If this were a bona fide correction, then the sequence to this point would comprise an incorrect answer followed by two post-expansions: an enthusiastic thumbs up from the class and a teacher-provided correction. This hypothetical sequence would be highly marked, given that neither of these post-expansion types is attested following an incorrect answer in the video corpus, so the students can tell that Ms. M’s apparent correction is not legitimate, and they immediately contest it in lines 11–13. Ms. M has used the preference structure of the thumbs technique to unambiguously activate the students’ voicing-of-misconceptions schema, avoiding the uncertainty that Dr. C faced in Example 9 (*Does this look fishy to you?*). While her statement in 9–10 is not clearly phrased as a first pair part of a post-expansion, the students’ knowledge of the thumbs technique allows them to recognize it as an initiation move, and to provide responses to it. Her
evaluation comes in the form of a sequence-closing third in line 14, confirming that the thumbs-up consensus was accurate and that Alberto’s initial response was correct.

The SCT in line 14 comes in the middle of a highly animated stretch of interaction. Beginning in line 10, while Ms. M is writing the alternative solution on the board, a number of students attempt to jump in and explain that this solution is equivalent to the one previously voiced by Alberto, and this brief period of overlap continues through the end of Fernando’s response in 18. Perhaps, by voicing a solution that is not meant to be considered as authoritative, Ms. M has suspended not only her typical participant role as the animator of mathematical truth, but also the classroom interactional norm according to which a student’s bid to speak must be ratified by the teacher; whatever the reason, the video recording shows not only Fernando but also Ricardo, Nayan, and possibly Tiana all giving responses more or less simultaneously. Okay in in line 14 does not actually accomplish the closing of the sequence, as Ricardo and Fernando do not cede the floor until they have completed their responses. Instead, Ms. M must wait until they have finished before closing the sequence with an authoritative explanation in lines 19–24.

As we saw previously in Example 15, when Ms. M asks the students to justify their thumbs-up or thumbs-down response, this grants them representational agency by providing an opportunity to reason with mathematical representations. The students justify their thumbs-up response on the grounds that \(1 \frac{1}{4}\) and \(1 \frac{25}{100}\) are “the same” (Nayan, line 12) and therefore “both ways are right” (Ricardo, line 17). This reasoning is accurate and therefore offers Ms. M another opportunity to end the sequence with a minimal SCT, but instead she articulates a lengthier explanation. This move in lines 19–24 fulfills a number of functions. First, it justifies the use of the detail sequence structure rather than the no-follow-up structure to raise the point that even though “both ways are right,” \(1 \frac{1}{4}\) is “more right” (line 20); we can infer from this that some students have been giving unreduced fractions as answers, and Ms. M wants them to understand why those answers might be marked wrong even though they are not mathematically inaccurate. This
distinction can be viewed as a kind of representational preference, and the fact that Ms. M considers it to be worth a minute of class time reinforces the “one right answer” aspect of “traditional” practice: even if there are multiple correct answers, students must choose one as being more right than the others. In line 23, Ms. M even refers to when we’re testing you as a speech context in which students would be sanctioned for choosing the dispreferred representation. This is a question of residential agency: even though the sign $1 \frac{25}{100}$ successfully composes a sign-object relationship, a student who writes $1 \frac{25}{100}$ cannot commit to the interpretant, that is, they cannot expect their answer to be marked correct.

Unlike the correction sequence in Example 15, this sequence includes only one post-expansion following the thumbs-up display: Ms. M’s initiation in lines 8–10, a number of student responses in 11–18, and an evaluation in 19–24. Ms. M’s evaluation of student reasoning in this example makes it clear that student agency is restricted, in a way that is best understood in terms of residential agency. When Nayan and Ricardo articulate their reasoning, they control the sign by successfully participating in class, and they compose a sign-object relation by making their reasoning clear. Their ability to commit to an interpretant is limited, however, as we see through Ms. M’s evaluation move treating their reasoning as flawed or incomplete. By reserving this level of agency for herself, Ms. M provides a reminder of her own privileged epistemic status. While students may sometimes reason with mathematical representations, not only is their reasoning restricted to certain topics and certain sequential contexts, but it is always provisional until ratified (as in Example 15) or amended (as in this case) by the teacher.

By looking at common patterns of realization of the thumbs technique, I have identified an underlying ideology of mathematical authority: any student utterance is considered “correct” only to the extent that the teacher treats it that way. While this instructional technique is particular to Ms. M’s class, the ideology holds widely in mathematics education. Turning to the calculus class, the next section considers a different discursive practice, but reveals a similar allocation of authority.
4.4. **STUDENT-INITIATED REPAIR SEQUENCES IN THE CALCULUS CLASS**

In Dr. C’s work with self-identified weak math students, one key component of his strategy was to put them at their ease so that they would feel comfortable to speak up and participate rather than sit in intimidated silence. I came to notice during field work that students sometimes felt so comfortable that they actually attempted to correct things that he had said during lecture. I first became interested in this practice in the hope that it would help me to identify difficulties in multimodal semiosis; for example, the spoken phrase $x$ plus one over $x$ would refer ambiguously to either of the expressions $x + \frac{1}{x}$ or $\frac{x+1}{x}$, and it seemed likely to me that repair might be used to clear up this sort of confusion. On looking at the video record, however, I saw no examples of this type, but instead, I found that these repair sequences provided insight into the use of “traditional” teaching practices. If students can correct their instructor, what does this mean for his institutional status as the arbiter of mathematical claims and unquestioned local authority of content knowledge?

In thirteen recorded class sessions totaling approximately twenty hours of video, I identified eleven instances in which students initiated repair sequences. A few cases were ambiguous—if the answer is one thirtieth and a student says I thought it was one third, is he admitting that he was wrong or insisting that he was right?—and Heritage’s (2012; 2013) theory of action ascription suggests that these would not be classified as repair initiations, due to the student’s lower epistemic status. While it seems like circular reasoning to make this kind of assumption, in fact these ambiguous repair initiations tended not to give rise to clear repair sequences, and for this reason, they were excluded from the analysis. We should keep in mind, however, that if the student had been correct and the answer had in fact been one thirtieth, then a repair sequence would likely have ensued. As a result, this methodological decision would tend to lead to the overrepresentation of cases in which the professor errs or misspeaks, although the
difference between an error, an imprecision, and a valid alternative is to some degree co-
constructed.

As the review of the conversation analysis literature showed (e.g. Schegloff 2007), repair
initiations typically occur in first pair parts of insert or post-expansion sequences. In the student
repair initiations in the calculus class recordings, the professor’s following second pair part tends to
accomplish one of three distinct actions, each of which accounts for three out of eleven instances of
repair. Having heard the student’s proposed correction, he may accept it, reject it, or reframe it as
something he had planned to say in future talk. The remaining two cases do not fit as well into the
typology, because in these instances, it is not clear that the professor has understood the proposed
correction. This section will present a microanalysis of one example each of accepting, rejecting,
and reframing, and of one sequence that did not fit into the typology. By bringing the theory of
representational agency into the interpretation of these examples, the analysis will show the degree
to which students actually acquire semiotic agency over mathematical representations as they
attempt to identify errors in the professor’s talk, thereby taking up an ostensibly agentive stance.

4.4.1. ACCEPTING

The first strategy for responding to student corrections is to accept the correction and
incorporate it into the stream of the lecture. This occurred in three out of eleven cases, but two of
these three were for corrections unrelated to the material of the lesson: one happened when the
professor had calculated a homework grade incorrectly, and another when he was assigning a
problem about a conical solid but wrote canonical on the blackboard. In Example 17, we will
consider the only example in the corpus in which Dr. C accepts a correction from a student about a
point of mathematical content. This occurred early in the semester, during a lesson on different
techniques for finding the derivative of a function. The correction locates a trouble source on the
blackboard, reproduced here as Figure 18.
Dr. C has made an error here. On the third line of Figure 18, he writes:

\[ f(x) = \cdots = x^{1/2} - x^{9/2} \]

He has represented the function as \( x \) to the one-half power, minus \( x \) to the nine-halves power. In the next stage of the solution, when he differentiates the function, the minus sign should be carried forward. Instead, however, he writes:

\[ f'(x) = \frac{1}{2} x^{-1/2} + \frac{9}{2} x^{7/2} \]

The plus sign should have been a minus sign, and Brad points this out.

**Example 17: 'It should be minus' (22 January 2014)**

1. **Dr. C** | Question.  
   | RH gesture toward the back row
2. **Brad** | Is it minus or plus.
3. **Mickey** | Yeah.
4. | (Shouldn’t we make)?
5. **Dr. C** | Oh it should be minus °I’m sorry°
6. | Erases minus sign, writes plus sign

Brad is off camera at this point, but by the way Dr. C calls on him in line 1, we can presume that he was bidding for the floor. In line 2, Brad initiates the repair sequence by asking “Is it minus or plus,” a candidate question in which the ordering of constituents is marked. Typically, this three-word phrase would be rendered “plus or minus” for both semantic and prosodic reasons: it places the “positive” term first, and it maintains a trochaic rhythm. Moreover, in her examination of the
use of candidate questions in other-initiated repair, Koshik (2005) identifies error correction among their functions, but in her data “one alternative, the second one, [is offered] as a candidate correction of an utterance targeted in the first alternative” (Koshik 2005, 203). Brad’s repair initiation does not follow the pattern identified by Koshik—he voices the candidate correction first, followed by the trouble source—and therefore, it is marked in this pragmatic respect as well.

So how can we account for Brad’s “minus or plus”? Aside from his marked ordering of candidates, line 2 is the same form that might be used if the chalk were smudged or Brad were otherwise having trouble making out Dr. C’s writing, which classifies this utterance as a deferential off-record strategy for performing a face-threatening act: “there is more than one unambiguously attributable intention so that the actor cannot be held to have committed himself to one particular intent” (Brown and Levinson 1987, 69). In this ambiguous utterance, the fact that Brad has chosen a word order that is prosodically, semantically, and pragmatically marked can be seen as an indication of its marked communicative function (Sicoli et al. 2015). In an additional show of deference, Brad locates the trouble source not with Dr. C personally, but in the depersonalized form of the blackboard text; he says, “Is it minus or plus,” not “Did you mean minus or plus.” In his response in line 5, Dr. C aligns with this depersonalization—“it should be minus”—but also apologizes, indirectly accepting responsibility for the error.

The epistemics of this brief exchange falls in line with the teacher-centered understanding of content knowledge and classroom authority. Within the classroom setting, Dr. C is positioned as the locus of mathematical truth, despite the fact that he is incorrect and Brad is correct; the function is to repair an error, but the form is that of a clarification question. Even though Dr. C is the authority within the classroom community, however, the mathematical knowledge itself is construed to reside in the blackboard inscriptions, which, despite being produced by Dr. C, are then depersonalized and treated as independent phenomena. What is at issue is Dr. C’s control of the mathematical notation, as a semiotic resource; Brad wishes to question his control in this one
specific instance, but to do so, he disguises this action pragmatically (through off-record realization of the face-threatening act), referentially (by alienating the inscription from the person who produced it), and interactionally (by initiating a repair for Dr. C to complete, rather than doing it himself).

4.4.2. **Rejecting**

The next strategy commonly used by Dr. C in response to student-initiated repair is to reject the proposed correction and insist that he was right. This type of response occurred in response to three out of eleven student initiations of repair; Example 18 provides a typical example. In this day's lesson, the professor is demonstrating a technique for graphing a function by first identifying its critical points and inflection points, which are places where the curve changes shape in salient ways. If a point is a critical point, then the first derivative of the function is equal to zero at that point, but the converse is not necessarily true, so the procedure for identifying critical points is to first identify all the points at which the first derivative equals zero, and then check each of those candidates to verify whether it is in fact a critical point. As Example 18 begins, Dr. C has already completed this procedure, and is going over the resulting graph on the blackboard to highlight all of the critical points and inflection points.

**Example 18: ‘That’s not a critical point’ (26 March 2014)**

1 Dr. C and here's a critical point,
2 (3.1)
3 labeling the graph
4 and here's another: uh critical point.
5 (2.8)
6 NRP ((omitted))
7 Mickey → Nuh- that's not a critical point,
8 Dr. C → Oh it is
9 Mickey → ts a poTEnTial,
10 right?
11 Dr. C → No it's actually a critical point,
12 → it's not an inflection point.
13 Mickey → Oh inflection,
14 sorry, r°sorry°.
Dr. C

14  Dr. C

4But it is a critical point.

Unlike Brad in Example 17, Mickey initiates the repair sequence with no show of deference, contradicting the professor in line 6. Dr. C’s response in 7 is oh-prefaced, which, as Heritage (1998) shows, indicates that the speaker is responding to something he found problematic; then, his affirmation “it is” insists not only on the correctness of his initial statement, but also accomplishes an epistemic “claim to primacy” (Stivers 2005) over the relevant mathematical content. In lines 8–9, Mickey initiates his correction a second time, proposing a candidate repair item: the point under discussion has not been confirmed as a critical point, but so far is only “a potential.” This claim echoes much of the lecture that occurred earlier that day, in which Dr. C stressed the importance of verifying each potential critical point, and several examples were provided of functions with points where the first derivative was equal to zero, but that turned out not to be critical points. Mickey is still orienting to this distinction between critical points and potential critical points, and he seems to miss the retrospective character of this part of the lecture, in which these issues have already been resolved.

When Dr. C once again rejects Mickey’s correction in lines 10–11, there seems to be a gap in intersubjectivity. He does not orient to the distinction between potential and confirmed critical points that Mickey had referenced, but rather to the distinction between critical points and inflection points. This response presupposes a different misconception of the material than the one Mickey had voiced in his proposed repair term. Inflection points are identified through a different process from the one outlined above for critical points; while Mickey’s re-initiation in line 8 suggests that he is oriented to the wrong step in the procedure, Dr. C’s response in 11 would indicate either that Mickey is oriented to the wrong procedure altogether, or that he has simply mixed up the terminology.

Next, in the third-turn uptake position, Mickey does not recognize that the professor has misconstrued his point, but instead in line 12 aligns with the professor’s framing of critical vs.
inflection point, and continues in 13 with an apology. It’s difficult to know what to make of this move. He may not notice that the professor’s response does not specifically take up his claim that the demonstration has skipped a step, or if he does notice, he may decide it isn’t worth pursuing further, or he may simply give the preferred response of agreement without evaluating the content of Dr. C’s talk. In any case, the exigencies of the interactional moment have led Mickey to abandon his attempted correction, and it seems fair to say that he was prepared to back down when faced with any resistance from Dr. C, especially when compared with what we saw in Example 17, where Dr. C showed no resistance, but immediately incorporated the repair into the flow of the lecture. The ellipsis in line 8 (“potential critical point”—is that really “not a critical point,” as stated in line 6?) and hedge in line 9 (“right?”) point to a lack of confidence on Mickey’s part, which reaches its climax with his half-whispered “sorry” at the end of 13.

While I would hesitate to make essentializing claims about “Mickey’s mathematics ideology” on the basis of this one example, his participation in this strip of talk is consistent with a lack of agency with regard to mathematical communication. With respect to representational agency, we see that Dr. C in line 3 thematizes a point on the curve, and in line 6, Mickey characterizes that point as “not a critical point.” In line 8, he attempts to reason with this proposition by providing evidence for it, but he ultimately cedes this agency to Dr. C even though Dr. C’s reasoning does not really respond to his own. Notably, Mickey considers himself to be a good mathematics student who stated in an interview that he “always found math to be [his] strong suit,” but even so, he abandons his attempt at mathematical reasoning in the face of his professor’s expression of authority. Mickey and Dr. C collude to locate mathematical authority and representational agency firmly in the person of the professor.

4.4.3. REFRAMING

In an additional three of the eleven student-initiated repair episodes, Dr. C neither accepts nor rejects students’ corrections, but reframes them as anticipations of upcoming material; one of
these is reproduced below as Example 19. This sequence occurred earlier in the same class session as Example 18, while the professor was outlining the different kind of curve shapes that are used in graphing. As the transcript begins, Dr. C has just wrapped up a discussion of concavity, which concluded by providing formal definitions of \textit{concave up} and \textit{concave down}, shown here in Figure 19.

\textbf{Def.} Let $f$ be differentiable on interval $(a, b)$

1. If $f'$ is increasing on $(a, b)$ then $f$ is \textit{concave up} on $(a, b)$ \hspace{1cm} ← tangent lines are below graph

2. If $f'$ is decreasing then $f$ is \textit{concave down} on $(a, b)$ \hspace{1cm} ← tangent lines above graph

\textit{Figure 19: Concave up, concave down}

According to the definition, the concavity of a function $f$ is determined by whether its first derivative $f'$ is increasing or decreasing. As Example 19 begins, Corey questions this definition.

\textit{Example 19: ‘Isn’t it the second derivative?’ (26 March 2014)}

1. Corey \hspace{0.5cm} Both of 'em say, <the derivative>, is it

2. \hspace{0.5cm} → s- isn't it the second derivative?

3. Dr. C \hspace{0.5cm} °That's a good point.°

4. (2.1)

5. \hspace{0.5cm} So.

6. \hspace{0.5cm} I haven't said anything about the second derivative yet.

7. But we used the first derivative

8. to define what it means to concave up,

9. and to concave down.

10. \hspace{0.5cm} → And we'll talk about

11. \hspace{0.5cm} → what second derivative has to do with this in a second.
So but let's think about that I guess Corey,

if your function is increasing
then what does that tell you about the derivative of the function is it positive or negative.

Corey If the function is increasing?

Then the, derivative would be positive,

Dr. C The derivative would be positive.

"Okay."

S:o. The derivative itself is a function, right?

And if the derivative is increasing,

then what do you know about the sign of the second derivative.

Corey Then, that would be increasing as well?

or that would be >positive as well<

Dr. C It would be positive, right.

So, uh if the second derivative is positive, then:

u:h the graph is concave up.

So you've just, uh proved,

|shrug shoulders

RH "finger quotes"

quote unquote,

the n- the next uh

kinda theorem that I'm gonna write.

