TWO ESSAYS ON DYNAMIC OPTIMAL PRICING

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THE TWO ESSAYS ON DYNAMIC OPTIMAL PRICING

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ABSTRACT

The first chapter studies the dynamic model in hotel demand estimation. This chapter analyzes a new database following a luxury hotel market over a 37-month period. It focuses on the pricing decisions of one of these hotels, which uses a popular commercial revenue management system (RMS) to set its prices. There is a presumption that a RMS can help the hotel set optimal dynamic prices, but so far there have been no empirical studies that have been able to test this hypothesis. I consider two alternative approaches to demand estimation that can deal with the endogeneity problem and associated censoring problems. Both estimation approaches use the method of simulated moments (MSM) but differ in the auxiliary assumptions used to identify the parameters of the stochastic process governing demand. One approach assumes that the hotel sets its prices optimally as a best response to the prices of its competitors. The other approach relaxes this optimality assumption and estimates demand with a two-step semi-parametric approach. These two approaches produce different parameters, resulting in alternative price policies. I compared them by conducting the Wu-Hausman test. From this test, I concluded that the assumption of optimality may cause distorted estimates of demand.

In the second chapter, I consider the static model in hotel market to see whether it is possible to identify consumer preferences and arrivals when assumptions a) optimality, and b) equilibrium are relaxed. We establish the global, non-parametric identification of preferences and consumer arrival probabilities in a simplified static setting but show via examples that the identification of unobserved types of consumers is
very challenging, in contrast to the more optimistic conclusions from theoretical analyses that prove that the random coefficient logit model (which is a component of our overall model of demand) is non-parametrically identified.

INDEX WORDS: price discrimination, semi-parametric estimation, dynamic programming, hotel revenue management
DEDICATION

To my parents and fiancé for their support, love, and sacrifices.
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# Table of Contents

## Chapter

1. Hotel Demand Estimation With and Without the Assumption of Optimality
   - 1.1 Introduction
   - 1.2 Data
   - 1.3 Demand Estimation With the Assumption of Optimality
   - 1.4 Demand Estimation Without the Assumption of Optimality
   - 1.5 Result
   - 1.6 Conclusion

2. Semi-parametric Instrument-free Demand Estimation: Relaxing Optimality and Equilibrium Assumptions
   - 2.1 Introduction
   - 2.2 Hotel Data
   - 2.3 Identification of a Static Model of Hotel Demand
   - 2.4 Empirical Application to the Hotel Market
   - 2.5 Conclusion

## Appendices

A. Estimation Result
   - A.1 Parameters of Consumer’s Preference
   - A.2 Weekdays Demand
   - A.3 Weekdays Group Arrival and Cancellation
   - A.4 Weekends Demand
   - A.5 Weekends Group Arrival and Cancellation

B. The Law of Competitors’ Average Price

C. Irrational Phenomenon Caused by Competitors’ Feedback Effect

D. Evidence of Strategic Cancellation

E. Computational Evidence of a Reduced State Variable

F. First-step Semi-parametric Price Rule

G. Maximum Likelihood Estimates of Hotel Regression Model
**List of Figures**

1.1 Weekly average best available rates (BAR) for all hotels .......................... 9
1.2 Data example for November 18, 2010 .................................................. 12
1.3 Demand simulation by sub-optimal and optimal (busy weekday) ........ 30
1.4 Optimal price trajectory (busy and busiest weekday) ....................... 32
1.5 Parameters estimates ($\beta_s$) with suboptimal price policy ............. 38
2.1 Booking and price dynamics over the week and year ....................... 65
2.2 Annual price dynamics for all seven hotels ........................................ 66
2.3 Co-movement in average daily rates (ADR) and occupancy rates for all seven hotels .......................................................... 68
2.4 Price scatterplots for hotel 0 and its competitors ............................. 71
2.5 Censored multinomial distribution of occupancy .............................. 78
2.6 Occupancy distribution, $A = 120$ ...................................................... 83
2.7 Predicted versus actual average daily rates (ADR) for hotels 0 and c .. 110
C.1 Value function by various given occupancy is 0 and $\rho$ is $300$ ........ 123
E.1 Optimal price by different average daily rates (ADR) ......................... 125
### List of Tables

1.1 Data sources used in this study ............................................... 7  
1.2 Hotels in the local market in our study ................................. 10  
1.3 Expected simulation result of two-step semi-parametric estimation . 35  
1.4 True parameters ($\theta_0$) for validity test ............................... 37  
1.5 Wu-Hausman statistic .......................................................... 40  
2.1 Hotels in the local market in our study ................................. 59  
2.2 Data sources used in this study ............................................. 62  
2.3 Ordinary least squares regression with dependent variable $\text{ADR}_0$ . 69  
A.1 Estimates of choice parameters $(a_\tau, b_\tau)$ with optimality ($\hat{\theta}_1$) .... 113  
A.2 Estimates of choice parameters $(a_\tau, b_\tau)$ without optimality ($\hat{\theta}_2$) .... 114  
A.3 Estimates of zero-inlated negative binomial (ZINB) demand parameters $(\gamma, \phi, \mu)$ with optimality ($\hat{\theta}_1$) ............................. 115  
A.4 Estimates of zero-inlated negative binomial (ZINB) demand parameters $(\gamma, \phi, \mu)$ without optimality ($\hat{\theta}_2$) ............................. 116  
A.5 Estimates of group demand and cancellation with optimality ($\hat{\theta}_1$) .... 117  
A.6 Estimates of group demand and cancellation without optimality ($\hat{\theta}_2$) . 117  
A.7 Estimates of zero-inlated negative binomial (ZINB) demand parameters $(\gamma, \phi, \mu)$ with optimality ($\hat{\theta}_1$) ............................. 118  
A.8 Estimates of zero-inlated negative binomial (ZINB) demand parameters $(\gamma, \phi, \mu)$ without optimality ($\hat{\theta}_2$) ............................. 119  
A.9 Estimates of group demand and cancellation with optimality ($\hat{\theta}_1$) .... 120  

x
A.10 Estimates of group demand and cancellation without optimality ($\hat{\theta}_2$). 120
B.1 Regression result: Law of competitors’ price, $\rho_{t-1}$ ............... 121
D.1 Strategic cancellation ........................................ 124
F.1 First-step pricing rule - regression learning ......................... 127
G.1 Maximum likelihood estimates of hotel regression model ........... 129
H.1 Optimality and equilibrium-constrained maximum likelihood estimates of hotel regression model ........................................ 131
I.1 Maximum likelihood estimates of mixed trinomial demand model . 132
Chapter 1

Hotel Demand Estimation With and Without the Assumption of Optimality

1.1 Introduction

The problem of endogeneity is a frequent problem in demand estimation and it sometimes leads to a positive correlation between price and quantity. The standard approach to handle this endogeneity problem is to use instrumental variables (IV) estimation. However, it is hard to find instrumental variables in many cases and there is an abundance of literature on the search for instruments (Berry et al. [4], Berry and Haile [6]). MacKay and Miller [24] suggest an alternative estimation method where they estimate demand parameters without relevant instrumental variables.

The instrumental variables estimation and instrument-free estimation by MacKay and Miller [24] assume that the demand curve is linear or semi-linear. However, when linearity of demand is not present, both methods are inapplicable. Hotel pricing is an example where the demand function is not linear or semi-linear, as described by Yu [35]. He modeled hotel demand as a stochastic process generated from customers randomly arriving to book rooms in hotels that are included in a local luxury hotel market. Using the method of simulated moments (MSM), he succeeds in dealing with endogeneity and estimates a downward sloping demand curve. At the same time, his methods are useful when facing a data censoring problem since the hotel in question is only able to access limited information about its competitors. He assumes that the
existing price set by the commercial revenue management system (RMS) is optimal for parameter identification. Although his method was successful at recovering hotel demand, it relies highly on the assumption of optimality. Cho et al. [9] suggest an alternative in demand estimation but relaxes the optimality assumption with a two-step semi-parametric estimation. By adopting these two approaches, this paper tests the validity of their hypotheses by comparing the estimation results.

The analysis of this study is based on a luxury hotel market in a major US city using a very detailed dataset of this market, enabling structural model analysis based on three years' of daily level data. There are seven hotels in this market with varying attributes and prices. I focus on one of these hotels, hotel 0, which is at the lower end of the price/quality range of this set of hotels. Since the hotel uses an RMS which provides a recommended price, the manager can either set their own price or use the recommended RMS price. Not only must they set the price for each day, but they also must post prices for future days as well. If the hotel has 11 room types and takes reservations 100 or more days in advance, in principle it needs to adjust over 1100 prices each day. But as Phillips [28] noted “The Internet increases the velocity of pricing decisions. Many companies that changed list prices once a quarter or less now find they face the daily challenge of determining which prices to display on their website or to transmit to e-commerce intermediaries. Many companies are beginning to struggle with this increased price velocity now — and things will only get worse." (p.11). The high demands for intelligent real time price setting is a key reason why hotels increasingly rely on commercial RMS to help set their prices.

We do not know exactly what information the RMS uses and how it learns demand in setting its recommended prices. The system also considers several factors such as hotel loyalty programs, the quality of hotel services and facilities, and convenience of the location (Sturman et al. [31]). RMS is used widely in other areas besides hotels,
particularly in airlines, where we do not know how it works either. McAfee and te Veld [25] note “At this point, the mechanism determining airline prices is mysterious and merits continuing investigation because airlines engage in the most computationally intensive pricing of any industry." (p.437).

Demand prediction is a key factor for setting the price since a hotel maximizes its revenue with a fixed room supply. It is crucial to consider the expected demand up to check-in day as there is a trade-off in setting the price. If a hotel sets the price high, each reserved room has a large markup, but is not expected to reach revenue maximization due to a higher number of unoccupied rooms. If the hotel sets the price too low at the beginning, it may fill up due to the low prices, leaving very few available rooms for late arriving business customers who are willing to pay a higher price. Besides this trade-off, the pricing system must also consider the time-varying demand patterns by different customer types. Factors such as these relating to hotel demand, bring up the need for a dynamic programming (DP) framework in the pricing system. Yu [35] developed an optimal hotel pricing model with DP assuming that RMS produces an optimal recommended price.

Hotel room rentals are perishable goods and there are plenty of studies on price discrimination of perishable goods such as flight tickets (Escobari [12]), concert tickets (Courty and Paglieri [10], Leslie [22]), and food products (Bhattacharjee and Ramesh [7], Liu et al. [23]). Many studies of optimal pricing have been developed in the operation management field where they focus on how revenue is maximized under conditions of a perishable good on a theoretical basis (Gallego and van Ryzin [15], Anderson and Xie [1] and Zhao and Zheng [36]). However, only a few empirical studies use dynamic programming in a perishable good market since there is a huge computational burden in estimation and so many factors need to be considered. Recent
studies such as Lazarev [21], Williams [33], Yu [35] and Aryal et al. [2] analyze optimal pricing model empirically.