Corey's repair initiation in line 2 is more indirectly phrased than Mickey's in Example 18. Rather than contradicting Dr. C directly, Corey depersonalizes the trouble source, locating it on the blackboard rather than in Dr. C's spoken utterances, and he phrases it as an interrogative ("Isn't it...?") rather than a declarative ("That's not ... "). As a negative polarity question, though, Corey's initiation is best read as a suggested repair of Dr. C's earlier utterance rather than a neutral request for information; he is proposing that while Dr. C wrote $f'$, he had really intended to write $f''$. Like Brad in Example 17, Corey shows deference to Dr. C by phrasing his repair in the form of a question.
In his response, Dr. C begins by validating Corey’s question as “a good point,” but continues in lines 6–9 with what looks like a rejection. The option is available to Dr. C to reject Corey’s correction at this point, as he does with Mickey’s in Example 18; one way of understanding Corey’s suggestion is that he is simply mistaken, as concavity has been defined in terms of the first derivative, not the second. In 10–11, however, Dr. C deviates from this strategy, casting Corey’s proposed “second derivative” as something that will become relevant “in a second.” And then, in 12, the discourse markers “so but” mark a footing shift as he decides to immediately take the opportunity to talk about the second derivative. Two IRE sequences in 14–21 and 23–30 explain the relevance of the second derivative while also keeping Corey engaged, and in lines 32–34, Dr. C voices his conclusion; these three discursive units are bounded by the discourse marker “so” in lines 13, 22, 31, and 35, marking each element as a successive “warranted inference” (Schiffrin 1987) in a chain of logical reasoning.

The point that Dr. C reaches in lines 32–34 could be viewed as a correction of Corey’s proposed repair term. That is, the definition of concavity should not be phrased in terms of the second derivative, because the definition talks about the derivative increasing or decreasing, which corresponds to the second derivative being positive or negative; this fact is not technically a part of the definition, but is rather a corollary of it. Instead of framing it as a contradiction of Corey, however, Dr. C’s retrospective evaluation in lines 35–40 positions Corey as the source of mathematical knowledge; in Goffman’s (1974) terms, he is the principal of the mathematical ideas that Dr. C has just animated. As he accomplishes this positioning, however, Dr. C’s talk is marked by numerous indicators of uncertainty: hesitations in lines 36 and 39, a significant pause in 37, hedges “quote unquote” in 38 and “kinda” in 40, a shrug of the shoulders, and a gestural icon of quotation marks. On one level, these displays of uncertainty mitigate Dr. C’s imprecise use of terminology; in mathematics, “prove” and “theorem” have specific technical meanings that do not apply to the preceding strip of talk. At the same time, the attribution of these mathematical ideas to Corey is also
imprecise. In terms of representational and residential agency, Corey’s initiation move does characterize concavity as being somehow related to the second derivative, but he cannot commit to the interpretant of this utterance, depending instead on Dr. C to validate it. In the resulting exchange, Corey reasons with properties of the first and second derivative, but only after these properties have been thematized by Dr. C. As a learner, Corey’s participation in mathematical reasoning is peripheral, and while he does display some agency through his classroom participation, this only extends as far as the professor will allow it, not only on the residential level of interaction—he only speaks when called upon—but also on the representational level of reasoning.

4.4.4. Deviation from the Pattern

To this point, we have considered three common strategies used by Dr. C for responding to student-initiated repairs: accepting, rejecting, and reframing. In Example 20, in contrast, Dr. C never expresses a clear enough understanding of the proposed correction to even evaluate it as right or wrong. This strip of interaction occurred as Dr. C was walking through a sequence of algebraic operations, factoring a polynomial as part of a demonstration of a technique for solving a problem. As the transcript begins, the blackboard appears as in Figure 20, and Sara proposes that Dr. C may have made a mistake.

*Figure 20: x on the left side*

*Example 20: ‘Corey’s head was just in the way’ (5 March 2014)*

1 Dr. C And, I can also factor out “small interval” under + sign tap-tap-tap on (x-1)^3
2 point back and forth between two (x-1) terms
3 Sara → Wait shouldn't that be x
In line 3, Sara uses a reverse polarity question, similar to Corey in Example 19, to initiate a repair sequence, although unlike Corey, she breaks into the middle of Dr. C’s utterance at a point where he has paused to gesturally highlight a bit of notation on the board. His response in line 5 is markedly different from what we saw in earlier examples, perhaps due in part to Sara’s having misconstrued his pause as a transition relevance place: rather than characterizing her repair attempt as “a good question” as in Example 19, or even saying “Well yeah, it SAYS $x$” along the lines of the rejection move in Example 18, he asks Sara to rerun her sequence initiation. At the same time, he changes the framing of where the trouble source is located. While Sara in lines 3–4 identifies a problem with “that … on the left side” of an equation written on the blackboard, Dr. C responds in 5–6 by asking her to repeat or explain what he did wrong; he positions her as having negatively evaluated him personally, a weightier face threat than identifying a depersonalized mistake on the blackboard. When Sara reruns her repair initiation in lines 8–9, she takes up this shift in framing, saying “you left an $x$ squared” rather than “it says $x$ squared.”
In line 10, when Dr. C responds to this second repair attempt, the status of his utterance *I left an x here* is uncertain. One interpretation is to view it as a negative response to Sara's candidate repair item: “Should it be x?” “Well, yes, that’s what I wrote.” Alternatively, Dr. C may still be attempting to achieve intersubjectivity with Sara. She first identifies a trouble source with a deictic *that* whose referent is uncertain; while Dr. C’s proximity to the board allows him to disambiguate this kind of talk through indexical gestures, Sara’s movement of her finger in line 7 is too vague for an indexical referent to be retrieved. When she reruns the repair in 8, the trouble source “an x squared” is represented through symbolic rather than indexical reference, so its object must be inferred from the deictic field in which Dr. C and Sara have coordinated their joint attention. As it occurs, however, there are several instances of x and x^2 in the relevant blackboard space, so Sara’s reference is still ambiguous. In this reading, Dr. C’s utterance in 10 is an attempt to “rewind” to an earlier point in his demonstration so that he can reestablish intersubjectivity with Sara and continue from there. If this is the case, then Sara has not reached the point of attempting to make mathematical claims; rather, she has experienced a breakdown of her residential agency in which she can control both the pointing gesture and the verbal sign “an x squared,” but does not manage to compose a sign-object relationship with either of these.

Following Dr. C’s response to Sara’s second repair attempt, her subsequent uptake move in lines 11–12 resembles Mickey’s in Example 18: when faced with the least indication of resistance from the professor, she abandons her repair attempt and begins to apologize. This move is consistent with either reading of line 8: either she is acknowledging Dr. C’s rejection of her repair attempt, or she has repaired her joint attention with Dr. C and realized that her repair attempt was mistaken. In either case, she retracts the correction by first claiming a lack of understanding in line 11 to account for the repair attempt’s having occurred, and then in 12 switching to a claim that she does understand: *OK I got it*. Sara is speaking quickly and it’s possible that she says “I didn’t understand” rather than “I don’t understand” in line 11, but either way, the transition between 11
and 12 displays a rapid epistemic shift. The combined effect is to make further discussion of this topic unnecessary—she didn’t understand before, but now she does—so it functions as a bid to close the repair sequence.

Although Sara may be satisfied at this point, Dr. C is not. In line 13, rather than taking up her bid to close, his utterance “And you’re OK?” initiates a post-expansion in which he pushes her to verify her understanding. She begins to reply in line 14, but he does not wait for her to complete her response, rerunning his understanding check in 15–16 in a more restricted formulation: “Do you understand what I’m doing, or do you not.” While Dr. C’s “and you're OK?” in 13 is designed to prefer an affirmative response, the rerun in 15–16 not only paraphrases “being OK” more specifically as “understanding,” but also offers Sara the option “or do you not,” mitigating the preference structure of his earlier utterance and presupposing a greater likelihood that Sara has not in fact understood, despite her prior claim of understanding in line 12. Sara’s response to this in line 17, “Corey’s head was just in the way,” not only minimizes her moral accountability for the error, placing the blame instead on Corey for blocking her view; it also minimizes Sara’s semiotic agency. While the breakdown in lines 8–9 was construed at the level of composition, Sara’s account in 17 claims an inability to even perceive the sign in question, that is, a failure at the level of control of the sign.

4.5. **Agency, Received Knowledge, and the Traditional Classroom Ideology**

This chapter is primarily concerned with students’ conception of mathematical knowledge, and the roles that are constructed for them as they go about acquiring it. We began with Christopher Danielson’s (2014) shapes book, in which truth is constructed between children and their caregivers in the course of talking about geometric images, and compared it with Example 9, in which the exchange *Do you guys buy this?—Is it right?* positions the professor as the authoritative source of mathematical truth. This difference can be described as a contrast between two alternative ways of knowing (Boaler and Greeno 2000): *connected knowing*, in which knowledge
emerges from interaction, and *received knowing*, in which knowledge is provided by an authoritative source and acquired by a learner. In practice, students’ “ways of participating are adaptations to the constraints and affordances of the environment” (Boaler and Greeno 2000, 172–173); a class that stresses interaction will lead students to become “connected knowers,” while students who independently work through textbook problems tend to become “received knowers.”

The level of interaction in the two classes considered here falls somewhere in between the “didactic” and “discussion-based” AP Calculus classes studied by Boaler and Greeno. While the students I observed did not typically “discuss the different questions and consider the meaning of possible solutions with each other” (Boaler and Greeno 2000, 181), they did more than just work through pages of problems and check their answers against an official answer key. Although I did observe this practice in both settings, the middle school students were also often asked to come to consensus with a partner, and the calculus students, like those in Boaler and Greeno’s discussion-based classes, “described their involvement [in class] in terms of community participation and family relationships” (Boaler and Greeno 2000, 189). How, then, can we account for the observation that both Ms. M’s and Dr. C’s students seem to position themselves as received knowers?

A consideration of the agency granted to students may indicate their conception of their own epistemic status, in line with Boaler and Greeno’s finding that “In the discussion-based classrooms students were, quite simply, given more agency” (Boaler and Greeno 2000, 189). Considering the allocation of representational agency in both Ms. M’s use of the thumbs technique and Dr. C’s response to student repair initiations, we saw that students do not *thematize* mathematical objects, but only *characterize* objects and processes previously thematized by their instructor, suggesting that one way to increase student agency would be to give them freedom to choose which mathematical objects are used to solve a problem. Students do have the ability to *reason* with mathematical representations, but this reasoning is restricted to certain sequential contexts, and must always be validated by the instructor. Importantly, this validation is not merely
contingent on whether the student’s reasoning is correct; on the contrary, we saw students’ reasoning sometimes lead to mathematically correct statements that the instructor nevertheless declined to validate. Students accurately claimed in Example 16 that $\frac{13}{4}$ and $1 \frac{25}{100}$ are equivalent, and in Example 19 that the second derivative of a function indicates its concavity, but neither of these statements corresponded to the authoritative “right answer” with which the instructor ultimately concluded the sequence. As a result, both claims were treated by the instructor as being, not incorrect, but insufficient.

To understand this observation more thoroughly, recall that the processes of thematizing, characterizing, and reasoning reflect different aspects of representational agency, as viewed through Peirce’s phenomenological categories of first, second, and third. If students can characterize, but not thematize, and only reason in a limited way, this means that their agency over mathematical objects exists not at the level of firstness, but only as a secondness, and to a lesser extent as a thirdness. In other words, they have the discursive capacity, or the epistemic rights, to put mathematical objects in relation with one another, but not to get at the essence of the objects themselves; at the same time, these relations remain largely unsystematic, only rarely solidifying into more general principles. This is the level of agency that allows students to work problems quickly and accurately, relating a question to its intended answer, without ever reaching a deeper understanding of the mathematical objects themselves, or of the ways they may be used to represent natural phenomena.

While Boaler and Greeno (2000) cast this lack of student agency as a consequence of a didactic rather than discussion-based pedagogical approach, the present analysis suggests that what makes the difference is not practice, but ideology. This is not to deny that cultural beliefs are implicit in social practices and technologies; Danielson’s shapes book and a textbook answer key clearly reflect different ideas about how mathematics learning happens. Nevertheless, when students position themselves as received knowers, they are responding to the way they have been
positioned by other participants and aspects of the environment, as positioning functions to assign
roles within a “storyline” (Van Langenhove and Harré 1999) through which they understand the
mathematics classroom and their role in it. A practice like the thumbs technique is ambiguous in
this regard. In Ms. M’s use of it, her way of eliciting student corrections makes space for students to
reason with mathematical representations, so it is more than just “checking in” (Lanier 2015), but
in principle, a thumbs-up / thumbs-down display could also be used to prompt students to defend
and justify alternative solutions or methods for a given problem, or to evaluate one another’s
reasoning rather than just their final answer. Progressive mathematics educators such as Boaler
have come to view open-ended interaction as a matter of educational equity, a way to involve
students who feel disempowered in traditional classes, and there is experimental evidence as well
suggesting that an emphasis on received knowing contributes to the gender bias in mathematics
(Schommer-Aikins and Easter 2006). To enact this reform, however, would require teachers to
change their way of thinking about the practice of school mathematics. The sort of discussion that
supports connected knowing would only be possible in a classroom environment where more than
one answer may be correct, and where correct answers may come from sources other than the
instructor, because otherwise, like Wendell, the students will surrender their agency and simply
ask, is it right?
Chapter 5

THE USES OF IDENTITY IN THE MATHEMATICS CLASSROOM

The episode I recounted in Chapter 1 first caught my attention because of Alberto’s “body language,” the way he carries himself as he walks to the board and writes up his answer to the problem. My first instinct was to view this embodied performance as conveying something about Alberto’s identity: that he struggled in school, was perhaps a bit of a “class clown,” but certainly not a “good student.” This association, based on my decades of experience as a student and teacher, in this case steered me wrong. If Alberto lacked confidence, it was not in his math ability, but in his English language proficiency; perhaps what seemed to me to index “not a math person” is better interpreted as indexing Alberto’s self-concept as “not an English speaker.” This observation motivated the question, How is the “good math student” identity performed, and how does it relate to other aspects of students’ identities, as they are expressed in the classroom? In this chapter, I take up that question through an analysis of interview responses as well as classroom interaction. I present a description of what “good math students” do in the middle school class, how ethnicity becomes relevant in the calculus class, and how students who identify in diverse ways try to create roles for themselves in the classroom community.

5.1. LOCATING IDENTITY BETWEEN STRUCTURE AND INTERACTION

In the conclusion of Talk and Social Theory, Frederick Erickson (2004b) highlights a difficult question facing social theorists: how does social change happen? The sociological theories of Durkheim and Parsons show that social actors are not autonomous individuals, but are socialized or enculturated to carry out specific roles in preexisting social structures; at the same time, Garfinkel and Goffman describe ways in which social actors co-create interaction and even recreate structure itself through moment-to-moment improvisational choices. Considering these theoretical traditions as addressing distinct levels of analysis, it is relatively easy to see the influence of
structure on interaction; in Erickson’s examples, because of the mid-1970s economic recession, the price of household goods becomes available as a dinnertime conversation topic, and because of the Vietnam War, a community college student and his advisor conspire to delay the student’s graduation and thereby keep him ineligible for the draft. It is much more difficult, from a methodological perspective, to turn the arrow of causality the other way—if we wanted to document how public opinion changed and the war ended, how many dinnertime conversations would we have needed to observe?—but everyday instances of situated interaction must have some influence on social structure, because this is the only possible way for the structure to change over time.

To address Erickson’s question and attempt to make this influence visible, it seems that an intermediate level of magnification is called for between the micro-level of interaction and the macro-level of structure. One potential candidate is Pierre Bourdieu’s (1977; 1991) notion of *habitus*, defined as “structured structures predisposed to function as structuring structures, that is, as principles which generate and organize practices and representations” (Bourdieu 1990, cit. in Grenfell 2012, 66). Because of the simultaneously backward- and forward-looking nature of the *habitus*—it obtains structure through exposure to existing practices, but at the same time provides structure to generate new actions—there is a potential here to connect the micro and macro scales of interaction and structure. However, Bourdieu’s theory is often critiqued on the grounds that it is too deterministic, modeling situated social action as a relatively straightforward consequence of the actor’s *habitus*, which in turn follows from their primary socialization according to social class membership; linguistic creativity and other forms of social agency are difficult to explain in Bourdieu’s terms (Collins 1993; Erickson 2004b).

Another theoretical construct with a similar structured-and-structuring nature is the concept of identity. In sociocultural linguistics, identity has been theorized not only as membership in a macrosociological category such as gender or ethnicity (e.g. Labov 1966) or as “an enduring
sense of self” (Bamberg 2011, 100) that an individual carries across interactional events, but also “as the emergent product rather than the pre-existing source of linguistic and other semiotic practices” (Bucholtz and Hall 2005, 588). That is, identity is viewed as a sort of performance that is built up from particular interactional choices; this is Butler’s (1990) insight regarding gender, which resonates with Goffman’s (1959) more general treatment of the presentation of self. At the same time, identities are built out of pre-existing semiotic materials; as Agha (2005) writes, drawing on Bakhtin’s (1981) concept of dialogicality, individual identity performances become recognizable because they incorporate and recombine identifiable voices in a dialogic fashion. Motivating and resulting from these performances is the fact that they provide or restrict access to structurally defined social roles. For example, mathematics student identities normatively index whiteness and maleness, and so coexist uneasily with Black ethnicity (Stinson 2013) or feminine gender (Solomon 2012). The relationship between moment-to-moment performative choices, perceptions of gender and ethnicity, and expectations of mathematical competence can be understood as orders of indexicality (Silverstein 2003) which, in an institutional setting, give rise to semiotic processes of authorization and illegitimation (Bucholtz and Hall 2005, 603ff) through which access is granted or denied. Giddens wrote that social structure “is always both constraining and enabling” (Giddens 1984, 25), and because social structures are implicated in performances of identity, this observation is true of identity as well; any performative choice forecloses some social roles and actions even as it provides access to others.

If we think of identity as a way social actors categorize themselves and others, perhaps the most obvious set of labeling practices attends to macrosociological categories such as gender and ethnicity just mentioned. Looking deeper, we see that this is only one source of identity designations; “identities [also] encompass … local, ethnographically specific cultural positions; and … temporary and interactionally specific stances and participant roles” (Bucholtz and Hall 2005, 592), all of which “emerge in interaction through several related indexical processes [including]
displayed evaluative and epistemic orientations to ongoing talk” (Bucholtz and Hall 2005, 594). In fact, Kiesling (2009) argues that this evaluative and epistemic function, which he analyzes under the rubric of stance, is in fact the foundation of all sociolinguistic style; and following Tannen’s (2009) synthesis of Bateson, Becker, and Bakhtin, I understand style to mean the taking on of enregistered voices in situated interaction. In this formulation, the choice of a style from among the options available in a speaker’s repertoire is essentially motivated by the affordances that the style provides in a particular discursive moment for expressing approval or disapproval, certainty or uncertainty. From this, it follows that the various types of identity performance, from those involving footing shifts and participant roles (Goffman 1981) up to situationally relevant categories, institutional designations, and even more transportable, higher-order indexes of geography, race, class, and gender (Eckert 2008), are all rooted in stance. For example, “speaking as a teacher” is a way of taking a highly certain epistemic stance toward academic content knowledge, as well as a privileged epistemic status relative to those positioned as students (as discussed in the preceding chapter). Kiesling argues that this connection between stance and style persists even at the macrosociological scale:

First, personal style is often described by people (or novelists) in terms of their habitual stances: “she’s very full of herself,” “he’s very touchy-feely,” and so on. Second, we find that what tends to differentiate census-like groups—in the discourses of the society that defines them, real or imagined—are the stances they habitually take. This is especially true with linguistic differences that are found between men and women (e.g., “men are confrontational,” “women are servile”). (Kiesling 2009, 175)

Considering Kiesling’s description of this style/identity nexus—including an individual’s “personal style” as well as stylistic choices that index social group membership of various kinds—we find identity to be a kind of ultimate interpretant, defined by Peirce as a result of an individual’s choices and experiences that is not itself directly perceptible (Kockelman 2005, 277). Ultimate
interprets, such as beliefs, predispose actors to behave differently on future occasions, but they cannot be observed directly, only inferred through observation of those subsequent actions. For example, if my experiences as a mathematics student are supportive and engaging, I may decide that doing well in math class is worth the effort, and come to think of myself as a “math person.” This self-concept may then lead not only to increased effort and confidence in class, but also to linguistic behaviors such as telling “math jokes” (Bucholtz et al. 2011) and even hypercorrect pronunciation (Bucholtz 1999). This latter sociophonetic example brings us back to style; if I wish to be seen as a certain type of person, I can accomplish this by expressing a characteristic set of stances (e.g. “I’m confident I know the answer,” “precision is important”), which I instantiate by using an enregistered set of linguistic practices (e.g. pronunciation).

I mentioned above that identity can provide access to social roles through the semiotic process of authorization, and that roles themselves can also be viewed as types of situationally relevant identities. For example, the classroom teacher’s role indexes a privileged epistemic status regarding the lesson content (Heritage 2013) as well as a certain level of institutionally legitimated authority over the students themselves. Over time, a process of stance accretion occurs through which “stances accumulate into more durable structures of identity” (Bucholtz and Hall 2005, 596); that is, stance choices solidify to produce an ultimate interpretant. Applying this to the case of the mathematics student, each student’s answer to the question *What kind of mathematics student am I?* is one such ultimate interpretant, a belief that influences the student’s participatory practices while also being continually modified and recreated through each day’s experience in the classroom. I understand this resulting identity as a mediating factor between, on the one hand, the structures of school and the institutional practices of mathematics education, and on the other, the specific interactional choices made by students and teachers. If we can infer and describe in detail the way particular individuals answer this question for themselves, it may lead to an understanding of why they orient to the process of mathematics learning as they do, and eventually suggest ways
that the institution may change as a result of the actions of the people within it. This approach is in line with the work of Dorothy Holland and her colleagues, who write that we “are not just products of our culture, not just respondents to the situation, but also and critically appropriators of cultural artifacts that we and others produce” (Holland et al. 1998, 17); what is of interest is neither structure nor interaction per se, but the dialogic relationship between them, and the improvisations that occur as a result.

This chapter presents an investigation of the ways that situationally relevant identities are constructed, and macrosociological identities are made relevant, in the course of mathematics classroom interaction. First, I develop a semiotically grounded theory of stance and positioning, allowing us to observe how ultimate interpretants such as identity are leveraged in ways that are appropriate and effective in discourse. Next, I present one episode of classroom interaction from each of the two field sites, drawing additional evidence from ethnographic interviews, to understand how a particular sort of identity is performed in each setting: the way a “good mathematics student” style is constructed in middle school, and the way ethnicity is made relevant in the calculus class. Finally, I will consider these observations in light of the principle that identity is one kind of mediating factor between interaction and structure, and consider the possibility for social change at the institutional level.

5.2. STANCE, POSITIONING, AND SEMIOTIC THEORY

In order to describe the situationally specific identity work accomplished by students and instructors with respect to the domain of mathematics education, as well as the connections they make to their personal biographies and their membership in macrosociological demographic groups, this chapter will pay particular attention to acts of stancetaking. To accomplish this, a theory of stance will be developed with reference to the Peircean semiotic framework used throughout this dissertation (see Table 5), allowing us to view stancetaking utterances in their dual
function, both as interpretants of preceding signs, and as signs with following interpretants, occurring within a chain of signification. To begin with, John Du Bois defines stancetaking as a public act by a social actor, achieved dialogically through overt communicative means, of simultaneously evaluating objects, positioning subjects (self and others), and aligning with other subjects, with respect to any salient dimension of the sociocultural field.... *I evaluate something, and thereby position myself, and thereby align with you.* (Du Bois 2007, 163; original emphasis)

Du Bois’s model is situated as a response to formerly distinct theories of evaluation, positioning, and alignment, providing in their place a unified model of stancetaking in which all of these actions are present, whether implicitly or explicitly. The model begins with *evaluative* utterances such as “that’s ideal” or “that’s nasty” (Du Bois 2007, 142) which are pragmatically incomplete unless a stance object can be identified to indicate what the speaker finds ideal or nasty. Once the stance object is known, the speaker is reciprocally positioned as the one who holds that opinion of that object; this act of *positioning* is foregrounded in stance utterances such as “I’m glad” or “I’m disgusted,” as well as epistemic stances such as “I know.” *Alignment* is considered to arise in the response to a stancetaking utterance such as “I agree” or “I’m glad too,” in which the speaker aligns with a preceding stance, and so implicitly takes up the same positioning and evaluation that were voiced by the preceding speaker; such an utterance “foregrounds ... the dimension of alignment almost exclusively, [yet] for its interpretation it must still indexically incorporate a prior stance content, including the relevant object of stance” (Du Bois 2007, 149).
Du Bois goes on to consider the consequences of stancetaking acts, identifying a social consequence of each of the three functions of evaluation, positioning, and alignment. First, evaluation is an act, and one that “may be among the most broadly consequential of social actions in its cumulative effects” (Du Bois 2007, 175); as the discussion of Kiesling (2009) made clear, styles and identities are ultimately built up out of evaluative actions. Next, by attributing the evaluation to a specific social actor, positioning introduces the element of responsibility, leading an individual act of stancetaking to have potential consequences for the social actor that transcend the immediate occasion of utterance. Finally, by aligning or disaligning with other individuals and communities, a stancetaker acts within a broader ecology of value, understood as ideology or symbolic capital. In this way, Du Bois situates stancetaking as a key component in the production and reproduction of sociocultural norms and practices.

Despite Du Bois’s synthesizing the three formerly distinct theoretical constructs of evaluation, positioning, and alignment within a single framework, he leaves us with the impression that the theory has even further-reaching implications yet to be dealt with:

- what about cases which don’t seem to involve a shared stance object?... the argument can be made that the stance triangle applies even in such less-than-transparent cases.... It has been claimed that all meaningful use of human language, from the age of about one year, presupposes shared orientations, for example toward a word's referent (Tomasello 1999;
Tomasello et al. 2005; see also Hobson 1993). It will be an important task for future research to show how the stance triangle extends naturally to incorporate such observations. (Du Bois 2007, 168)

To extend Du Bois’s framework beyond cases with an identifiable stance object, we can draw on one of its inspirations, the theory of positioning associated with Rom Harré (Davies and Harré 1990; Harré and Van Langenhove 1999). Rooted in the field of discursive psychology, this theory is concerned with issues related to stories, primarily understood not as actual discursive units that display narrative structure or function, but as the “personal stories that make a person’s actions intelligible and relatively determinate as social acts” (Van Langenhove and Harré 1999, 16); that is, metapragmatics understood as a more or less transportable narrative. With this broader focus, positioning, rather than evaluation, is treated as primary: “One can position oneself or be positioned as e.g., powerful or powerless, confident or apologetic, dominant or submissive, definitive or tentative, authorized or unauthorized, and so on” (Van Langenhove and Harré 1999, 17), and institutionally defined social roles such as teacher and student are considered to be positions as well. These positions are then taken to index a range of available speech acts, as positioning is used to determine the social force of an utterance, similar to the process of action ascription described by Heritage (2013). For example, a question from a student would be hearable as a genuine request for information, while a teacher’s asking the same question might be understood as the initiation move in an IRE sequence (Mehan 1979a; see previous chapter). As the interactive sequence unfolds, such positions both fit into and construct the storyline of the conversation; if positions are appropriate and effective in discourse (Silverstein 2003), then the storyline is the metapragmatic frame in which their appropriateness and effectiveness reside.

To integrate Du Bois’s theory of stance with the positioning theory of Harré, it will be useful to step back and consider both frameworks as alternative framings of a single semiotic process of stancetaking, which will be described using the neo-Peircean semiotic theory developed by Paul
Kockelman (2005; 2006a; 2006b). Du Bois’s model, in which a stancetaking subject evaluates a stance object, and a second subject then aligns with this evaluation, offers a neat illustration of Kockelman’s definition of intersubjectivity, in which

a self (or "subject") stands in relation to an other (or “object”) on the one hand, and an alter (or “another subject”) on the other, in such a way as to make the alter stand in relation to the other in a way that corresponds with the self’s relation to the other. (Kockelman 2005, 237)

To think about Du Bois’s model using Kockelman’s terminology, then, a first attempt might be, stancetaking is a semiotic process whose sign is an evaluation, whose object is a position, and whose interpretant is an alignment. That is, by expressing an opinion about a stance object, I orient to an affective or epistemic stance, and invite the hearer to align with this stance; for example, my saying “That’s amazing” indexically positions me as feeling amazed by some stance object, and thereby leads you to consider whether you are also amazed by it.

There are some problems with this figuration, however. Du Bois gives examples of positions as well as evaluations that are expressed discursively—not just That’s amazing, but also I’m amazed—and there is no reason to treat either of these utterances as phenomenologically prior. On the contrary, if the stance object is considered as a previously occurring sign to which I am responding, then both utterances are potential representational interpretants of it, variably representing my feeling of amazement, which is an affective interpretant. For this reason, rather than considering a position to be a verbalization of epistemic or affective stance, I will provisionally define it as an intentional status reflecting opinion, affect, or relative epistemic certainty, which arises as an affective interpretant of a stance object, and is indexed by a stancetaking utterance.

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3 It should be noted that while the treatment of stancetaking provided here does build strongly on Kockelman’s work in semiotic theory, it is quite distinct from his own treatment of stance and its morphosyntactic realization (Kockelman 2004).
Kockelman discusses social and intentional statuses at length (2005, 278ff; 2006a), treating them initially as ultimate representational interpretants, which, once established, are then indexed as the objects of embodied signs: actions, roles, and identities. An action, such as accomplishing a speech act or crafting an artifact, is considered to be a sign whose object is a purpose. Purpose, then, can be understood as the intentional status that interpreters attribute to the actor on perceiving the action: “The teacher asked me to solve a problem; she must be testing me.” Indeed, if someone were to explain the intention behind their own action, this would be understood in semiotic terms not as a psychological state that exists prior to the action, but as a representational interpretant of one’s own preceding action. As for roles, they are defined in a more open-ended fashion as “the enactment, or performance, of the rights and duties one is assigned by virtue of one’s status…. any behavior that can be considered the enactment of a right or responsibility is a role” (Kockelman 2006b, 50). Status is understood here as similar to Bucholtz and Hall’s conception of identity in that it comprises discursive participant roles such as “questioner,” locally emergent cultural positions such as “teacher,” and transportable macrosociological categories such as “African American,” as articulated in the positionality principle of identity (Bucholtz and Hall 2005, 592). Thus the bulk of contemporary sociolinguistic research on the indexical performance of identity can be understood as an attempt to catalogue, analyze, and interpret the range of linguistic signs (enregistered styles, etc.) that are understood as roles in Kockelman’s sense. Finally, identities for Kockelman index basically the same categories indexed by roles, but where roles are considered to be enacted in a particular interactional moment, identities are framed at a broader time scale: any role can be ‘bumped up’ into an identity if its status is treated as a value. In everyday speech, this is often phrased as identifying with a role—as in, ‘he really identifies with being a father’ (that is, he takes being a father as relatively more important that the other roles he inhabits, thereby usually inhabiting it across a wider range of contexts.) (Kockelman 2006b, 57)
This understanding leads to the conclusion that “an identity is just a relatively complex and composite set of roles” (Kockelman 2006b, 59).

Crucially for our purposes, although Kockelman argues for the epistemic and normative salience of these distinct categories, he does acknowledge that “one could unite some of these divisions and produce fewer constituents” (Kockelman 2006b, 24). In particular, what counts as a “role” is sometimes ambiguous; the category overlaps with identities at one end, as we have seen, and with actions at the other (is “requester” a role, or is “request” a speech act?). This observation allows us to expand Du Bois’s model of stancetaking to account for cases where there is no definite shared stance object, thereby addressing an admitted limitation of the Du Bois framework, and also better incorporating Harré’s positioning theory. Recall that the frameworks outlined in Table 5 were developed to address subtly different phenomena; while Du Bois defined a position as essentially corresponding to an affective or epistemic stance, van Langenhove and Harré (1999) defined it more broadly to comprise not only locally emergent, mutually determining roles such as “dominant/dependent,” but also enregistered voices such as “teacher/student”—a whole range of social and intentional statuses. Transplanting this definition into Du Bois’s framework, we can consider a position as the object of a stance, and we may define stance (following Kockelman’s definition of role) as any behavior that can be considered the enactment of a position.4 As for the “stance object,” recall that in Du Bois’s formulation, this is not really an object in the semiotic sense, but is better understood as a preceding sign that gives rise to a stancetaking utterance as its representational interpretant; thus, in “cases which don’t seem to involve a shared stance object” (Du Bois 2007, 168), the position is a social or intentional status indexed by the stancetaking utterance, and the stance object is the preceding state of affairs that has given rise to this status as ultimate representational interpretant.

4 “If you think this is circular, you’re right; if you think circularity is bad or somehow avoidable, you’re wrong. Indeed, if there is any sense to the slogan ‘meaning is public,’ this is it.” (Kockelman 2005, 281)
One point deserves further clarification. In the summary provided in Table 5, *position* was listed as a secondness under Du Bois’s framework—for Du Bois, a position is the object indexed by an evaluation—but as a firstness under Harré’s. The difference here is primarily a question of *semiotic framing*, defined as “the ‘orientation’ of the interpreter relative to the sign being interpreted” (Kockelman 2005, 269). In this instance, rather than understanding semiosis in terms of sign events (such as speech acts) that presuppose certain context (as their ‘roots’) and create certain contexts (as their ‘fruits’), [an interpreter may] focus instead on embodied signs (such as social and intentional statuses) that presuppose certain sign events (as their ‘roots’) and create certain sign events (as their ‘fruits’). (Kockelman 2005, 271)

In this understanding, Du Bois’s model, as revised here, is of the second type: *a position is a social or intentional status that emerges as the ultimate interpretant of a state of affairs, is indexed by a stancetaking utterance, and gives rise to an alignment as its representational interpretant* (Appendix A, column 6). Van Langenhove and Harré, however, present a model of the first type, viewing the “social force” of a speech act as appropriate to the speaker’s social positioning, and effective in discursively establishing an ongoing storyline. Understood as a difference of semiotic framing, we see that these two distinct models of stance and positioning are in fact easily commensurable.

Stancetaking was defined above as *any behavior that can be considered the enactment of a position*, and for this reason, detailed analysis must account for a range of stancetaking utterances. Evaluative language is considered, as in Du Bois (2007), to index a corresponding *affective stance* on the part of the evaluator. Evidential language, such as the expression of degrees of certainty or of epistemic rights (Raymond and Heritage 2006), is considered to index the sort of intentional status commonly referred to as *epistemic stance*. The sequential context of talk provides insight into a speaker’s alignment or disalignment with preceding utterances. Utterances in which speakers describe states of affairs in the world and then evaluate them, such as instances of narrative
practice, express not only an affective stance with regard to the state of affairs, but also an alignment or disalignment with the purpose for which that state of affairs came about. Finally, to consider instances of role-as-identity rather than role-as-action, we may consider intertextual reference to enregistered types that are familiar to participants from outside the immediate social environment. In the following analysis, all of these types of stancetaking actions will be seen to index positions, align with individuals and groups, and instantiate speakers’ performances of style and identity.

5.3. **Participation, Confidence, and the “Good Math Student”**

One of the motivations for this study was the observation that students (as well as educated adults) often make comments such as “I’m not a math person,” thus essentializing mathematics aptitude as something that other people possess, and excusing themselves from any responsibility for knowing or feeling comfortable with mathematics. To accomplish this, they presuppose that there does exist a social type of “math people” to which they do not belong, and so we may ask how this type is constructed, how a “math person” can be recognized. Given the nature of the evidence available here, the question will be modified slightly to be fit within the classroom environment; focusing on the middle school field site, this section addresses the question of what it means to be a good mathematics student, and how stancetaking actions function to position students in this role.

5.3.1. **Nayan Raises His Hand**

To ground this discussion in an instance of classroom interaction, we will consider Example 21, which occurred during the homework review portion of class. As she regularly did for homework review, Ms. M projected on the Smart Board an image of the previous night’s homework on which the answers had been written in by hand, and left time for students to ask questions. In this way, if all the students got a question right, a minimum amount of time would be spent on it;
and if any student wanted an explanation of the answer, they could ask for it. As Example 21 begins, homework problems 9 and 10 had just been displayed on the board, as reproduced in Figure 21.