A key challenge to effective pricing is having a good understanding of customers’ demand for hotel rooms. There are several aspects to this: identifying the different types of consumers in this market (i.e. business versus leisure travellers) who have differential willingness to pay for the different hotels in a local market, and understanding the patterns of customer arrivals, i.e. how far in advance of the date of occupancy do they choose to book their rooms? However learning about demand is greatly complicated by the strong endogeneity of prices in many hotel markets: a scatterplot of prices and occupancy rates will typically be upward sloping (e.g. positive correlated) due to time varying demand shocks that cause the hotels in the market to sell out on some days and not others. Some of these demand shocks are relatively predictable, so hotels adjust their prices in a manner that is endogenous to these demand shocks, raising prices when prices and occupancy rates are high and lowering prices on low demand days where occupancy rates are low. Thus, the endogeneity of hotel pricing in response to tie varying demand shocks can easily produce what might appear at first glance to be an “upward sloping demand curve” for hotel rooms. Unfortunately beyond incorporating observable demand shifters in the econometric model (such as estimating separate demand/arrival parameters on days that are expected to be busy and days that are not busy) there are often few relevant instrumental variables that can be used to solve the price endogeneity problem and generate sensible downward sloping predicted demand curves.

Structural models that impose the hypothesis that firms set their prices optimally in response to their beliefs about demand shocks and the pricing behavior of their competitors can deal with the endogeneity problem by adding moment conditions that fit predicted prices to actual prices. These extra moment conditions effectively enforce
reasonable (e.g. downward sloping) behavior on the implied demand curves, since if demand curves were truly upward sloping, an optimizing firm would have an incentive to set its prices much higher than it actually does. Thus, the structural model requires reasonable estimated preferences and arrival rates of customers that imply demand curves that are *ceteris paribus* downward sloping in order to rationalize the prices the hotel actually charges. However, if the assumption of optimality is incorrect, the estimated demand parameters could be inconsistent, since the structural estimation algorithm will try to find demand model parameters to rationalize the observed set of prices as optimal prices. This is a familiar problem in econometrics where the imposition of an constraint on an econometric model can result in more efficient estimator if the constraint is valid but will result in an inconsistent estimator if the constraint is invalid.

There are several studies in the marketing field which estimate a demand without an optimality condition. Cho et al. [9] describe a novel “two-step semi-parametric” method to demand estimation that estimates demand parameters outside the dynamic optimal price model. Despite the many advances in the field of dynamic structural model since Rust [30], only a few models come close to the work of this study. The key point of their study is relaxing the optimality assumption. Regardless of whether the current price is optimal, their methods enable the model to obtain demand parameters by acquiring the effects of the price on hotel demand.

This study first implements two-step semi-parametric estimation empirically following Cho et al. [9]. Due to the complicated structure of the hotel demand model, price elasticities and other parameters of demand are estimated through the method of simulated moments (MSM) rather than direct estimation. Once two-step estimation is finished, I plug in the estimated parameters into the optimal pricing model developed by Yu [35] to obtain the optimal pricing rules under the assumption that the
hotel’s pricing behavior is not necessarily optimal. By comparing the outcome based on the two approaches, I relate the effects of two assumptions to the estimated price elasticities and other demand parameters for different types of consumers. Further, I suggest a validity method to test those assumptions.

Section 2 presents the dataset and its stylized facts underpinning the model. This will contain the underlying problems in the dataset that I had to confront. Section 3 introduces the demand estimation with the optimality assumption and describes the dynamic structural model of hotel demand, which is used in both approaches. Section 4 describes the demand estimation without the optimality assumption through a two-step semi-parametric estimation process and presents the identification problem of the process. Section 5 explains the results and validity test of estimations, while section 6 is the conclusion of this study.

1.2 Data

1.2.1 Overview of Data

A hotel (hotel 0) in a major US city provided its reservation details, aggregate daily reports, and records of competitors’ price between October 2010 and October 2013, a total of 37 months. From Smith Travel Research (STR), which handles hotel market data and benchmarking technique, we get competing hotels’ aggregate occupancy information within the same time period. The summary of data source I used is shown in Table 1.1. Hotel 0 tracks its competitors’ prices in setting its own prices. A commercial service Market Vision, which is in the first row of Table 1.1, provides hotel room prices to put its client hotels in a competitive position, is purchased by hotel 0 to set the room rates. This data set includes the lowest room price for competing hotels and the channel information such as hotel website or travel websites.
Table 1.1: Data sources used in this study

<table>
<thead>
<tr>
<th>Data</th>
<th>The first day of occupancy</th>
<th>The last day of occupancy</th>
<th>Observations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Vision</td>
<td>2010-09-21</td>
<td>2014-08-13</td>
<td>609,181</td>
<td>competitors’ price</td>
</tr>
<tr>
<td>reservation raw</td>
<td>2009-09-01</td>
<td>2013-10-31</td>
<td>201,176</td>
<td>reservations detail information</td>
</tr>
<tr>
<td>cancellation raw</td>
<td>2009-09-01</td>
<td>2013-10-31</td>
<td>29,241</td>
<td>cancel detail information</td>
</tr>
<tr>
<td>daily pick-up report</td>
<td>2010-09-16</td>
<td>2014-05-21</td>
<td>475,187</td>
<td>daily revenue report</td>
</tr>
<tr>
<td>STR market data</td>
<td>2010-01-01</td>
<td>2014-12-31</td>
<td>1,731</td>
<td>competitors’ occupancy</td>
</tr>
<tr>
<td>Data range</td>
<td>2010-10-01</td>
<td>2013-10-31</td>
<td></td>
<td>37 months</td>
</tr>
</tbody>
</table>