<table>
<thead>
<tr>
<th>Evaluate when $x = -2$ and $y = -6$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>9) $xy$</td>
</tr>
<tr>
<td>10) $x - y$</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

**Figure 21: Number nine**

**Example 21: ‘I got it wrong’ (20 February 2014)**

1. Ms. M | All right, display problems 9 and 10, as in Figure 21
2. (6.7)
3. (Anyone (↑not get it).↑)
4. Nayan | raising RH
5. (I get it)
6. (RH point to blackboard)
7. (wrong)
8. (the first one)
9. (n- n- number nine.)
10. Ms. M | Do you know why you got it wrong,
11. (5.0)
13. (1.0)
14. Ms. M | What's negative two times negative six. turn to board circle -2 circle -6
15. (5.0)
16. Nayan | It’s times?
17. Ms. M | Isn't this times? circle xy
18. Nayan | ↑O:h | it is times shift forward in chair
19. Ms. M | When they’re together, ↑ BH palms together
20. Nayan | ↑y:eah↑
21. Ms. M | It’s times. (between x and y)
22. Ms. M | I did subtract. look at his paper on desk
23. Ms. M | OK good, I'm glad you brought that up.
This moment first caught my attention because of what Nayan does in lines 4–6: he bids for the floor and volunteers the admission that he had got the problem wrong, accomplishing a self-directed face-threatening act when he could just as easily have chosen to remain silent. The evidence is mixed as to whether Nayan is cognizant of the face-threat. The long pause in line 2 suggests that he, like the rest of the class, is reluctant to answer. During the pause, Ms. M cocks her head at one point as if to elicit a volunteer; on the video, it is not clear whether her eye gaze is directed at Nayan when she does so, but when she finally verbalizes the question in line 3, he is already beginning to slowly raise his hand. As for the content of his utterance, the error in Nayan’s mathematical procedure—failing to recognize that the notation $xy$ represents multiplication (line 14)—is a common one among students at this level, and it is unlikely that he was the only one to make that mistake in this instance. Ms. M’s evaluative comment “I’m glad you brought that up” (line 22) also points to a belief that the preceding discussion would have been useful not just to Nayan, but also to other students (i.e. *I’m glad you asked for a public explanation of this point*), and it may also reflect an acknowledgement of the face-threat involved in asking the question (*I’m glad you weren’t too embarrassed to bring this up*).

To elicit the students’ understanding of what it means to publicly admit a wrong answer, this clip was included in the middle school interview protocol (Appendix B), along with the questions: Why did Nayan raise his hand? Why didn’t anyone else? These questions were meant to explore how the students balance, on the one hand, social concerns such as saving face in front of their peers, and on the other, academic concerns such as understanding the mathematics content. In sequence, this can be considered as a positioning triad, using the framing associated with van Langenhove and Harré (1999): the position is the intentional status of knowing you have a wrong answer, the speech act is admitting it publicly, and the storyline is the social status that other students ascribe to you as a result.
My intuition was that a student’s willingness to speak up in class would correlate with their being perceived by their peers as a good student. For this reason, in all of the interviews, I asked, “Aside from the students present here, who in the class would you say is best in math?” Aside from one group that refused to name names and said that everyone was equal, the responses were quite consistent: every group named Fernando, Nayan, and Tiana, including the interviews where these students were participants, and two groups named Jamie and Ricardo as well. The responses are summarized in Table 6.

*Table 6: Who is best in math?*

<table>
<thead>
<tr>
<th>Interview date</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>27 May 2015</td>
<td>Fernando, Nayan, Tiana</td>
</tr>
<tr>
<td>29 May 2015*</td>
<td>Fernando, Nayan, Tiana</td>
</tr>
<tr>
<td>3 June 2015**</td>
<td>Fernando, Nayan, Tiana, Jamie, Ricardo</td>
</tr>
<tr>
<td>5 June 2015</td>
<td>Fernando, Nayan, Tiana</td>
</tr>
<tr>
<td>10 June 2015</td>
<td>Fernando, Nayan, Tiana, Jamie, Ricardo, Rita, Alberto</td>
</tr>
<tr>
<td>12 June 2015</td>
<td>Don’t want to say; “everyone is equal”</td>
</tr>
</tbody>
</table>

* Interviewees: Tiana, Jamie, Sandra
** Interviewees: Fernando, Nayan, Ricardo

Notably, the students commonly considered to be “math people” were all exceptional in some way, and all had a higher level of English language proficiency than the typical student in this class. While most students in the class were Central American—out of the eighteen students I interviewed, nine were from El Salvador, two from Honduras, and one from Guatemala—the “best students” were not. Nayan is from Bangladesh and Tiana is from Madagascar; as the only two non-Spanish-speakers in the class, they had needed to rely on English as a means of communication, with their peers as well as their teachers, since their arrival in U.S. schools. Fernando had attended English-medium schools in Colombia, and while students named him as a particular resource when they had trouble with math, they also described him as taking the teacher’s role in these interactions, perhaps inappropriately; as Tiana said, “He does too much ... if you’re doing something he stays there to see if it’s correct.” While Jamie and Ricardo are both Salvadoran, Jamie was a long-
term English learner who had arrived in the U.S. at age three, and I do not know why she had not been exited from ESL services. I never learned Ricardo’s language-learning background, but classroom observations made it clear that he was very comfortable speaking English and participating actively in class, and Ms. M also perceived his language proficiency to be exceptional, as she said to me: “Except for Ricardo, the other kids here? They take so long to learn English.”

These “best students” also seemed to have close social ties with one another. Participants were free to choose their own groups for the group interviews, and one consisted of Fernando, Ricardo, and Nayan, while another included Tiana, Jamie, and Sandra. These are the two interviews from which examples are taken for analysis, because these students were most articulate in verbalizing their interpretations of the video and the classroom social dynamic it reflects. Perhaps these students’ positive orientation to school helped them to feel comfortable talking to me as a perceived authority figure, and at the same time, their classmates may have felt comparatively ill at ease with me despite my familiar presence in the classroom, my use of Spanish in the interview, and the pizza that I offered as an incentive to participate. This is all speculation, however, and the best I can do is to note the methodological challenge of conducting ethnography across age and status boundaries, mention that no other student said anything in contradiction of what is presented here, and maintain my awareness of the potential limitations of the analysis.

To begin, consider Sandra’s response to the video, provided in Example 22. In this interview, Jamie and Tiana both participated actively in English, and at times I switched into Spanish to encourage Sandra to speak up as well; in Example 22, the left-hand column provides a transcript, which I have translated on the right. While Sandra was not considered to be one of the best students, she and Tiana sat next to one another and worked together consistently throughout the semester. Despite her social and physical proximity to one of the “best students,” however, her reflections on the video were broadly representative of what other classmates said as well.
In this example, Sandra first suggests that Nayan raised his hand because he was interested in the content (lines 6–7) in order to be prepared for the test (13–14). When I ask her in lines 19–21 why that makes him exceptional, she draws two contrasts between Nayan and his silent peers: maybe they were embarrassed to ask (27), or maybe they just weren’t interested (32–33). In Sandra’s account, a student’s relative willingness to ask questions is seen as an individual character
trait. Either you’re interested, or you aren’t; either you’re mainly concerned with avoiding embarrassment in front of your classmates, or you care more about doing well on the test.

Following this, Jamie proposed (in English) that maybe Nayan really was the only student to get the wrong answer, and then Tiana returned to the topic of embarrassment—or maybe, since she did not speak Spanish, she had independently come up with a similar explanation to Sandra’s. The transcript continues in Example 23.

Example 23: ‘They’re waiting for someone else’ (Continues from Example 22; 28 lines omitted)

62 Tiana Well, I think that
63 (1.2)
64 that some-
65 the others just want to know like
66 they di- they want- don’t want to raise their hands
67 but they want to know the answer
68 so they're waiting for someone else to
69 Daniel a:h.
70 to: raise their hands and then they can
71 answer.

Tiana proposes that Sandra may have been partly right: maybe the students are embarrassed to admit they’re wrong, but they do actually want to know the answer. To resolve this conflict, they can rely on their knowledge of their classmates and how they are likely to respond to a given situation, and by taking advantage of one another’s habits, they can get what they want both socially and academically. Tiana’s explanation is more sophisticated than Sandra’s in that it reflects not only students’ individual preferences, but also their knowledge of the group, and so Nayan’s willingness to speak becomes an affordance that the other students can use to elicit information from Ms. M without having to admit their own ignorance. It seems likely that Ms. M is aware of this dynamic as well, which is implicit in her saying “I’m glad you brought that up.”

When I interviewed Nayan, Ricardo, and Fernando the following week, I also showed them the video transcribed in Example 21. Example 24 reproduces their response when I asked them to summarize what they had seen on screen.
Example 24: ‘Nayan learned from his mistakes’ (Interview, 3 June 2014)

1. Daniel So
2. Fernando what happened in the video.
3. Ricardo We all got the wrong k- tm.
5. Fernando L@@@ L@@@
6. Ricardo No ah-
7. Daniel mhm
8. Nayan yeah.
9. Fernando Nayan learned
10. from @@his mistakes,

In these few lines, we see a complex dance of alignment and disalignment regarding the question of whether making a mistake carries with it a loss of face. In Line 4, Ricardo answers my question “What happened in the video?” in a relatively straightforward way, but in Nayan’s response in 5, we see that Ricardo’s simple recount is also hearable as a mild accusation. Implicit in this weakly defensive utterance is the perception that there is something to be defensive about; that is, Nayan not only construes Ricardo’s utterance as a negative evaluation, and thereby imputes to Ricardo the stance that getting wrong answers is bad, but he also aligns with this stance. And yet, Ricardo goes on to disalign from the stance that Nayan has just attributed to him. The discourse marker “No” in line 7 signals a footing shift, followed in 9 by the assertion that “from mistakes you learn.” Ricardo uses the generic “you” to align with Nayan-in-the-video, whose utterance “I got it wrong” had put into practice a belief that mistakes are valuable for learning, a belief that Ricardo verbalizes here. In this alternative framing, the student’s goal is not to be right, but to learn. Making an error is acceptable—or even, perhaps, a good thing—if it leads to learning, but for this to happen, the error must be made public.

There is a tension here, however, between the desire to be right (which shows that one has learned) and the desire to learn (which sometimes involves being wrong), as Fernando makes clear.
in 12–13. This utterance revoices Ricardo’s earlier comment, but replaces the generic “you” with a specific personal reference to Nayan. In this way, Fernando simultaneously aligns and disaligns with both Ricardo and Nayan; he evaluates Nayan positively, as having learned, but also negatively, as having made a mistake. Considering Chafe’s (1987) view that humor results from incongruity, I read Fernando’s laughter as a response to this ambivalence: despite the commonplace understanding that “from mistakes you learn,” still nobody wants to be the one who made a mistake.

5.3.2. Students Reflect on “Feeling Comfortable”

Unfortunately, soon after the sequence transcribed in Example 24, Ricardo spilled his drink, and after we cleaned up the mess, we moved on to other issues. As a result, Nayan never got to explain why he had raised his hand. In place of this, I will take a broader view of how the “best students” feel about being publicly wrong, looking at some more generic comments they made in earlier modules of the interview that preceded the video playback. First, in Example 25, Tiana talks about the importance of class participation. This strip of talk occurred several minutes into a discussion that I had prompted by asking, “So what do you think, just sort of overall, of your class?”

Example 25: ‘I feel comfortable because I can raise my hand’ (Interview, 29 May 2014)

1 Tiana And I feel comfortable because
2 (0.7)
3 um
4 I can
5 raise my hand (and remind) and
6 Daniel mhm
7 Tiana when I was in the elementary school
8 the teacher just
9 choose whatever people
10 uh if you don't know the answer you have to go but
11 you can't
12 raise your hand whenever you want.

In this example, Tiana describes two different kinds of classroom participation structure, which index two distinct ideologies of student participation and its pedagogical purpose. First, in
lines 1–5, she describes Ms. M’s practice, in which a student may volunteer to participate “whenever you want” (12). This allows students a greater degree of agency, as they may use their turns at talk to ask questions or to demonstrate what they have learned, or they may choose not to respond. In contrast, she describes her experience “in the elementary school” in Madagascar, where students were cold-called and required to answer (7–12). This practice reduces student participation to a form of assessment; through their turns at talk, students are only able to demonstrate correctness or incorrectness, and crucially, they are not free to choose the time and place at which this demonstration is made. Between these two options, Tiana takes a strong stance in preference of the first one. As a student who participates actively in class, and is seen by her peers as a competent mathematics learner, she values the right to control when and how she will demonstrate her competence.

In his reflections, Fernando also talked about “feeling comfortable” to participate in class. Example 26 occurred after I had asked which students were best in math, and Nayan had listed Tiana, Jamie, Fernando, Ricardo, and himself (as shown in Table 6). I asked, “How do you know that these other students are good in math?” Nayan replied, “Because they all all always answer correctly and they always get good grades,” but then Fernando offered another explanation.

Example 26: ‘They don’t care if they have wrong answer’ (Interview, 3 June 2014)

1  Fernando  No I- I (the) because they participate too much, and
2        they feel comfortable, for example
3        they don’t care if they have
4        wrong answer or right answer, they just show it?
5        so I like that’s people because
6        they know how to,
7        how to,
8        how to learn of their mistakes.

First, in lines 1–4, Fernando explains that good students can be identified based on their style of participation in class. Not only do they participate a lot (“too much” in line 1 is probably not meant as a negative evaluation, but rather is a common learner error reflecting a mistranslation of
“mucho”), but they “feel comfortable” to speak up even when they are uncertain of the correctness of their answer. Paradoxically, this suggests that voicing a wrong answer, or admitting a mistake as Nayan did in Example 21, indexes a “good student’s” participation style rather than a general difficulty in understanding. Next, in 5–8, Fernando gives a positive evaluation “I like that’s people,” and aligns with the stance that by making one’s mistakes public, one may learn from them.

Recall that it was Fernando who in Example 24 ambiguously seemed to be laughing at Nayan’s mistake. In Example 26, which occurred earlier in the interview, Fernando aligns much more clearly with the belief that mistakes are for learning, which Ricardo later verbalized in response to the video. Given this additional evidence, I would tend to view Fernando’s utterance “Nayan learned from his mistakes” in Example 24 less as an expression of the belief that making errors is shameful, and more as an instance of solidarity-building through gentle teasing among friends.

What is it, then, that sets the “best students” apart from their peers? When I asked them this question directly, students often made reference to grades and correct answers, but in fact they were generally unaware of one another’s grades. While correct answers do become public knowledge if they are articulated publicly, students who generally solved problems correctly but kept their answers to themselves would not be perceived as above-average mathematics learners; as it happens, Alberto fit this profile, but was only named as one of the “best students” once in six interviews, toward the end of a long sequence of names that suggests the participants in that interview wanted to include as many classmates as possible among the “best.” Ms. M’s perception of Alberto was similar; when I asked her about types of students that stood out to her, her first thought was those who are “pretty smart but lazy, (.) Alberto.” Being perceived as a good student—by both teacher and peers—has surprisingly little to do with one’s actual facility with academic content, and is much more an interactional accomplishment achieved through an active, agentive style of participation in classroom discourse. This underlies the perception common among Ms. M’s
students that the best math learners also happen to be those who speak more fluent English, which is implicit in the identification of “best students” in Table 6, and was also verbalized more explicitly in interview responses not presented here.

I conclude this section by observing that the connection observed here, that more class participation correlates with higher status, is likely to vary significantly from one academic environment to another. In fact, in the calculus class I observed, a very different dynamic held sway, which became explicit in my interview with Mickey, one of the calculus students; the relevant section is reproduced in Example 27. During this interview, I asked, “How do you judge whether the other students are good students?” similar to what I had done with the middle school groups. At first, he mentioned test scores, but then he continued:

Example 27: ‘The types of questions they ask’ (Interview, 20 March 2015)

```
1 Mickey and then <another way I'd probly judge is>
2   (1.7)
3 u:m
4   (0.7)
5 just the question
6 the,
7 types of questions they ask?
8 Daniel mhm
9 Mickey and
10 probably the amount
11 of questions they'd ask.
12 Not like
13 that they're
14 like y'know
15 less smart 'r
16 Daniel mhm
17   (1.0)
18 Mickey anything like that it just
19   y'could tell that they struggled more.
```

Unlike his middle school counterparts, Mickey sees a student's asking questions, or at least asking certain types of questions, as an indication that they “struggled.” There are a number of potential explanations for this contrast, beginning with 20-year-olds’ more sophisticated ability to
reason about other people’s intentional statuses; recall that Sandra considered a student’s not
asking questions to indicate that they were not interested in knowing the answer, and Jamie
suggested that “best student” Nayan might have been the only one to ask about the homework
because everyone else had got the problem right. Another consideration is the denseness of social
networks. While the calculus students might only see one another during calculus lectures, and
never again in following semesters, middle school students, especially those in a high-intensity
ESOL program, are likely to be in class together all day, every day. As a result, there would be less
willingness to ask questions that display a lack of understanding, and more likelihood that the
students who ask such questions would be those who have established high status in the group.

This last point deserves further discussion. Wouldn’t it be just as likely to say that low-
status students would have the least to lose, as their wrong answers would only fulfill expectations,
while students who have attained a higher status would be at risk of losing it? (Think of the trope of
the “class clown,” whose responses are not intended as attempts to be correct, but rather as claims
on others’ attention for its own sake.) While this sort of dynamic is at least plausible and may obtain
in other classrooms, my own observations are better explained if we consider public displays of
understanding, such as correct answers, to convey a sort of classroom social capital. Nayan’s being
consistently correct—and, unlike Alberto, doing so aloud—has led to his being perceived as one of
the “best students,” and (to push the economistic metaphor a little) he has amassed enough social
capital that he may now engage in speculation. Not only does he have enough self-confidence to
offer answers when he is uncertain, but he can also publicly admit it when he knows he is wrong,
investing his social capital for the benefit of all those students who, in Tiana’s words, “are waiting
for someone else to raise their hands.” For those students who stay silent, if they were to speak up,
they would risk becoming like the students Mickey describes, who so consistently show a lack of
understanding that they come to be positioned as “struggling” and, potentially, as “less smart.”
5.4. **Race and Ethnicity in the Calculus Class**

Mathematics education is often seen as a racialized field, “a discipline that is first and foremost a White, middle-class, male domain” (Stinson 2013, 71; cf. also Clark, Badertscher, and Napp 2013; Ladson-Billings 1997; D. B. Martin 2006; D. B. Martin 2013). Students in a broad range of countries share a stereotype of mathematicians as being overwhelmingly white, male, and inaccessible “absent-minded professor” types (Picker and Berry 2000), so mathematics students of color, particularly at the post-secondary level, have reason to think of themselves as being atypical or somehow not belonging.

Given this broad understanding of the link between ethnicity and mathematics education, the calculus class presents an opportunity to investigate non-stereotypical students’ identities. Approximately one in four students in the calculus class identified as Black or African American,\(^5\) compared to 6% in the institution as a whole.\(^6\) A macrosociological understanding of race might focus on the overrepresentation of African American students in a quasi-remedial mathematics section and claim that this reflects and perpetuates the belief that students of color are not prepared for college math. By using ethnographic methods, however, we can take a deeper look at the emic significance of race and ethnicity to members of the community as they go about their day-to-day business. Synthesizing a number of classroom ethnographies, David Bloome writes:

> Descriptions of how race was played out, its significance, and the meaning constructed, were different in [different classroom ethnographies], which emphasizes that, within a classroom ethnography, it is not enough simply to label students in terms of broad demographic variables as if the implication of such labels can be known a priori. At one level, while ethnographic descriptions may suggest similarities across classrooms at one

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\(^5\) This is based on my count using the class roster at the end of the semester, but it is almost certainly an underestimate; a number of Black students dropped the class mid-semester, so earlier in the term, the accurate figure may have been closer to one in three.

\(^6\) According to the Integrated Postsecondary Education Data System, provided by the National Center for Education Statistics (http://nces.ed.gov/ipeds/datacenter); to maintain the anonymity of the field site, a direct link to the relevant page is not provided here.
level (e.g. in each classroom race was a meaningful issue and social marker), at other levels there were significant differences in how its meanings and their practical consequences were socially constructed. (Bloome 2012, 15)

This section presents a discussion of the “meanings and practical consequences” of ethnicity in the calculus class. I begin by profiling Dr. C and the ways and reasons that he made his African American identity relevant in classroom interaction. Following this, I present interview transcripts from two calculus students in which they describe what they see as the link between ethnic identity and mathematics aptitude.

5.4.1. **Dr. C’s Use of African American Culture**

Dr. C’s teaching philosophy is rooted in a tradition of culturally relevant mathematics pedagogy that stresses not only students’ responsibility for their own learning, but also their responsibility to one another. As an undergraduate, he participated in an “emerging scholars program” designed, as he put it, “to provide more support for students that kinda traditionally struggle” in mathematics (cf. Treisman 1985; Treisman 1992). In this program, he first became interested in mathematics through his role as a peer tutor, and only later became aware of academic mathematics research as a field of study.

As a researcher, Dr. C has worked with high school students who were participating in the Algebra Project, an organization with roots in the 1960s Civil Rights Movement, which is concerned with access to mathematics education as a matter of social justice (Moses and Cobb 2001; Dr. C’s own work in this area is not cited here for reasons of confidentiality). His university teaching, in traditional sections as well as the quasi-remedial section where my fieldwork took place, also incorporates Algebra Project principles: an emphasis on classroom community and mutual aid, as well as a belief that mathematics content should be grounded in students’ personal experience. For example, he talked to me about making a “connection between the language that students speak and ... this formal symbolism ... and so I feel the freedom, in the math class, to try and unpack that in
terms of everyday language,” a register-shifting flexibility that is also a part of the Algebra Project approach. He also offers all of his calculus students extra credit in exchange for forming study groups and working together outside of class, and he sees benefits from this practice not only for his students’ understanding of mathematics content, but also for their developing sense of responsibility and ability to work collaboratively in groups. Reflecting on his personal history, he sees clear links between his experience as a non-stereotypical mathematics learner and the principles that guide him as a teacher: “The more I think about the type of stuff that I see, like, that’s most effective for the broadest group, I think a lot of it comes from that [emerging scholars] experience.”

Dr. C’s mere presence in the classroom, to say nothing of his particular experience as a teacher and learner, as well as the African American cultural roots of his pedagogical approach, stand in opposition to the “white male math myth” (Stinson 2013), a commonly observed ideology that indexically links mathematics ability and “math person” identity to male gender and white (or Asian) ethnicity. For this reason, I wanted to identify particular episodes in the video corpus where Dr. C’s being African American became situationally relevant in the course of interaction. Perhaps the clearest such episode is reproduced here as Example 28 and Example 29. Example 28 occurred toward the beginning of class, as Dr. C was reviewing the product rule and quotient rule for differentiation. These are formulas or techniques for calculating the derivative of a function by breaking it down into simpler components; for example, if a function can be written as a second function divided by a third, then the Quotient Rule offers a way to proceed with differentiating that first function. To accomplish this, however, requires the use of a formula that students often do not find intuitive or easy to remember. Earlier in the class, Dr. C had written that formula on the blackboard, as shown in Figure 22.
Figure 22: The quotient rule

To make this formula more accessible to students, Dr. C offered them a mnemonic device:

“There’s different shortcuts that I’ve seen to remember the quotient rule. My favorite, probably, we can remember it because it’s in the form of a rhyme.” He explains that if the numerator function \( f(x) \) is called “high,” the denominator function \( g(x) \) is called “low,” and the letter “d” is used to stand for “derivative,” then the quotient rule can be recited as a rhythmic chant, as in Example 28.

Example 28: ‘You got your beat goin’ (22 January 2014)

1 Dr. C *so* | you got your beat goin
2 and then you go
3 | low, d | high,
4 | \( g(x) \) | \( f'(x) \)
5 minus | high d | low,
6 | \(- \) | \( f(x) \) | \( g'(x) \)
7 and | don’t forget to square down below
8 | \( (g(x))^2 \)
9 Mickey r@@@@
10 student | Oh man
11 class | ((general chatter))
12 Dr. C | low d | high minus high | d low,
13 | \( LH 5\)-hand to head height | lower hand
14 and | don’t | for | get to square down below
15 | \( LH \) out | in | out ((keeping rhythm))
16 Charles I like that
17 Dr. C | (ibby ibby ibby) ((falsetto record scratch noise))
18 | pantomime record scratch ((Figure 23))
19 student roh my god
20 class | ((general laughter))
In the course of this example, Dr. C uses several tactics to position himself as African American, in Michel de Certeau’s (1984) sense of tactics to mean social actors’ improvised ways of “making do,” repurposing institutional or otherwise normative practices for their own particular ends. In this case, while Dr. C’s African American ethnicity is already familiar to all participants in the scene, there are particular aspects of his semiotic behavior in this strip of talk that index this social status and thereby make it interactionally relevant in the moment. To begin with, through all of my observations and interactions with Dr. C, I perceive him as a speaker of Standard Black English, a relatively high-prestige variety of African American English that incorporates Standard American English morphosyntax with some distinctly African American phonological features (Grieser 2014; Hoover 1978; Rahman 2008; Taylor 1970), and in this example, he seems to stress these aspects of his speech. For instance, in line 5 he pronounces “forget to square” as [fəːɡtʰ tu skweɪə], merging /ɛ/ with /i/ in “forget,” and breaking the vowel in “square” across two syllables. While the first of these features is associated with both AAE and Southern U.S. English, the second is similar to what has been described as a “Southern drawl” (Allbritten 2011; Feagin 2003). Dr. C is not a Southerner, however, and while “breaking” is not generally listed in inventories of AAE
features, it seems likely that this nonstandard phonology in Dr. C’s repertoire reflects his African American heritage, given the well-established overlap between Southern U.S. and African American varieties of English (e.g. Wolfram 1974). Notably, in the same utterance with these nonstandard features, he produces a released /t/ in a highly marked environment in “forget to,” a hypercorrect pronunciation that has been identified as an index of a “nerd” identity (Bucholtz 1999). Even on the phonological level, Dr. C finds ways to simultaneously perform African American and mathematician identities.

Other aspects of Example 28 relate to African American identity by bringing in aspects of hip hop culture. It is a common instructional technique to use music or rhythm to support students in memorization tasks; for example, Ms. M’s middle school class sang the rules of integer addition and subtraction to the tune of “Row, Row, Row Your Boat,” and I have seen high school teachers sing the quadratic formula to the tune of “Pop Goes the Weasel,” but these children’s tunes might feel out-of-place and perhaps insulting to undergraduates. Instead, Dr. C uses the sort of rhythmic speech characteristic of hip hop music to accomplish the same pedagogical purpose without being seen as age-inappropriate. To accomplish this, he first characterizes the mnemonic chant as rhyming over a beat (“you got your beat goin,” line 1), which he accompanies with an unusual rhythmic body movement that seems to suggest a dance move. He then recites the rhyme, while simultaneously gesturing at the board to show the correspondence between spoken and written-notational channels. The students’ response in lines 6–8 suggests that they are engaged, or at least intrigued. Next, Dr. C repeats the rhyme in 9–10, this time accompanying it with a rapper-like hand gesture in 10 and a pantomime record scratch in 12. (Co-occurring with these indexical icons of the rap music genre, Dr. C’s nonstandard phonology as well may be heard not only as an instance of African American language generally, but also more specifically as Hip Hop Nation Language (Alim 2009a).) The students seem to accept this performance, and Charles in 11 (“I like that!”) gives a particularly positive response.
Following this transcript, Dr. C repeats the rhyme a third time, this time transcribing it word for word onto the blackboard. Then, Jason offers a response, transcribed here in Example 29; note that Jason and Charles are both Black, while Brad and Siobhan are white.

**Example 29: ‘I don’t wanna be Eazy-E’ (22 January 2014)**

1. Jason  Dr. C wants to be Eazy-E
2. Siobhan  @@@
3. Dr. C  
4. Brad  Never gets ((inaudible))
5. Dr. C  \No I don’t wanna be Eazy-E
6. class  ((general chatter))
7. Dr. C  @@@
8. Jason  yeah Eazy-E passed away,
9. Dr. C  te- a decade or so ago
10. Dr. C  I don’t wanna be Eazy-E
11. Charles  ‘Cuz he died uh AIDS,
12. that’s why you don’t wanna ((inaudible))
13. Siobhan  @@@
14. Dr. C  yeah

Jason has clearly understood Dr. C’s performance of the quotient rule rhyme as a reference to hip hop culture, which he displays in line 1 through comparison with the rapper Eazy-E (“The Godfather of Gangsta Rap,” best known as a founder of the influential 1980s–90s group N.W.A.).

Jason positions Dr. C not as a rapper himself but as someone who “wants to be” a rapper, which can be read as dismissive of Dr. C’s rapping skills, and reciprocally positioning Jason as well-informed about hip hop music and culture. At the same time, the form of his utterance expresses a knowledge of what Dr. C “wants”—that is, the speaker assumes the right to make claims about the addressee’s intentional status, which has been interpreted elsewhere as a power move (cf. the discussion of “B events” in Labov and Fanshel 1977). With this remark, Jason shifts the footing of the event to a highly familiar tone, in a way that seems to contrast with his subordinate student status and, perhaps, to save face if he finds Dr. C’s rhyming exercise infantilizing.

In his response, Dr. C begins by contradicting Jason (“I don’t wanna be Eazy-E,” line 5), pushing back against Jason’s claim of familiarity. At the same time, through his laughter in lines 3
and 7, as well as his uptake of Eazy-E as a discourse topic in 8–11, he also validates Jason’s digression into hip hop history. By adding new information about Eazy-E to the conversation, Dr. C authenticates himself as a hip hop listener, and also establishes co-membership with Jason in the community of hip hop listeners. “Co-membership is established within the local conduct of interaction as interlocutors reveal to one another aspects of common background” (Erickson 2001, 164); this digression functions to make relevant Dr. C and Jason’s shared identity as hip hop listeners, and perhaps their shared African American identity as well. In lines 12–13, Charles also claims these shared identities with both Jason and Dr. C.

What is the function of these displays of co-membership in the context of a mathematics lecture? It is worth quoting Erickson at length:

When one or more of these diverse kinds of commonalities are revealed during the course of talk, a change in footing can take place. Before the moment that co-membership is revealed the encounter may be bureaucratically neutral in tone, and after that moment the encounter may become more cordial. The relationship of superordination-subordination between the interlocutors may shift, not probably to reverse itself, but to become less extreme than in the moments before the revelation of co-membership.

A relationship of increased solidarity among interlocutors seems to obtain after co-membership has been established. The addition of co-membership features to those features of social identity already salient in the scene is a kind of innovation accomplished locally by the interlocutors—it is a bricolage construction within just that occasion of interaction and for just those moments, not something that will be done in the same way again with other interlocutors. It is a “value-added” feature of the scene at hand. (Erickson 2001, 164–165)
Talking about Eazy-E and engaging with Jason is a way for Dr. C to downplay the status differential between himself and his student, counterbalancing the power of a college professor with the solidarity of a fellow rap aficionado. While solidarity through co-membership is “accomplished locally by the interlocutors,” as Erickson writes, it is not the case that it is only accessible “within just that occasion of interaction;” rather, what we observe in Example 29 is contextualized within Dr. C and Jason’s ongoing teacher-student relationship, and also Dr. C’s rapport with the rest of the students, such as Charles, who are all not only ratified overhearers of their banter, but also members of a generation that has grown up with an awareness of hip hop culture. Jason often behaved in ways that could be taken as oppositional to the academic setting, such as wearing large over-the-ear headphones during lecture, and for this reason Dr. C may have considered it a strategic goal to build solidarity with him in particular. I found it notable that his tactics for establishing solidarity seemed grounded in their shared African American identity, and I asked about this in the interview; Dr. C’s response is transcribed in Example 30.

Example 30: ‘Those more cultural references’ (Interview, 8 May 2014)

1 Dr. C <I don’t know if it’s particularly>  
2 ta, connect with black students in the class,  
3 Daniel mm  
4 Dr. C um I definitely wanna try and reach  
5 y’know a broad, as broad a range of students as possible,  
6 Daniel mm  
7 Dr. C but my feeling is that, um  
8 even, like with some of those, more cultural references  
9 Daniel mm  
10 Dr. C might still be accessible  
11 to a lot of the other students too,  
12 Daniel mm  
13 Dr. C and um  
14 and so I think I maybe try to use that as a way  
15 ta get to,  
16 all the, ta get to a wider range of students and  
17 um  
18 in some, in some way?  
19 Daniel mm  
20 Dr. C y’know and in a, in some kind of inclusive way maybe?
My perception, as a white observer, was that Dr. C’s performance in Example 28 was meant to index his African American identity and thereby establish co-membership with Black students. What we see in the interview data is that Dr. C rejects this interpretation, and prefers instead to describe his “cultural references” as a tool that allows him to “try and reach as broad a range of students as possible” (lines 4–5). Later in the interview, he observed that among his students, “none of them fit into that role where they feel like they’re math people,” and the white male math myth would tend to suggest that people who look like Dr. C, who listen to hip hop, are also unlikely to be “math people.” In response to this, Dr. C positions himself as a math person (by virtue of his institutional status), but an atypical one (through his pop-cultural references); in this way, he hopes to establish co-membership with his students across the “math person”/“not-math-person” divide, not trading on Blackness per se but rather using pop culture “in some kind of inclusive way” (line 20). That is, if hip hop indexes Blackness, and Blackness indexes “not a math person,” then Dr. C intends hip hop references to establish co-membership with all of his students as being atypical mathematicians. African American identity is simply one tool that is available to him to help him accomplish this act of solidarity.

Considering my initial interpretation and Dr. C’s reflection to be two alternative interpretants of the quotient rule rap, we can say that his indexing hip hop culture both is and is not about African American identity. Our different understandings of his hip hop-inspired performance resonate with the nature of hip hop itself as an originally African American cultural form, closely tied to African American language (Alim 2009a; Smitherman 1997), that has nevertheless grown beyond its origins and given rise to a global youth culture of unprecedented reach (Alim 2009b). While these “cultural references might still be accessible to a lot of the other students” (lines 8–11), I would not feel comfortable making them if I were teaching this class; Dr. C’s performance is authenticated not only by his knowledge of rap music and its history, but also by his being recognized as African American. At the same time, the broader appeal of hip hop allows Dr. C to
deny that his Black identity is relevant even as he uses it to construct co-membership across racial lines. This apparent contradiction follows a history in which Black culture (slang, fashion, music) becomes youth culture becomes American culture, and takes place in the same historical moment as the liberal ideology of color-blindness, which holds that racism can be ended if we collectively choose to believe that race does not exist; operating within these fluid and contested boundaries, Dr. C takes the semiotic and cultural materials that are available to him, and uses them to accomplish his professional goal, to “reach as broad a range of students as possible.”

5.4.2. STUDENTS REFLECT ON “DIVERSITY”

The topic of the calculus class’s ethnic diversity came up in my interviews with students as well. First, I will present my interview with Mickey, who is Latino, a business major, and was one of the top students in the class. He talked about finding it easier to understand the material than many of his classmates, claimed that he had “always found math to be [his] strong suit,” and told me about the college-level math classes he had taken in high school. This was unexpected, given that he was a student in a quasi-remedial calculus section, and we will consider excerpts from his interview in which he provides one way of accounting for that mismatch. Following the excerpts from Mickey’s interview, I will present sections of my interview with Sara, a white American student who was majoring in government. A weaker mathematics student than Mickey, she told me she had worried about failing the class, and was surprised and gratified to earn a B.