According to the revenue manager of hotel 0, its prices are the same regardless of channel. However, since other hotels do not have a uniform price policy, Market Vision monitors different websites and records their rates. It also contains room rates up to 90 days in advance of the arrival date, so the number of observations of Market Vision is the largest in the table. When it comes to room rates in Market Vision, it means the best available rate (BAR), the currently quoted price, and it is the base rate before any discount takes place. Since BAR is the rate before any discount, BAR is often different from a rate hotel customers actually pay for their stay. While average daily rate (ADR), the average of rates customers actually pay, is important for computing revenue maximization, BAR is important when we compare hotel 0’s rate with others’. Therefore, I use both rates in the hotel pricing model in section 3.

A huge strength of our data is the detailed information on reservations and cancellations records in Table 1.1. It contains all the information about each individual booking of hotel 0. A reservation identification number is created when the reservation is made and becomes the permanent ID for that reservation, which is in chronological order, thus it is useful when tracking cancelled bookings. Besides primary date information such as when the booking was made and when it is for, other significant information included are the room type, booking channel, rate code, and rate paid.
There are around 200 rate codes in the reservation data and they explain the detail of the contract, including discount rates and its condition of application. I categorized customer types into “leisure”, “business”, and “group” based on rate code information.

Although the data set provided by hotel 0 is very detailed, the model shown in the next section requires more data on other hotels’ reservations. When consumers compare prices between hotel 0 and its competitors, it is essential to know the reservation records for the hotels, along with its rates. We do not have access to the revenue data of hotel 0’s competitors, so I used an alternative data which enables inferences on the booking trajectory. Smith Travel Research (STR) provides data on the final occupancy rate of competitors with their average daily rate, revenue information. By using STR data, I find market share at arrival date. Without it, the model would have difficulty deriving the outcome of the price effect even if I have detailed data for one hotel. Though STR data does not contain the detailed information of competing hotels, it facilitates further steps in this analysis.

1.2.2 Stylized Facts of Hotel Data

Since the properties of a hotel market vary by region, the starting point of this study is to understand the characteristics of this hotel market. Hotel 0 and competing hotels are located in a major U.S. city where tourism demand is high and has stable visits by business customers. The considerable bookings by business customers drive higher weekday prices compared to the weekend. Meanwhile, a strong seasonality is observed for leisure customers. According to Figure 1.1, demand peak occurs during the commencement season of May, fall foliage of October while August and January are the slowest months of the year. A hotel manager is able to observe the reservation history of the hotel, and it is a known fact that hotel managers use past year data to predict seasonality. Figure 1.1 illustrates weekly average walk-in BAR for hotel
0 and its six competing hotels. This shows a consistent and high seasonality in the hotel demand. The bold line plots the rate of hotel 0, while other lines indicate rates of the competitors. Overall, all hotels have strong co-movement in the prices and price fluctuation is huge in all hotels. During the peak season, BAR of hotel 0 is over $400, which is almost twice as high as the lowest weekly BAR within a year, which is consistent with other hotels. Due to the varied price fluctuations, there is plenty of motivation to examine this further, as setting a proper price level based on accurate demand prediction is important.
The seven hotels in this study are summarized in Table 1.2. All of them are classified as upper-up or luxury hotels and they are all located in the same neighborhood, so I assume that they share the same potential customers. Hotel 0 occupies 15.2% of the total market supply and this ratio is close to $1/7$ (14.3%). Hotel 1 and 2 have very similar levels of price (BAR) compared to hotel 0 and are the primary competitors. Hotel 3, 4 and 6 have higher price levels while hotel 5 is the only one with a clearly lower price level than hotel 0. Since the average BAR of hotel 4 and 6 are over $100$ higher than that of hotel 0, it is likely that these two hotels are not actual competitors of hotel 0. However, their activity, including their price setting practices, can be a benchmark for the luxury hotel market of this city. Thus, it is worth considering them as competitors of hotel 0 once we find that their price pattern is alike as Figure 1.1.

Hotel 0 is an independent hotel with less market power and less renowned to customers. To make it more attractive, the hotel focuses more on services such as generous cancel policies, and competitive prices. I presume that hotel 0 is not a price-leader in the market and instead, it is more likely to be a price-follower. But, I consider the feedback effect of hotel 0’s price on other hotels in the next section.