Example 31 occurred at the end of my interview with Mickey. I had completed all of my prepared questions, and I concluded, as I typically do, by inviting Mickey to ask questions of me. He asked two: first, “Are you gonna like present this?” and then, following my answer to that question (and my assurances that I would protect his privacy), he asked, “I guess the only question I would have is, um. What’s the topic, again?” I responded by summarizing my research as an investigation into “how students learn to talk about math,” in which Dr. C’s class had served as “a field site for doing anthropology,” “a cultural group” in which “people get along, and get done what they need to
get done, by interacting with one another.” As I said this, I gestured toward my laptop monitor, which displayed a paused video of Dr. C’s classroom from a playback phase earlier in the interview. This seemed to bring Mickey’s attention to the image on the screen, and to the observations transcribed in Example 31.

Example 31: ‘Our backgrounds’ (Interview, 20 March 2015)

1 Mickey  I will say that
2 | (0.7)  
3 point to monitor 
4 Xander's from [here],
5 Daniel  Uh huh
6 | (0.5)  
7 Mickey  |Jason's from [here],  
8 raise and lower pointing finger 
9 Daniel  Uh huh
10 | (0.6)  
11 Mickey  and 
12 that's- we get along, 
13 | (0.7)  
14 Daniel  pretty well cuz,  
15 Mickey  sit back in chair 
16 our backgrounds? 
17 Daniel  uh huh
18 | (0.6)  
19 Mickey  whereas: 
20 | (0.5)  
21 Wendell would always talk about skiing, and,  
22 shake head  
23 | (0.8)  
24 Mickey  it was just some very different,  
25 lean forward, point to screen, sit back 
26 background 
27 and so I noticed though that a lot of 
28 | (0.8)  
29 Daniel  kids who came  
30 | (0.8)  
31 Mickey  lean forward, two finger point at screen 
32 who at least, had, somewhat similar background than mine, 
33 lean back, look at his hands on the table 
34 who at least, had, somewhat similar background than mine, 
35 lean back, look at his hands on the table 
36 Daniel  uh huh 
37 Mickey  whether it’d be our socioeconomic, or, 
38 educational, system?
39 Daniel  Uh huh uh huh
40 | (0.8)  
41 Mickey  or educational level,  
42 | (0.8)  
43 Daniel  u:h 
44 Mickey  were in that class.
Mickey’s reflections in this example can be seen as a description of the “cultural groups” in his class, responding to my description of my dissertation project. He constructs a subgroup of students with “somewhat similar background than mine” (line 20) who “get along pretty well” (10–11), and he specifically names himself, Xander, and Jason, but not Wendell, as members of this group. This act of social grouping can be seen as a narrative practice through which Mickey establishes his similarity and difference to others (Bamberg 2011), a stancetaking action in which Xander, Jason, and Mickey-in-the-story are positioned as sharing a certain social status, and the positive evaluation “we get along pretty well” is the interpretant through which Mickey-the-teller aligns with this positioning. As for how exactly this status is to be understood, Mickey mentions “socioeconomic [class] or educational system” in 22–23, and we can infer that he considers his group to be underprivileged in these areas; earlier in the interview, Mickey stated that he had grown up in one of the “worst ten education counties in the nation,” and while the university he attended was geographically distant from Mickey’s home county, it is located in a city with a similar reputation for troubled urban schools. (The city is transcribed as “[here]” in lines 3 and 6, but Mickey called it by name.) In contrast, Wendell’s “talk about skiing” (line 15) seems to function here as an emblematic index of higher socioeconomic class. In addition, it should be pointed out that Mickey’s group of students consists of young men of color: as mentioned above, he himself is Latino and Jason is Black, and I believe Xander is biracial, but Wendell is white, and if there are white working-class students in his calculus section, Mickey does not mention them. To understand this observation, we must take into account the pervasiveness of the “white male math myth;” even if Mickey does not mention ethnicity specifically, his self-concept as a mathematics learner has been constructed in settings where students like himself and Jason are not seen as prototypical “math people,” so we can read into this grouping at least a potential implicit orientation to ethnicity as a salient category of social identification.
Crucially, what sets this group of students apart is not that they find it harder to learn math or do well in class relative to their more privileged classmates. Earlier in the interview, Mickey said of himself, “I always found math to be my strong suit,” and conversely, when I asked him to “name names” of students who had difficulty keeping up with the material, he quickly identified Wendell and then took a long pause to think of others, going on to name Sara and then, reluctantly, Jason: “Maybe Jason? But me and Jason usually, at least in the first semester and ’bout half of the second semester, we sat next to each other so I would help him.” Mickey is a good math student, and he is available as a support to Jason because of their co-membership in this subgroup, but Wendell is not able to understand the content as well as Mickey and Xander do, his more privileged background notwithstanding. To the extent that Mickey does negatively evaluate his own mathematical ability or aptitude, he is clearly not positioning himself as less able than Wendell and Sara, but rather seems to be comparing himself to students in the traditional sections of introductory calculus.

Now that Mickey has introduced this group of students, we can consider what he goes on to say about them. Following up on what Mickey had said about his relatively poor preparation for college calculus, I asked, “Did you see that play out also in like how well people seemed to understand stuff?” Example 32 begins with his answer.

Example 32: ‘I think it is a factor’ (Continues from Example 31; 43 lines omitted)

<table>
<thead>
<tr>
<th>72</th>
<th>Mickey</th>
<th>I feel like where we come from</th>
</tr>
</thead>
<tbody>
<tr>
<td>73</td>
<td></td>
<td>(1.8)</td>
</tr>
<tr>
<td>74</td>
<td></td>
<td>I don't know why?</td>
</tr>
<tr>
<td>75</td>
<td>Daniel</td>
<td>Ah.</td>
</tr>
<tr>
<td>76</td>
<td>Mickey</td>
<td>But I think it is a factor.</td>
</tr>
<tr>
<td>77</td>
<td></td>
<td>(1.0)</td>
</tr>
<tr>
<td>78</td>
<td>Daniel</td>
<td>In what way</td>
</tr>
<tr>
<td>79</td>
<td></td>
<td>(0.8)</td>
</tr>
<tr>
<td>80</td>
<td>Mickey</td>
<td>Maybe just</td>
</tr>
<tr>
<td>81</td>
<td></td>
<td>where we're at,</td>
</tr>
<tr>
<td>82</td>
<td>Daniel</td>
<td>Uh huh</td>
</tr>
<tr>
<td>83</td>
<td>Mickey</td>
<td>Or our con- &lt;cognitive&gt; abilities,</td>
</tr>
<tr>
<td>84</td>
<td></td>
<td>whether it be our &lt;educational levels&gt;,</td>
</tr>
<tr>
<td>85</td>
<td></td>
<td>our backgrounds,</td>
</tr>
<tr>
<td>86</td>
<td></td>
<td>(1.7)</td>
</tr>
</tbody>
</table>
and just if we're prepared.
Because like I said like for me like
you're expected to at least get to algebra two?
Daniel  Uh huh
Mickey  And if you got to trig,
then,
trig 'n yeah, if you got to trig,
'tr precalc,
then that was like
a good thing for kids.
Daniel  Mm.
Mickey  Um
(1.1)
Thing- th- thing about algebra two
that's like freshman (.) year level.
Daniel  Mhm.
(0.7)
Mickey  So.
Daniel  So some of these other students: like
came from a background where the people had higher-
expectations?
Mickey  Yeah, yeah, that's what it was.
Yeah.

This example begins with an evaluation of "where we're at" (line 81), that is, the level of mathematical aptitude that is common to members of Mickey's group. Specifically, he characterizes this aptitude in terms of "cognitive abilities" (83), "educational levels" (84), and preparation (87). Along with these negative evaluations, Mickey takes a cautionary epistemic stance regarding his own statements: "I feel like" (72), "I don't know why" (74) "I think" (76), "maybe" (80). Even his listing of multiple potential explanations, "whether it be" (84) this, that, or the other factor, expresses a reluctance to speak definitively about the reasons for his group's mathematical underperformance. This is an embarrassing claim to make, that he is not as well prepared for college as his more privileged peers in the mainstream calculus sections, and perhaps his
uncertainty serves to mitigate the self-directed face-threat. At the same time, it is difficult to connect causes and effects in educational outcomes, and the best Mickey can do is to offer a guess.

In lines 89–99, we see a shift in footing into a narrative-like sequence in which Mickey describes his experience in high school. In this sequence, which is framed as a concrete illustration of his preceding reflection (“Because like I said,” line 89), he voices a stance that for students to complete anything beyond Algebra II would be considered exceptional, “a good thing for kids” (line 99). In 103–104, having articulated this position, he disaligns from it, characterizing Algebra II as “[high school] freshman year level” content, that is, something that should be a bare minimum requirement rather than a standard expectation of all students. During this stretch of talk, the epistemic markers have vanished; speaking from personal experience, rather than conjecturing about underlying causes, allows Mickey to speak with more certainty. At the same time, the agentless passive voice “you’re expected” in line 90 and the unattributed positive evaluation “that was like a good thing” in 98–99 position “you,” the generic student in Mickey’s high school, as subject to the policies of an unnamed authority whose position Mickey is voicing. As a student, he demonstrates epistemic certainty here, but not institutional agency; the claim is that he understands the forces that are disempowering him.

In lines 108–110, I offer an evaluative coda to Mickey’s narrative, suggesting that the best explanation for his peers’ lack of preparation is low expectations rather than cognitive deficiency. In this utterance, I was disaligning from a racialized view of mathematical competence that had seemed to me implicit in Mickey’s reference to “cognitive abilities;” I cannot say what this term means to Mickey, but to me, it suggested something innate that would tend to vary across racial groups, as in the scientific racism of The Bell Curve. By mentioning expectations in line 110, I was trying to give Mickey a way of accounting for his presence in a quasi-remedial calculus section.

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7 The belief that Algebra II is a reasonable graduation requirement is one that Mickey distanced himself from not only through explicit disalignment, but also through self-positioning in narrative; earlier in the interview, he claimed to have been the only student in his graduating class to take the Advanced Placement exams in both calculus and statistics.
without relying on a sense of himself as cognitively deficient. What is clear in Mickey’s account is that even having been accepted to this university does not put him on a level playing field with students like Wendell. At his school, it was considered “a good thing” for kids to graduate high school at what’s really “freshman year level” math, and as he states a few lines later, this “just carries on, carries over.”

The overall effect of this extended discussion, comprising Example 31 and Example 32, is to position Mickey as underprivileged for reasons of socioeconomic status and perhaps ethnicity as well, thereby explaining his presence in this calculus section rather than a traditional section. At the same time, despite his constructing and aligning with a group of his peers who suffered from a poor secondary education, he also positions himself as a relatively successful mathematics learner. Despite the circumstances of his upbringing, he is able to understand the content and earn grades that exceed those of the more privileged Wendell and Sara.

The topic of ethnicity came up in Sara’s interview as well, as shown in Example 33. In this case, I had asked her specifically about diversity; she said that, as a government major, she tended to be in class with a lot of international students, but the calculus class did stand out in one respect.

*Example 33: ‘American minorities’ (Interview, 8 April 2015)*

1. Sara Um, the difference in this class is, there are a lot of,
2. Daniel div- there are a lot of, minorities, American minorities?
3. Sara mhm
4. Daniel who,
5. Sara I n- like can tell?
6. Daniel went to, a, maybe a bad school that didn't prepare them?
7. Daniel mhm
9. Daniel mhm
10. Sara And,
11. Daniel like I mean I can think of like,
12. Daniel I- I'm pr- pretty sure that's true
13. Daniel for a lot of people in my cla,
14. Daniel in our class.
15. Daniel mm
16. Sara Who, would say, like,
17 y'know, we- I would talk to them,
18 I had precalc and stuff like that,
19 and I talked to them and they said they like never have,
20 Daniel mhm
21 Sara done precalc, regular calc, any calc.
22 Daniel mhm
23 Sara Um,
24 y'know, they got to, like, geometry.
25 So
26 (1.0)
27 that would be a huge problem.
28 Daniel mhm
29 (0.6)
30 Sara At this school.

In this example, Sara is not primarily positioning herself, but refers instead to a subset of the other students in her class consisting of “American minorities” (line 2), a social status that she links indexically to attendance at “a bad school” (6). Looking at her epistemic markers, at first she makes this link based on her own perception of her classmates (“I like can tell,” line 5), but then she downgrades her certainty (“I’m pretty sure that’s true for a lot of people,” 12–13), and ultimately reframes her opinion as based on hearsay rather than direct knowledge (“[They] would say ... I would talk to them ... I talked to them and they said,” 16–19). The story that she recounts, taking on the voice of her “minority” classmates, seems similar to what Mickey described in Example 32—graduating from high school without having taken the courses prerequisite to college calculus—but where Mickey talked about a school culture of low expectations, Sara only reports that “they like never have done” (19–21) the coursework, without attempting to explain why the school “didn’t prepare them” (6). What Sara does provide, unlike Mickey, is an explicit connection between this sort of high school experience and the students’ ethnic minority status, but then she rethinks that as well, as shown in Example 34.

Example 34: ‘More about being poor’ (Continues from Example 33)
31 Sara Um,
32 and the other thing wi- like- I- and I know this because,
33 last semes-
and- and i- I think it might be less, like, about, being a minority, and more about being p-
poor.

Daniel    mhm
Sara    Because last semester,
I was in,
what, er I mean,
	f:all semester,
when I took like the first half of the math thing?
Daniel    mhm
Sara    I was in um
(0.8)
a class that had
that was like, a pretty diverse class?
but >some of the< worst people in the class were white,
but, they were from a poor background.
Daniel    mhm
Sara    and I know this because
one of the girls in my class was also from [my hometown],
Daniel    mhm
Sara    but was- went to like a really,
like bad school in [my hometown], and,
Daniel    mhm
Sara    my class was
significantly,
athletes who got full ride scholarships
for being athletes.

Sara’s claim in lines 31–36, marked with numerous false starts and hedges (“I think it might be,” “like”), is that the lack of preparation she has observed in her classmates is indexically linked less to ethnicity than to socioeconomic status. In line 32, she begins to offer evidence before she has even made the claim, but cuts herself off in 33, and then reinitiates the evidential grounding through narrative beginning in 38. The relative importance of race versus class in predicting educational outcomes is a sensitive topic, especially for an outsider like Sara, who not only is white but also, elsewhere in her interview, aligns with high-SES peer groups; both her hesitation and her premature offering of evidence (anxious to prove, perhaps, that her opinion is not mere stereotype) can be read as acknowledgements that she is edging toward statements that might be considered politically incorrect.
Within the narrative section that begins on line 38, Sara describes a new social status in contrast to the “American minority” position described in Example 33. Where her previous description linked “minority” ethnicity with lack of preparation for college calculus, this example constructs a group of students who were “white, but they were from a poor background” (lines 48–49); and in this case, rather than connecting this category with a lack of preparation, describes these students as “some of the worst people in the class” (48)—a bare negative evaluation with no attempt to excuse or explain it. Sara also claims greater epistemic rights to talk about this group than about the “American minorities;” in 51 she makes the strong epistemic claim “I know this,” grounded in her privileged knowledge of her hometown and ability to recognize a “bad school in [her hometown]” (55), and her following mention of scholarship athletes functions as an implicit claim that these students were not prepared for college math, or perhaps for college at all, but were offered admission so that they could play on sports teams. Overall in this example, not only does Sara position poor white students as having worse preparation than their peers from underserved ethnoracial groups, but she also positions herself as being able to speak about them with more authority. She finds it difficult to make claims across lines of ethnicity, as shown by her hesitation to state that low math performance is “less about being a minority,” but she shows no such reluctance to talk about poor white students.

Comparing Mickey and Sara’s interviews, it becomes apparent that the meaning of ethnicity in Dr. C’s classroom is ambiguous and complicated. Both students seem more comfortable to make claims about students of their own ethnicity (construing Mickey’s ethnicity broadly as “non-white,” based on his claimed co-membership with Jason and Xander). Both observe that students of color tend to be less well prepared for college mathematics than their white peers, but qualify that observation by citing examples of white students who actually do worse in class or have a harder time understanding the material. Like Dr. C, they recognize that while students of color are not viewed as prototypical mathematicians, and may be expected to see college mathematics as
something that is not “for them,” this correlation does not determine students’ educational trajectories. On the contrary, both Mickey and Dr. C demonstrate that students of color can use their ethnic co-membership as a resource, taking on mutual responsibility for one another’s learning; and Dr. C takes this a step further, encouraging his students to find co-membership in their being “not a math person,” and to support one another as peers, regardless of ethnicity.

5.5. STANCETAKING, POSITIONING, AND THE POSSIBILITY OF SOCIAL CHANGE

This chapter began by highlighting the subtle connections between interactional practices and social structures, and proposed that a focus on identity would help to make these connections visible. What we have seen is that, while roles and identities such as “good math student” or “African American” may often be understood in an essentializing way as perduring traits of biological individuals, microethnographic analysis reveals them to be interactional accomplishments, produced and reproduced through social actions such as stancetaking, and achieved within actually occurring interactions. Nayan is recognized as a good student not only because he can solve problems quickly and accurately, but also, perhaps more importantly, because he participates confidently in classroom discourse. Dr. C’s Blackness is not only about his physical appearance, but is also expressed through his pronunciation of “forget,” his familiarity with rap artists, and even, I would argue, his belief that students learn better if they feel responsible for one another. And these identities are further solidified through expressions of co-membership with others who identify in similar ways; the banter between Nayan, Fernando, and Ricardo, or between Dr. C, Jason, and Charles, seems to say, whatever I am, so too are you, and we understand one another.

At the same time, these social positions become affordances that make space for particular kinds of engagement within the institutional setting, in ways that may seem unexpected or even paradoxical. Having achieved universal recognition as a good student, Nayan is able to be publicly wrong without worrying about losing face or seeing his status diminish. Dr. C’s being African
American opens the door for him to be an atypical mathematician in other ways, such as his intentionally fostering a learning community among his students, and his prioritizing pedagogy over pure mathematics in his research program. These sorts of engagement demonstrate the process of social creativity that Erickson (2004b), following Michel de Certeau (1984) and Claude Lévi-Strauss, terms *bricolage*: the recombination of existing means, in a particular interactional moment, in order to accomplish novel ends. If university mathematics departments are normatively understood to be populated by white men whose primary interest is pure mathematics research, then an African American mathematician who wishes to work with “students that kinda traditionally struggle” must be a bricoleur.