Table 1.2: Hotels in the local market in our study

<table>
<thead>
<tr>
<th>Property</th>
<th>Avg. BAR</th>
<th>Star</th>
<th>Class</th>
<th>Chained Brand</th>
<th>Rate</th>
<th>Relative Capacity</th>
<th>Distance to mass transit</th>
<th>Cancel Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>hotel 0</td>
<td>$293.26</td>
<td>4</td>
<td>Luxury</td>
<td>No</td>
<td>4.4</td>
<td>79%</td>
<td>3 min</td>
<td>1 day before</td>
</tr>
<tr>
<td>hotel 1</td>
<td>$282.64</td>
<td>4.5</td>
<td>Upper Up</td>
<td>No</td>
<td>4.4</td>
<td>81%</td>
<td>5 min</td>
<td>3 day before</td>
</tr>
<tr>
<td>hotel 2</td>
<td>$285.16</td>
<td>4</td>
<td>Upper Up</td>
<td>No</td>
<td>4.4</td>
<td>63%</td>
<td>3 min</td>
<td>1 day before</td>
</tr>
<tr>
<td>hotel 3</td>
<td>$338.29</td>
<td>4</td>
<td>Upper Up</td>
<td>Yes</td>
<td>4.2</td>
<td>99%</td>
<td>8 min</td>
<td>2 day before</td>
</tr>
<tr>
<td>hotel 4</td>
<td>$397.09</td>
<td>4</td>
<td>Luxury</td>
<td>No</td>
<td>4.6</td>
<td>100%</td>
<td>10 min</td>
<td>Strict</td>
</tr>
<tr>
<td>hotel 5</td>
<td>$253.51</td>
<td>4</td>
<td>Upper Up</td>
<td>No</td>
<td>4.2</td>
<td>47%</td>
<td>8 min</td>
<td>3 day before</td>
</tr>
<tr>
<td>hotel 6</td>
<td>$454.30</td>
<td>5</td>
<td>Luxury</td>
<td>Yes</td>
<td>4.7</td>
<td>52%</td>
<td>10 min</td>
<td>1 day before</td>
</tr>
</tbody>
</table>

Using reservations raw and cancellation raw data, I discovered an interesting pattern in inflow and outflow of hotel 0 when looking at customer type. Individual cus-
Customers like business or leisure seem to increasingly look for a hotel in this city as it approaches arrival day; one day before arrival is the highest in new reservations. Meanwhile, many group customers tend to make reservations 15 - 20 days before arrival. In terms of cancellations, the number increases until two or three days before arrival, then it abruptly decreases until arrival day. Since hotel 0 has a 24-hour free cancellation policy, customers tend to make their mind up early to avoid any penalty.

Figure 1.2 offers a sample price/occupancy trajectory on a specific arrival date. For each arrival day, I have this form of data containing the price trajectory of hotel 0 and its competitors and the occupancy records for hotel 0. For simplicity, the average of competing hotels’ rate, denoted by plus-solid line (+), is used in the figure and the model. The rate of hotel 0 is denoted by star-solid line (*) and the occupancy rate is marked by the shaded area located at the bottom of the figure. The figure shows the effect of price disparity on the occupancy rate. In particular, the day of Figure 1.2, November 18, 2010 is the end of the peak period. Since then and before Thanksgiving day, it goes into a slack period for the hotel market. It is still classified as a busy day, so the starting price of hotel 0 in 45 days before the occupancy is also as high as $319 which is rather high and their occupancy rate of that day is also high as 36.7% of hotel rooms are already booked. There is a rapid increase in occupancy rate until 40 days out and it results in an increase in the price of hotel 0 to $339 after 37 days out. We observe direct competition at 31 days before arrival day when competitors increase their price due to popularity and hotel 0 sees an increase over 10% occupancy rate to 83.5%. After this point, hotel 0 increases its price gradually while competitors drop theirs slowly. This produces a quiet period in bookings and at 18 days prior to arrival day, the price point for hotel 0 and its competitors intersect. Such an updraft in price continues until it reaches $379 in price and a 91.1% of occupancy rate around 9 days out. However, since then, the price suddenly drops to $319 and it may cause the
overbooking of rooms 3 days out. In order to prevent overbooking at arrival day, hotel 0 increases its price, which prevents new bookings and ends up with an occupancy rate of 99.1% on arrival day. On the arrival day, hotel 0 is almost sold out with a 99.1% of occupancy rate, so this example shows a successful implementation by the hotel manager regarding occupancy rate.

Although the data set is incredibly detailed, I still face some issue on demand estimation. If I knew the total number of potential customers who are looking for their accommodation in seven hotels, I would succeed in finding the demand for hotel 0. However, it is impossible to find that information in the real world, and the only knowledge available is the occupancy of the final day for competitors (STR data).
Using the method of simulated moments (MSM), this study tries to overcome this data censoring problem. Another important issue in estimation is the endogeneity problem of the demand function. Since there are strong co-movement of price between seven hotels, it is hard to know the reason for a price change in hotel 0. For example, when there is a decrease in reservations for hotel 0, it is not distinguishable whether the decrease is caused by the high price of hotel 0 or a demand shock in the whole market. To deal with endogeneity problem, I have tried to find an instrumental variable which is related to the rate of hotel 0, but not to the demand shock. However, there are few instrumental variables available and I don't have confidence that IV method works in stochastic hotel demand model. Censoring and endogeneity problems are severe problems in demand estimation and I deal with them using dynamic structural model with MSM estimation. Once those inherent problems in demand are solved, then the model for pricing policy is applicable.

1.3 Demand Estimation With the Assumption of Optimality

This section introduces a structural dynamic optimal pricing model described in Yu [35] and explain the demand estimation with the assumption of optimality. It is in contrast with the semi-parametric pricing model I use for the estimation of the demand parameter. The goal of Yu [35] and this study are to find an optimal price for hotel 0 based on demand forecasting. Therefore, this structural, or parametric, the pricing model is used to derive the pricing function in the final stage. I will start by presenting some assumptions for the model, and then introduce each component of the aggregate structural model.
1.3.1 Assumptions

I assume that hotel 0 sets the price for each arrival day every morning. For simplicity, the manager of hotel 0 is assumed to set prices for up to 45 days in advance of arrival. In the reservation data, I looked for a reservation which is confirmed almost a year before arrival. However, this rarely happens and I find it difficult to have a robust demand estimation with a limited number of reservation records. So, the manager considers prices for 46 days which are posted on the same day. All rooms in hotel 0 are considered a standard room. The reservation data also contains price information on luxury suite rooms, which take up less than 5% of hotel 0’s total rooms. However, it is hard to compare them with other hotels’ suite rooms due to the lack of price information on other hotels. Another problem of demand estimation of suite rooms is the same with early booking cases. While the capacity of suite rooms are around 5%, their actual booking is around 1-3%. In this sense, the manager focuses on only the standard room price of 46 days every day.

Some reservations are made for a single day stay, but more than half of the reservations are for multiple days. Even though Market Vision collects price information of multiple days stay as well as a single day stay, it may bring computational difficulties in the model if it considers the price for multiple days stay. Fortunately, I did not find any evidence that there is an obvious discount for multiple days booking from Market Vision. Therefore, this study regards multi-night booking as a sum of several independent single-day bookings with the principle of decomposition. By focusing on the price of a single-day booking, the model can be simplified.

Every morning a manager needs to set the single-day price (BAR) of the standard room in advance of up to 45 days, or a total of 46 rates. In the meantime, he or she takes into account several factors which may affect the demand and price. Since the
actual booking process in the hotel market does not occur all at once, the manager needs to have the expectation of future arrival and cancel other than today's condition. First, days before arrival (DBA), $t$, is necessary to be considered in the sense that demand pattern highly depends on $t$. In particular, different types of customers have distinct demand patterns by $t$. Given the same number of occupancy, the price strategy must be distinguishable depending on how many days are left before arrival. Another key factor for setting price is how many reservations have been made so far, $n_t$. If the manager realizes current $n_t$ is less than his expectations, he may decrease the rate to fill up the hotel. However, he also needs to consider the capacity constraint of hotel 0, $\bar{n}$, so BAR set by the manager has to keep $n_t$ is less than $\bar{n}$ by controlling the demand of hotel 0. Furthermore, the manager must observe other hotels’ price in the morning and factor them for his/her own price. Though there are six rates of competitors denoted by $(\rho^1_t, \rho^2_t, \cdots, \rho^6_t)$, I assume the manager uses the average of those prices, $\rho_t$, instead of using all of them. It reduces the computation burden and makes the optimal price more stable since hotel 0's demand is less affected by the irregular price of other single hotels. Besides these key components of demand, the "demand shifter", $x$, which represents for seasonality or other factor shifting the demand, and average daily rate (ADR) are also used in the optimal pricing model. I will explain in detail in the next section.

Prior to the value function on how the optimal price is decided, the next subsection introduces the component functions which make up the value function. The factors the manager considers for the price are plugged into these component function, so the outcome of the function is affected by variables, $t, n_t, \rho_t$ and so on.
1.3.2 Stochastic Probability Distribution

I assume that customer arrival process in the market is stochastic and follows negative binomial distribution. Those negative binomial distribution parameters \((\phi_{t,s}, \mu_{t,s})\) vary by days before arrival (DBA), or \(t\). For convenience’ sake, the time-varying parameters are replaced with 3rd-degree polynomial function of \(t\). And I also allow various types of potential customers in the market denoted by \(s\). Different customer types such as leisure, business and group have different inflow pattern by \(t\) and can be fitted well by 3rd-degree polynomial function. So, type \(s\) customers’ arrivals in the market at \(t\) can be denoted by \(r_{t,s}\) and its distribution, \(\pi(r_{t,s}|\phi_{t,s},\mu_{t,s})\), is given by

\[
\pi(r_{t,s}|\phi_{t,s},\mu_{t,s}) = \left(\frac{r_{t,s} + \phi_{t,s} - 1}{r_{t,s}^{\phi_{t,s}} (1 - q_{t,s})^{r_{t,s}}}\right)\]

where \(q_{t,s} = \phi_{t,s}/(\mu_{t,s} + \phi_{t,s})\). The assumption behind this is that arrival process in the market is independent across arrival day and \(t\), and all hotels are assumed to know the distribution of market inflow though they are not aware of the exact number of potential customers.

The potential customers of type \(s\) then choose one of seven hotels in the market for their accommodation during a trip. They are assumed to compare the prices of all hotels and finally pick one. With this sense, hotel 0 expect its new reservations of customer type \(s\) at \(t\) days before arrival as follows,

\[
a_{t,s} = r_{t,s} \cdot P(p_t, \rho_t|\alpha, \beta) = \frac{r_{t,s}}{1 + \exp[\alpha - \beta(p_t - \rho_t)]} \]  

(1.2)

where \(r_{t,s}\) is given and \(P(p_t, \rho_t|\alpha, \beta)\) is choice probability that market customer finally choose hotel 0 as their accommodation; \(p_t\) is price of hotel 0 at \(t\) and \(\rho_t\) is average price of other hotels at \(t\). The price is not the only thing customers consider when they look for their accommodation. In reality, there are more factors to be considered such as a location of hotel, facility level, guest satisfaction rate and other services (parking,
WiFi, etc). Such a non-price attributes of the hotel are assumed to be represented by $\alpha$ in the choice probability. Therefore, the number of new reservation at $t$ depends on $p_t$ and $\rho_t$ while $\alpha$ and $\beta$ are given.