Understanding a particular instance of bricolage as a stancetaking act provides a way to begin seeing the influence of interactional choices on social structures. If Dr. C is positioned, or positions himself, not as a deviation from the prototypical way of being a mathematician, but rather as a different kind of mathematician, then his way of existing within the institution of higher education starts to be reified. If he continues to be positioned in this way, then through the process of stance accretion, a novel kind of status is constructed that is indexed by particular social roles and identities. And as mathematicians who position themselves in similar ways come to recognize and authenticate one another through expressions of co-membership, they may form a community of practice that ultimately comes to redefine the prototype from which they had initially deviated.

The same sort of process could conceivably refigure not only the types of persons who are legitimated as mathematical authorities, but also the nature of mathematics education itself. Chapter 4 explored the traditional ideology of mathematics education according to which there is always exactly one right answer to any question (Boaler 2002; Boaler and Greeno 2000), and if students align with this ideology, then uncertainty as to the accurateness of one’s answer would tend to inhibit confident participation and, as a result, restrict the range and diversity of students who are able to position themselves as good mathematics students. On the other hand, if
mathematics can be reframed as a process of reasoning and problem solving, a more creative and egalitarian endeavor than what the traditional ideology allows, then more students could “develop identities of mathematical competence” (I. S. Horn 2008): by being brought into active participation, they may thereby come to see themselves as good students, and eventually as “math people.”
Chapter 6

AFFORDANCES AND LIMITATIONS OF MATHEMATICS CLASSROOM INTERACTION

6.1. A REVIEW OF THE PRECEDING

Up to this point, I have been concerned with patterns of mathematics classroom interaction in which a particular set of semiotic resources, including talk, gesture, diagrams, and written notation, is used for a variety of interactional purposes. Chapter 3 considered how mathematical objects can be understood as either continuously changing or discretely subdivided, and how metaphors of participation—occurring primarily in gesture, but potentially in any medium—provide contextualization cues that may privilege a typological or topological understanding. In Chapter 4, a focus on students’ roles in initiating and completing conversational repair provided insight into the epistemic rights that they are granted with respect to these mathematical objects. Finally, Chapter 5 considered actions of stancetaking in discourse as realizations of intentional and social statuses, which solidify into classroom identities and co-membership groups.

At each stage of the analysis, the semiotic theory of C. S. Peirce has been helpful for modeling different objects and processes: the iconicity of mathematical formalism, the representational agency of students, and the realization of stancetaking as an embodied sign that is appropriate and effective in discourse. Looking back, we can also view the overall progression of the analysis as a development through what Richard Parmentier (1987, 31) calls “Peirce’s three degrees of reality”: Firsts, which are qualities; Seconds, actually existing objects; and Thirds, patterns or laws. In the mathematics classroom, the raw materials of mathematical communication such as equations and gestures are Firsts; actual utterances and sequences, the units of classroom interaction, are Seconds; and positions and identities arise from patterns of interaction, and therefore are Thirds.
To integrate the findings of the preceding analytic chapters, this chapter takes up the Firstness, Secondness, and Thirdness of mathematics classroom communicative practices in order to identify the *affordances* that these practices offer for the accomplishment of situated social actions—that is, how they enable students and teachers to “get things done” in the mathematics classroom. In the course of observation, I have noticed that certain semiotic practices are normatively legitimated, seen as “the right way” to use diagrams and equations, participate in discourse sequences, or perform social roles; at the same time, participants’ goals and interests often require them to improvise new semiotic practices that are different from the standard. In the space between the official and the improvisational, I identify *problems of practice*, social goals that neither normative nor improvised practices can accomplish. I begin by fleshing out this interpretive model and its intellectual history, and then apply it retrospectively to the preceding chapters’ analyses. The problems of practice identified here are among those that have been previously identified by mathematics education researchers, which can be viewed as a practical validation of the semiotic approach; at the same time, it is my hope that the methods of linguistic ethnography will provide useful insights into these problems by grounding them in theories of language and communication. I conclude by evaluating the study along these lines, and arguing for more focused interdisciplinary work between linguistic anthropology and the learning sciences.

### 6.2. **Semiotic Practices and Their Affordances**

The concept of *affordances* originated in the ecological psychology of James Gibson (1977; 1979; Costall 1984) in response to the cognitivist theory that the world “out there” is only perceptible to us in a mediated form, seen through an intervening layer of mental representation. Instead of this, Gibson proposed that our perceptions have a physical reality that corresponds to the perceptible aspects of the world around us, and therefore the proper unit of analysis is not the individual mind as a sort of ghost in the machine, but rather the relationship between organism and environment. Within this theory, affordances are defined as “the functions that objects serve for our
activities” (Costall 1984, 113), such as the surfaces of objects that allow us to perceive them visually, or the sequence of events that reveals the passage of time. Kockelman (2006b) transports this construct from a theory of perception to one of semiosis, understanding an affordance to be a semiotic process whose sign is a feature of the natural world, and whose object is the purchase that this feature provides for the accomplishment of action. In Kockelman’s formulation, Gibson’s directly perceived affordances are semiotic processes whose sign is a Firstness—the visible surface of a physical object is one of its qualities—but Seconds and Thirds may offer purchase as well, opening up the possibility for representational and social affordances.

Of course, human action takes place not only in the natural world, but also in semiotized cultural spaces and communities of practice, and so we can speak of manufactured artifacts and social practices as providing particular affordances. Unlike the natural world, however, cultural settings are regimented by systems of norms, so a distinction can be made between affordances that are normatively legitimated and those that are improvised according to the exigencies of the moment. Following Michel de Certeau (1984), I call the first of these a strategy and the second a tactic. For example, strategy is using the blade of a screwdriver to drive a screw; tactics is using its handle to drive a nail when you’ve lost your hammer.

Considering the use of the screwdriver to be a sign, we see that strategic and tactical action are semiotically quite different. Peirce’s semiotic begins with the division of signs into qualities (Qualisigns), actual objects and events (Sinsigns), and laws (Legisigns) (Peirce [1902] 1955, 101–102); while both uses of the screwdriver are Sinsigns whose object is the purpose of the worker’s action, the strategic use is a token of a type—a Replica of the Legisign “Screwdrivers are for driving screws,” which is embodied in the form of the tool—and also iconically represents prior instances of screwdriver use that the interpreter may remember. The tactical use, on the other hand, is seen as a unique occurrence with no such semiotic connection to norms or past experiences; even if the user is not the first person to hammer a nail with a screwdriver, the action’s status as a tactic
depends on its being understood as an improvisation rather than a habit. The loose formal resemblance that the screwdriver bears to the missing hammer is an affordance that makes tactical improvisation possible in a process that Certeau, drawing on Lévi-Strauss, calls *bricolage*.

The boundary between strategies and tactics is fluid, as today’s bricolage may be tomorrow’s norm; if the worker continues to drive nails with the screwdriver, this may come to be reflected in local practice (other members of the workshop ironically calling the screwdriver “the hammer”) and also in the form of the tool itself (tool marks on the handle of the screwdriver), and in other cases where improvisation leads to emically recognized improvement over the norm, more durable social change may result. Nevertheless, the distinction is a useful heuristic, particularly if we compare the interpretants of strategic and tactical action. As we have seen, strategic action indexes a norm through its iconic resemblance to similar instances in the interpreter’s prior experience. As a result, one affordance of strategic action is that in its interpretant, it provides an indication of the type, purpose, or character of action—its metamessage (Bateson 1956) or storyline (Van Langenhove and Harré 1999)—through a process that has been variously termed framing (Goffman 1974; Tannen 1993), contextualization (Gumperz 1992), metapragmatics (Silverstein 2003), or action ascription (Heritage 2013). In order for metapragmatic frames to function, however, participants must mutually recognize them as “proper;” this is the insight behind the cases of intercultural miscommunication that Gumperz presents, in which speaker and hearer’s conflicting norms result in divergent metamessages. In the case of tactical action, then, the metamessage is not recoverable because the action itself cannot be reliably abstracted beyond its immediate circumstances:

The “proper” is a victory of space over time. On the contrary, because it does not have a place, a tactic depends on time—it is always on the watch for opportunities that must be seized “on the wing.” Whatever it wins, it does not keep. It must constantly manipulate events in order to turn them into “opportunities.” (Certeau 1984, xix)
At the same time, "Many everyday practices (talking, reading, moving about, shopping, cooking, etc.) are tactical in character" (Certeau 1984, xix). So, while situated social action is inevitably tactical, strategy is needed to provide contextualization, and between the two levels of tactics and strategy, there are bound to be gaps, types of coordinated action that neither strategic nor tactical action can accomplish. In a community of practice such as a mathematics classroom, these gaps represent problems of practice, areas where new norms must be established to make particular social actions recognizable. These problems may be addressed on the local level, as in the habitual use of a screwdriver to drive nails if a hammer is for some reason unavailable, or larger-scale solutions may arise, such as the invention and mass production of a new sort of tool. By observing strategic and tactical action in the course of interaction, we will be able to clearly describe particular problems of practice, and thereby identify specific areas of focus that may be of practical use, as well as loci of social change in progress.

6.3. Mathematics Classroom Strategies and Tactics

Having developed an interpretive framework to characterize situated semiotic practices in terms of their strategic and tactical affordances, I now bring this perspective to bear on the preceding analytic chapters, viewed respectively as descriptions of the Firstness, Secondness, and Thirdness of mathematics classroom discourse. In this way, I will be able to articulate a synthesis of findings in which the tools of linguistic anthropology and semiotic theory provide insight into problems of practice.

6.3.1. Affordances of Technical Notation

Chapter 3 identified a paradox in the representation of certain mathematical objects—specifically, rational numbers in the middle school, and functions on the real numbers in the calculus class—in which the objects themselves are understood to be continuously variable, but the nature of symbolic reference forces these objects to be represented in discretizing ways. That is,
while the real number line itself may be thought to have no ultimate parts, we find ourselves unable to describe and reason with it unless we apply some sort of subdivision. Subdivisions may be standardized (e.g. subdividing the real numbers by powers of ten, as in decimal representation) or ad-hoc (e.g. dividing the domain of a function into intervals on which the graph of the function has a particular identifiable shape, as in the graphing episode), but some partitioning is necessary. To accomplish these partitions of continuous space, the disciplinary practices of mathematics include norms that conform with Goodwin's (1994) description of professional vision: aspects of a problem are highlighted and coded in ways that lead to standardized material representations of mathematical objects.

What mathematical notation provides in this respect—the affordance by which it extends symbolic reference beyond the capability of language and into the domain of topological meaning (Lemke 1999; Lemke 2003)—is not an icon of continuity like that provided in mathematical diagrams, but rather an approximation of continuity using symbolic reference. To see how this is accomplished, I highlighted the aspects of fractal recursivity (Irvine and Gal 2000) that go beyond social grouping and apply to semiosis and cognition more generally: if a continuous phenomenon has no ultimate parts, it may still be partitioned into subintervals, each of which is itself continuous and may be similarly partitioned. (It is no accident that fractals and recursion originated as mathematical concepts, and in retrospect it feels natural that they should be applicable to mathematics viewed as social practice.) The clearest example provided here is the Riemann sum episode, which showed that continuity may be approximated by constructing indefinitely many subdivisions of indefinitely small size. Riemann sums and decimal arithmetic are strategic uses of mathematical formalism, in Certeau's sense, and their institutional legitimation is demonstrated by their being included as a lesson or unit topic in a course of study.

If a use of mathematical notation is a sign with symbolic grounding, does it refer to a discrete or continuous object? Considering the object of a sign to be “that to which all interpretants
(of the sign) correspondingly relate” (Kockelman 2005, 242), we find that the same notational sign may be construed as referring to an object that is either discrete or continuous. Not only did we see continua like the real numbers broken into discrete subsections, but we also observed that the fraction \( \frac{5}{8} \), which seems to represent a selection of five out of eight parts of a whole, may be estimated as a location on a continuum, a point on the number line somewhere between 0.6 and 0.7, at a first approximation. To clarify this potential ambiguity, teachers used *metaphors of participation* as a sort of contextualization cue that frames a mathematical object as discrete or continuous. For example, in the Riemann sum demonstration, Dr. C characterized the same interval first as a partition, by using staccato gestures and the word “break,” and then as a continuum, indicated through smooth gestures and the question “how far is it?” The normativity of this metapragmatic framing rests not in established mathematical practice, but rather in the iconicity of the metaphor itself, as using chopping movements to refer to fluid surfaces would seem incongruous.

The affordance of fractal recursivity is that mathematical objects may be represented as more or less continuous at varying levels of magnification, and this optionality provides a tactical use as teachers improvise mathematical explanations. The approximation of a continuum as a series of subsections is useful not only for standardized algorithms like the Riemann sum, but also for pedagogical simplifications like the assistance Ms. M provided to Elena in writing the fraction \( \frac{5}{8} \) as a decimal. While these two moves seem similar, there is a difference in their metapragmatic framing, resulting from the normative status of the Riemann algorithm and the improvisational character of the teacher explanation. In the canonical understanding of a Riemann sum, the function really is continuous, and the sum is only an approximation; this is why the *definite integral*, the precise quantity that a Riemann sum estimates, is defined as the limit of the Riemann sum as the width of the subintervals approaches zero. As a learner comes to know these ostensibly continuous mathematical objects, however, they are presented as discretely subdivided, using the
mathematician’s professional vision to locate and name their salient features; the student is eventually expected to make a conceptual leap, comparable to the limit step in the derivation of the integral, that allows them to see the object holistically.

The move from discrete to continuous thinking is just one case of the way mathematical concepts are typically presented: first as operational processes such as counting, but later as structured objects such as the numbers. Mathematics education researchers (Sfard 2008; Sfard and Linchevski 1994) call this reification—from a semiotic standpoint, it might be termed rhematization—and it has been identified as a fundamental source of many difficulties experienced by mathematics learners:

most of the time algebraic formulae are for some pupils not more than mere strings of symbols to which certain well-defined procedures are routinely applied. In these students’ eyes, the formal manipulations are the only source from which the symbolic constructs may draw their meaning…. the sense of meaningfulness comes with the ability of “seeing” abstract ideas hidden behind the symbols. (Sfard and Linchevski 1994, 223–224)

This is the problem of practice created by the very flexibility of the notation. If a fraction or an interval may be treated as either discrete or continuous, then what kind of thing is it really? As long as the interpretants of the notation remain contradictory, there will be no “correspondence-preserving projection” from them, that is, no identifiable object. Contextualized within problem-solving events, the referential quality of the notation is a Firstness that students take on faith, as long as they can find the answers to particular problems, the Secondness by which their learning is typically evaluated. Moreover, “more often than not, the pupil cannot cope with problems which do not yield to the standard algorithms” (Sfard and Linchevski 1994, 223); the Thirdness in which different types of problems represent multiple realizations of more general laws remains imperceptible as well.
Is there a way out of this limited understanding of mathematical objects? In contrast to my observations, consider the discursive practices of physicists in laboratory meetings (Ochs, Gonzales, and Jacoby 1996), who deal with similarly imperceptible phenomena—in this case, the spin of atoms in a magnetic field—but speak about these processes as though they were imagining themselves among the atoms, subjectively present in the place of their objects of study. These professional researchers discursively construct themselves as participants not just in scientific inquiry, but also, metaphorically, in the phenomena being studied. In this way, the abstract becomes personal. Similarly, when research mathematicians appear on talk radio (Barwell 2013), they blend “everyday” and “mathematical” language, using concrete images such as “hyper-bagel” and “hyper-football” to nail down counterintuitive concepts. As Barwell points out, teachers often presume that “academic language” is maximally technical, but in fact actual academics do not talk that way, relying instead on metaphor to ground abstract referents in everyday experience. Perhaps what is needed is new metaphors of participation, designed to support students’ development of a mathematical ontology.

6.3.2. Affordances of Interactional Sequence

Chapter 4 presented an analysis of students’ roles in accomplishing conversational repair, and by interpreting their participation as an indication of the degree of semiotic agency that they were granted in classroom discourse, it became possible to see these discourse practices as realizations of a prevailing ideology of mathematical knowledge and classroom participant roles. The practices I observed were similar to those described by Jo Boaler as “traditional” or “didactic,” in which teachers explain algorithmic procedures and then assign problem sets that allow students to show mastery by replicating the procedures quickly and accurately. If students’ “ways of participating are adaptations to the constraints and affordances of the environment” (Boaler and Greeno 2000, 172–173), then in the idiom developed in this chapter, a clear description of the
environment itself should allow us to see which ways of participating are strategic and which are tactical, and to identify problems of practice inherent in these classroom discourse practices.

Boaler and Greeno’s (2000) concern is to relate students’ mathematical epistemologies to the instructional practices in which they participate. In the case of didactic teaching, they show that book work, algorithmic procedures, and a focus on speed and accuracy support an ideology of “received knowing” in which authorities such as the teacher and textbook are held to be in possession of certain knowledge that the student is responsible to acquire. Strategic actions are those that reflect this ideology, such as the prevalence of epistemic evaluations as sequence-closing thirds (the familiar IRE sequence), the use of thumbs-up / thumbs-down as an understanding check (rather than an invitation to debate), and the reframing of student corrections as anticipations of the next point in the lecture. In all of these patterns, the teacher alone reserves the epistemic right to ratify mathematical statements as correct or incorrect, or more precisely, as appropriate or not (e.g., incorrect, imprecise, premature, or off topic). This allocation of authority is the metapragmatic framing that supports “traditional” pedagogy in any discipline; in Mehan’s (1979b) prototypical example, we understand that when the teacher asks us what time it is, it is not because they want to know, but because they are checking whether we know.

While students are afforded a certain degree of residential agency in classroom discourse—Tiana favorably compared Ms. M’s teaching to the approach she experienced in Madagascar, because she is now able to decide where and when she will speak up—their representational agency is limited. In semiotic terms (Kockelman 2007), we see that “traditional” practices thematize mathematical objects in the questions that teachers ask, leading students to characterize these objects by answering the question, and teachers to incorporate the resulting arguments in an ongoing process of reasoning. For example, in Chapter 3, when Ms. M applied her professional vision to the interval from 0.5 to 1.0, she highlighted the lower bound of the interval, elicited the coding “zero point five” from Elena, and then incorporated that code into her gestural material
representation. So we have seen that not only students’ understanding of notation (a First) but also their classroom participation (a Second) is limited to the mathematical representations themselves, eliding the qualities of those objects as well as the patterns of relations among them.