Indeed, I am not aware of the precise process of price setting as it is unknown which hotel sets the price first and which one is next to set their price. So, I need additional assumptions to simplify the price setting process as follows. 1) All hotels set the price once a day. 2) hotel 0 is able to set the price for last time after observing all other hotels’ price. 3) All hotels have historical records on market price, so they reflect them in today’s posting. With 2), there is no feedback effect of $p_t$ on $\rho_t$, but it is expected that other hotels consider $p_t$ at $t-1$, so $\rho_{t-1}$ be affected by $p_t$.

For computation of optimal price solution, we need more details about feedback effect of $p_t$ as the manager of hotel 0 sets the price ($p_t$) based on the prediction of $\rho_{t-1}$ which is affected by $p_t$. As mentioned before, competitors set their own price ($\rho_t$) based on their previous price ($\rho_{t+1}$) and previous price of hotel 0 ($p_{t+1}$), and then hotel 0 observes the average of its competitors’ average price ($\rho_t$) to set the price ($p_t$). Given this timing of decisions assumption, the expectation of $\rho_t$ can be expressed as below,

$$\log[h_t(\rho_{t-1}|p_t,\rho_t)] = \begin{cases} 
\gamma_{1,t} + \gamma_{2,t} \log \rho_t + \gamma_{3,t} \log p_t + \epsilon_t & \text{if } p_t < \rho_t \\
\gamma'_{1,t} + \gamma'_{2,t} \log \rho_t + \epsilon_t & \text{if } p_t \geq \rho_t
\end{cases}$$

The error term in Equation (1.3), $\epsilon_t$, follows a normally distributed $i.i.d$ with mean 0 and variance $\sigma_t$. This equation shows a law of motion for competitors’ price ($\rho_{t-1}$) which is affected by $\rho_t$ as well as $p_t$ where $p_t$ is less than $\rho_t$. With 0.90 of $R^2$, the feedback effect of $p_t$ is approximately 14.8% compared to the effect of $\rho_t$, which is
very reasonable since the supply share of hotel 0 in the market is 15.2\%.\textsuperscript{1} However, I assume that there is no feedback effect where \( p_t \) is equal to and larger than \( \rho_t \). Intuitively, if other competitors always partly follow hotel 0’s price, then there is an incentive for hotel 0 to increase their price. As hotel 0 increases its’ current price, it is expected that other hotels will also increase their price in the future. In the long run, hotel 0 would have a chance to have a high markup since all hotels’ price are high as well.\textsuperscript{2} With such a restriction in place, we have a more reliable and realistic price rule when computing dynamic optimal pricing.

I use a deterministic probability of cancellation as \( e_t(c_t|n_t) \) with an assumption that cancellation is exogenous and has nothing to do with the change in price of hotel 0. It is expected that reservations are cancelled more when a price drops compared to at purchasing day. I find there is evidence that strategic cancellation exists, so cancellations are affected by \( \Delta p_t \) as well.\textsuperscript{3} However, I did not include strategic cancellation in the cancel probability function, \( e_t \). First, the evidence of strategic cancellation is weak and might have no effect based on a different perspective. I did not find that cancellations occur more with a price drop compared to a price rise. However, when I focused on only the cancelled cases, the ratio of cancellations with a price drop is significantly higher than cases with a price rise. Second, as adding an initial price of cancelled bookings to the model, there is a high computational burden. I am open to considering strategic cancellations in the model, however, it will have to be saved for the future once I have a better understanding of the cancellation mechanism.

\textsuperscript{1}See Appendix (B) for the regression result.
\textsuperscript{2}See Appendix (C) for the detail of computational evidence for irrational phenomenon caused by price feedback effect.
\textsuperscript{3}See Appendix (D) for evidence of strategic cancellation.
1.3.3 Computation of Optimal Pricing

This section introduces the Bellman equation developed by Yu [35] and extends it to include a price feedback effect on competitors’ price. This equation contains the value function of the optimal dynamic hotel pricing mode. The value function consists of several components, including those mentioned in the previous section, such as \(e_t(c_t|n_t), \pi(r_{t,s}|\phi_{t,s}, \mu_{t,s}), P(p_{t,s}, \rho_{t,s}), \) and \(h_t(\rho_{t-1}|p_t, \rho_t).\) In addition, I need to define further notations and equations to complete the value function. While hotel 0 posts a rate called BAR \((p_t)\) for a specific arrival day at day \(t,\) this does not imply that all customers who make a reservation of hotel 0 at day \(t\) are supposed to pay the same rate, \(p_t.\) Depending on which rate code each customer belongs to, he/she is provided a discounted rate of \(p_t.\) However, the variation in discount rate is huge, which makes it almost impossible to predict the exact discount rate. With this in mind, I use the average discount rate for each customer type \((s)\) and each seasonality \((x).\) Although it is not the perfect way to handle the discount rate, it makes the model more realistic. Eq. (1.2) is re-expressed as

\[
a_{t,s} = r_{t,s} \cdot P(p_t, \rho_t | \alpha_s, \beta_s) = \frac{r_{t,s}}{1 + \exp[\alpha_s - \beta_s(\sigma_s p_t - \sigma_s \rho_t)]}
\]