At the same time, students use their residential agency tactically. Tiana also stated that when students choose not to participate actively, they may have specific misunderstandings or questions about the material that they choose not to ask, but instead “they’re waiting for someone else to raise their hands.” But if high status students are the ones who feel empowered to ask potentially embarrassing questions, and status correlates to some degree with proficiency in the material (although only to some degree, as status is complicated; see I. S. Horn 2011), it seems inevitable that the students who wait will often wait in vain. As this waiting is a tactical move, there is no contextualization cue that signals it; if a student asks a question, it may be difficult for the instructor to gauge how many of their peers are silently wondering the same thing, and if a student remains silent, it is difficult to guess what questions they may be choosing not to ask.

The problem of practice is not how to get more students to speak up when they are wrong or uncertain, as this misses the point in two respects. First, it overlooks the fact that other avenues are open even to low-status students who have specific questions; several of the middle school students told me that they felt free to approach the teacher outside of class. More to the point, though, framing the problem in this way replicates the ideological position that “being right” is what is desirable above all. When practitioners, particularly teacher educators, write about “classroom discourse,” they tend to define it normatively as “the genuine sharing of ideas among participants” (Hancewicz 2005, 72) or the kind of teaching practices that can promote this kind of interaction. But if students believe that there is always exactly one right answer to any question, and that the teacher already knows it, what point is there to engaging in a “genuine sharing of ideas”? If this is the pedagogical goal, then a focus on discourse practices treats the symptom rather
than the cause; the real barrier to student participation is their lack of representational agency over mathematical objects and ideas, which is rooted in the one-right-answer ideology itself.

To move beyond this concept of mathematics education, perhaps we can begin once again by taking a cue from professional mathematicians. As Lampert (1990) writes, the community of mathematics researchers prizes intellectual courage and creativity over computational speed and accuracy; how can these values be reflected in classroom practice? One way is to pose problems that can be answered correctly in multiple ways (as in Danielson 2014), shifting students’ agency from characterizing—which is trivial in Danielson’s book, as any of the answers is potentially correct—to reasoning, explaining why their answer is true, and working to resolve differences of opinion with their peers. Another possibility is to allow students more agency to thematize mathematical objects, for example, by posing problems that have one right answer, but may be solved in various ways. As mathematician James Sandefur (personal communication) told me, in this scenario, all the solutions arrive at the same final result, but each provides a different understanding of the initial question; thus, by increasing students’ representational agency, we may also lead them to a deeper understanding of mathematical objects.

6.3.3. AFFORDANCES OF PERFORMED IDENTITY

As we have seen, when students participate in “traditional” mathematics classes, they are positioned as “received knowers.” Positioning theory (Harré and Van Langenhove 1999) relates a social actor’s position to the types of social force that their utterances may have, and to a storyline of what kind of interaction is taking place; in this case, within the classroom discourse storyline, a “received knower’s” answer to a teacher’s question is primarily hearable as an attempt to find the “right answer.” Within this dynamic, students may be additionally positioned as having a relatively higher or lower status, which leads to their questions having different social force. When Nayan admitted he had a wrong answer, this was taken as a sign of confidence, and the teacher thanked him for raising an important question, but in the calculus class, Mickey identified students whose
questions evinced a lack of understanding and, in his opinion, held back the progress of the group as a whole.

Within each classroom setting, there are norms through which “good students” are identified and status is allocated. The strategic action in this setting is to “do being-a-good-student,” as Nayan does: to build social capital by publicly volunteering correct answers, and to invest this capital through active, enthusiastic class participation. This was theorized in chapter 5 as a stancetaking action that may over time, through a process that Bucholtz and Hall (2005) call stance accretion, solidify into a “good math student” identity. As we observed, however, this strategy is not available to all students. Some are not quick enough with the correct answer, which led us to consider the “traditional” ideology itself as a problem of practice, but in other cases, even students who are proficient with the material may have trouble being seen as “good math students” by the teacher (like “smart-but-lazy” Alberto), or even in their own self-concept (like Mickey, who got A’s but still doubted his own “cognitive abilities”). In my observations, participants’ main tactical response to this disconnect was to appeal to non-academic identity categories to construct co-membership groups whose members are then available to one another for mutual encouragement and assistance. This is the sort of action that Dr. C hoped to foster by offering extra credit to study groups, and that Mickey enacted by sitting next to Jason. Rather than violating norms or acting independently of them, this tactic achieves a degree of metapragmatic grounding through intertextual references that make non-academic identities relevant in the classroom, and thereby appeal to alternative sets of norms from outside the classroom setting. Rhyming over a beat is not a mathematical practice but a hip hop practice, and Dr. C’s quotient rule rap signals that classroom norms regarding who is “in” and who is “out” may be to some degree suspended. This act of bricolage carries a certain amount of metapragmatic ambiguity, however. Is the quotient rule rap to be considered a flawed attempt at rapping, as Jason suggested; a sign of Black solidarity in the normatively white world of mathematics, as I assumed; or a more broadly inclusive move
irrespective of ethnicity, as Dr. C claimed? This is the gap between strategy and tactics at the level of identity: while different student identities may have a relatively clear realization in classroom discourse, and while being Black in a mathematics classroom clearly means something, exactly what it means is ambiguous, contested, and variable.

Like the technical notation and the discourse practices described above, classroom identities are also available to students in a limited way that restricts their agency to the level of Secondness. In this case we may consider the residential aspect of agency:

the rights and responsibilities that make up a social status (such as priest, sheriff, mother, doctor) may often be described in terms of semiotic rights and responsibilities: what kinds of actions and utterances one may or must make as a function of the social context one is in. In this way, a key constraint on our residential agency is the kinds of social statuses we relationally inhabit to the extent that these social statuses enable and constrain semiotic processes as to their when and where (control), their what and how (composition), and their why and to what effect (commitment). (Kockelman 2007, 382)

This claim of Kockelman’s is meant to characterize statuses such as student, and in fact the students I observed generally needed permission to speak, they were sanctioned for speaking off topic, and their utterances were subject to ratification by the instructor. At a finer level of magnification, we can also consider good student as an identity that enables and constrains in somewhat different ways. For example, Tiana expressed a preference for the greater degree of control she felt in Ms. M's class than in her home country, and Fernando expressed admiration for students who compose mathematical claims even when they cannot commit to the interpretant, who “don’t care if they have wrong answer or right answer, they just show it.” In this sense, we see that expressing a greater degree of residential agency is one way in which students can position themselves as “good students.”
At a different level of semiotic framing, however, we can also consider the expression of residential agency itself to be a sign whose object is the “good student” positionality. At this level of framing, students have limited agency to express what kind of student they are. Their control is limited in that they may only participate in certain ways, during certain phases of class, and among these phases, some sorts of participation (e.g. volunteering to read aloud from the board) confer more status than others (e.g. finding the correct answer on a worksheet). If students have any agency, it is in the composition of the sign-object relationship—during the phase when they do participate, students may take the opportunity to attempt being-a-good-student—but ultimately these attempts are evaluated by their teacher and peers as appropriate and effective in the whole history of their mutual relationship. Students often try and fail to be perceived as good students, showing that their ability to commit to an interpretant is limited. Dr. C’s “hip hop mathematician” performance occurs at this scale as well, and as a tactical rather than strategic action, this sign has a greater variety of potential interpretants, so the actor’s ability to commit to one is even more questionable.

This semiotic framing addresses the problem of practice of how students may come to see themselves as potential successful learners of mathematics—to construct “identities of mathematical competence” (I. S. Horn 2008). We heard Dr. C expressing an awareness that many of his students saw themselves as mathematically incompetent, but his chosen strategy for addressing this problem—the building of co-membership for mutual support—relies either on the imposition of new norms such as extra credit for study groups, which requires students to reframe “class participation” in unfamiliar ways, or on extra-academic communicative strategies such as rapping, whose meaning in the classroom is ambiguous at best. While these alternative approaches are difficult to enact, the existing normative approach is also problematic; the students who participate most actively are seen as “good students,” but class time and opportunities to speak up are finite, so only a minority of students can ever be seen as competent.
So, for all students to potentially see themselves as “good at math,” it seems that a redefinition of competence itself is in order, moving away from the zero-sum understanding in which students are explicitly or implicitly evaluated against one another. One approach to this problem is through the principles of complex instruction (Cohen and Lotan 1997; Cohen et al. 1999), which Ilana Horn (2011) has applied specifically to mathematics instruction. In complex instruction, teachers assign open-ended problems for students to complete collaboratively, but the activity is intentionally structured in such a way as to minimize status differences between students, for example, by ensuring that all group members have equal opportunities to talk. Notably, the open-ended problems characteristic of complex instruction are unlikely to be simple computations with a clear correct answer; rather, the creative tasks that grant students increased agency over mathematical representations may also lead them to see themselves as members of a community of successful mathematics learners.

6.4. Toward a Dialogue between Linguistic Anthropology and the Learning Sciences

Reviewing the findings of this dissertation, we have seen that the interactional practices of the mathematics classroom often restrict students’ semiotic agency to the realm of particular objects and events by rendering the objects’ qualities invisible and the patterns of relationships among them inaccessible. At the level of notation, the nature of mathematical objects, which is notoriously abstract and difficult to define, was obscured even further by the very pedagogical practices that are meant to make them comprehensible. At the level of sequence, as a result of the belief that there is always exactly one correct answer to any question, students’ representational agency was restricted to the right to answer questions, not to propose solution techniques or to observe more general laws. And at the level of performed identity, among the majority of students who question their own mathematical competence, this can be seen to result not only from their limited opportunities to show competence, but also from the conflicts they perceive between
mathematical competence and other, more transportable aspects of their identity such as ethnicity, geography, and social class.

At each stage in the analysis, the findings were seen to resonate with the work of mathematics experts working within the learning sciences. This is an approach to education research that does not treat learning as an acquisition of skills and information—a sort of psychological intervention that could be measured through pre-and-post psychometric assessment—but rather considers learning holistically as a social process, aiming “to better understand the cognitive and social processes that result in the most effective learning and to use this knowledge to redesign classrooms and other learning environments so that people learn more deeply and more effectively” (Sawyer 2014, 1). As Sfard and Cobb (2014) illustrate, research on mathematics education has been particularly influential within the learning sciences, taking the forefront in advancing a theory of learning-as-participation rather than learning-as-acquisition, based on prior research in situated learning and cultural-historical activity theory (Lave and Wenger 1991; Vygotsky [1934] 2012; Yamagata-Lynch 2010). Each of the problems of practice identified here has been studied in a more focused manner by learning scientists; for example, the preceding discussion cited Anna Sfard’s work on algebraic representation (Sfard and Linchevski 1994), Jo Boaler’s work on discussion-based approaches to reforming mathematics instruction (Boaler and Greeno 2000), and Ilana Horn’s work on students’ social and academic status in the classroom (I. S. Horn 2008; I. S. Horn 2011). While I did not set out to replicate these studies, my awareness of them cannot but have directed my ethnographic attention and analytic focus. One of my goals for this dissertation was to address issues of significance to practitioners, and to the extent that I have been able to do so, it is because I have been guided on some level by these researchers and their understanding of teachers’ practical concerns.

In the broader conversation about the social processes of learning, what this dissertation contributes is a theory of communication as multimodal semiotic, including all the diverse
meaning-making channels and modalities of the mathematics classroom, and incorporating 
varieties of meaning-making such as gestural metaphor, interactional sequence, and stancetaking / 
positioning. These are the insights of linguistic anthropology and discourse analysis, achieved 
through a focus on human communication in all its social and cultural variety, in Indigenous 
villages, friendly telephone calls, service encounters, Thanksgiving dinners, teenage cliques, press 
conferences, Samoan mission schools, and working-class Piedmont Carolina parenting practices. 
Having considered communication as something that happens not just in classrooms but in all 
social life, the field has arrived at a set of methodological practices that can then be applied to 
classroom interaction, allowing us to see learning scientists’ problems of practice as fundamentally 
discursive phenomena, and to provide a detailed description of the discourse practices in question. 

To push this interdisciplinary research program further, is there a role for the linguistic 
anthropology of education in “redesign[ing] classrooms and other learning environments so that 
people learn more deeply and more effectively” (Sawyer 2014, 1)? While the ethnographer’s 
observational stance does not canonically envision the researcher’s designing aspects of the field site under investigation, the approaches of applied, practicing, and public anthropology do reflect a 
practical or moral responsibility that the researcher bears to the community (Lamphere 2004), as 
does the sociolinguistic “principle of linguistic gratuity” (Wolfram, Reaser, and Vaughn 2008). In 
educational applications of sociocultural linguistics, social engagement most often takes the form of 
educating teachers about their students’ cultures of primary socialization (Charity Hudley and 
Mallinson 2010; Heath 1983), framing application as something that takes place subsequent to 
research. In the learning sciences, however, research and application are integrated as overlapping 
phases in an iterative process, “advancing theory while at the same time directly impacting practice 
... [using] the close study of learning as it unfolds within a naturalistic context that contains 
theoretically inspired innovations” (Barab 2014, 151). Hypothetically, I may educate teachers on a 
sociolinguistic topic of interest—say, student agency in mathematics classroom discourse, or the
linguistic structure and systematicity of African American Vernacular English—but if I want to know whether my training has any tangible benefits for students, then I must commit to a longer-term engagement in which my linguistic knowledge, teachers’ pedagogical expertise, and students’ lived experience are brought together, objectives are formulated, and outcomes are assessed in some way. By framing the classroom setting itself as a community of practice where fieldwork may take place, the theories and methods of linguistic anthropology can be the tools we use as we articulate the goals of our intervention in line with observed problems of practice, and ground our assessment in a detailed, systematic description of classroom interaction. In this way, it will become possible to conduct ethnographically informed learning sciences research that articulates and responds to teachers’ professional concerns, putting research in the service of practice, or even integrating the two altogether.
### Appendix A: Extensions of Peirce’s Phenomenology

<table>
<thead>
<tr>
<th>Definitions (Peirce [1902] 1955)</th>
<th>Areas of Inquiry (Ch. 1)</th>
<th>Professional Vision (Ch. 3; cf. Goodwin 1994)</th>
<th>Agency (Ch. 4; cf. Kockelman 2007)</th>
<th>Stance and Positioning (Ch. 5; cf. Table 5, p. 139)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Firstness</strong></td>
<td>Qualities</td>
<td>Technical notation</td>
<td>Highlighting</td>
<td>Residential</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Control</td>
<td>Representational</td>
</tr>
<tr>
<td><strong>Secondness</strong></td>
<td>Actual objects and events</td>
<td>Interactional sequence</td>
<td>Coding</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Compose</td>
<td>Position</td>
</tr>
<tr>
<td><strong>Thirdness</strong></td>
<td>Laws, patterns, regularities</td>
<td>Performed identity</td>
<td>Material representation</td>
<td>Commit</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Reason</td>
<td>Alignment</td>
</tr>
</tbody>
</table>

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APPENDIX B: STUDENT INTERVIEW PROTOCOL

1. Personal background
   a. What was school like in your (home country / high school)?
   b. What was your math class like in your (home country / high school)?
   c. (Calculus) What’s your major? Why are you taking this class?

2. Impressions of the class
   a. What do you think of this class? Compared to the other classes you’re taking, do you like it? Do you work hard at it?
   b. How would you describe yourself as a mathematics student?
   c. Aside from you, which students in your class are best in math? How do you know?
   d. What do you think of the (teacher / professor)?
      i. (Middle school) What do you think of her use of Spanish? What do you think of the hand gestures and songs that she uses to review material?
      ii. (Calculus) What do you think of his hip hop / pop culture references? What do you think about the way he makes it personal, talks about his family, etc.?

3. Video playback
   a. Introduction: Let’s watch a video clip that I recorded in your math class. This is really how I do my work—watching video and trying to understand why people do what they do—and it can be really helpful to talk about it with other people who were there. The first time I play the video, we can watch it all the way through, and then I'll ask you to summarize what happened. Then we’ll watch it again. The second time, if you see anything interesting, say “stop” and I'll pause the video so we can talk. After that, I'll ask you to comment on anything you thought was interesting. If nothing strikes you in particular, I may have a few more specific questions.
   b. Middle school video options:
      i. Nayan 'I got it wrong' (Example 21, p. 146)
      ii. Ricardo asks to read the objectives aloud at the end of class
      iii. Alberto shades 5% for 0.5
   c. Calculus video options:
      i. ‘Corey's head was just in the way' (Example 20, p. 125)
      ii. Xander raises his hand; Dr. C puts him off
      iii. Jason's headphones: “It’s not Katy Perry”

4. Conclusion
   a. Is there anything I didn’t ask that you were expecting me to ask?
   b. Do you have any questions for me?
APPENDIX C: TRANSCRIPTION CONVENTIONS

Participants

class  choral response or multiple minimal expressions
student speaker unidentifiable from recording
NRP   non-research participant (content of utterance omitted)

Multimodal transcription

new line intonation unit break
plain text talk
italics gesture; if mathematical notation, pointing to notation
**bold (bold)** blackboard writing (location of writing)
| word | simultaneous talk and gesture
| gesture |
| word | while speaking word, point to notation
| notation |
| word | while speaking word, write notation
| notation |
RH   right-hand gesture
LH   left-hand gesture
BH   bilateral ("both hands") gesture

Transcription of talk

word→ intonation unit continues over line break
word lengthened sound
word~ cut-off or self-interruption
word? high rising terminal intonation
word. low falling terminal intonation
word, mid-rising terminal intonation
↑word sharp rise in intonation
↓word sharp fall in intonation
word contrastive stress
>word< fast speech
<word> slow speech
°word°    quiet or whispered speech
@         one beat of laughter
@word     co-occurring speech and laughter
(word)    uncertain transcription; or, non-lexical speech sounds or ideophones
(.)       brief (~0.1 second) pause or hesitation
(#.#)      pause length in seconds (for pauses 0.5 second and longer)
[word]    modified for confidentiality purposes
((word))  transcriber's note or description
word_l    latching utterances
word_r    overlapping utterances
REFERENCES


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