(2')

where \(\sigma_s\) denotes the average discount rate (%) for reservation by customer type \((s).\) So, each customer type \((s)\) compare the discounted price of hotel 0 with the discounted price of other hotels, and then finally chooses either of them. It is also necessary to rewrite the Equation for the demand of hotel 0, \(a_{t,s},\) in the sense that customer arrival in the market is a stochastic distribution following a negative binomial rather than a deterministic process. Using the probability mass function of binomial distribution, \(a_{t,s},\) is derived from Eq. (1.4).

\[
f_t(a_t|p_t, \rho_t) = \sum_{s \in S} \sum_{r \geq a} \binom{r_{t,s}}{a_{t,s}} P(p_t, \rho_t | \alpha_s, \beta_s)^a [1 - P(p_t, \rho_t | \alpha_s, \beta_s)]^{(r-a)} \pi(r_{t,s}|\phi_{t,s}, \mu_{t,s})
\]

(1.4)
Given the number reservations of hotel 0 at time $t$, there is a various possible number of potential customers in the market. Thus, the probability of $a_{t,s}$ is expressed with a combination of the probability of market arrival and customer choice preferences. Since there is customer type set $(S)$, we aggregate them to derive $a_t$.

I need to separate the group reservations from other individual reservations and transient reservations since they book a block of hotel rooms at a time which differs from the process of individual reservations. Rather than being affected by BAR, group reservations may negotiate with the hotel directly and decide their booking block. The number group reservations of hotel 0 is denoted by $a_{t,g}$, $g \in S$ which follows the deterministic probability of $g_t(a_{t,g}|p_t, \rho_t)$. I also use $p_t$ and $\rho_t$ as inputs of function $g_t$ to prevent $p_t$ from increasing abruptly since the negotiated rate of group reservations are also bound to the level of BAR, $p_t$. Now $a_{t,-g}$ where $-g \in S$ represents the individual reservations without group reservations ($a_{t,g}$).

With the new reservation probability function $g_t(a_{t,g}|p_t, \rho_t)$, $f_t(a_{t,-g}|p_t, \rho_t)$ and cancellation function $e_t(c_t|n_t)$, we have an expectation of occupancy level $n_{t-1}$ for day $t-1$. When the current occupancy at $t$ is $n_t$, $n_{t-1}$ is expressed as

$$n_{t-1} = n_t - c_t + a_{t,-g} + a_{t,g} \quad (1.5)$$

Considering the case of binding constraint on $\bar{n}$, I need an assumption about the process of rationing rooms to customers. With the same manner of processes of price setting by seven hotels, it is hard to know who has priority when they arrive on the same day. Basically, the rule of rationing rooms is “first come, first serve” however this process is unclear in the model which has a stochastic probability on the number of each type of customer in a day. To proceed further in detail, I let the function $\eta_t$ denote the demand censoring rule that enables the hotel to prevent overbooking.
every $t$

$$n_{t-1} \equiv \eta_t(a_{t,-g}, a_{t,g}, c_t, n_t, \bar{n}) = \min(\bar{n}, n_t - c_t + a_{t,-g} + a_{t,g}) \quad (1.6)$$

Specifically, the realized components $(a_{t,-g}, a_{t,g})$ of $n_{t-1}$, which is denoted by $(a_{t,-g}^r, a_{t,g}^r)$, follows the below rule,

$$(a_{t,-g}^r, a_{t,g}^r) \text{ of } n_{t-1} = \begin{cases} (a_{t,-g}, a_{t,g}) & \text{if } \bar{n} > n_t - c_t + a_{t,-g} + a_{t,g} \\ (\bar{n} - n_t + c_t, 0) & \text{if } n_t - c_t + a_{t,-g} \geq \bar{n} \\ (a_{t,-g}, \bar{n} - n_t + c_t - a_{t,-g}) & \text{otherwise} \end{cases} \quad (1.7)$$

This rule is necessary when I compute the revealed demand keeping capacity constraint. Due to the difficulty of enforcing “first come, first serve” in computation, I assume that transient consumers have priority if there is overbooking in hotel 0. If there are rooms available, the hotel manager decides whether to accept a group reservation or not as expressed in Eq. (1.7). For group reservation contracts, there is a clause that rooms are subject to availability, which implies that the hotel manager is allowed to deny a group reservation under contract if necessary. Therefore, the underlying assumption of Eq. (1.7) is reasonable in this sense.

In reality, overbooking occurs often before arrival day and it occurs even on arrival day. According to the hotel manager, they try their best to prevent overbookings, but it still happens from time-to-time when cancellation rates are less than expected. The compensation to customers at overbooking is up to the manager’s discretion, so we are not able to measure the penalty of overbooking. With regard to this, I strictly enforce hotel 0’s capacity constraint $\bar{n}$ on every date $t$, thus overbooking never happens in the model due to a few observations of overbooking occurrences in the reservation data of hotel 0.

As the last step before moving on to value function, I define the law of motion for average daily rate (ADR), $\bar{p}_t$, for DP model. According to Yu [35], $\bar{p}_t$ can be excluded
in hotel DP model when cancellation is exogenous instead of strategic cancellations.\textsuperscript{4} However, by using $\bar{p}_t$ in DP, it is convenient to keep track of revenue information and grants flexibility in case of strategic cancellations in the model. The law of motion for $\bar{p}_t$ is as below,

$$ \bar{p}_{t-1} = \frac{(n_t - c_t)p_t + \sum_{s \in S} \delta_s p_t a_{t,s}}{n_{t-1}} $$  \hspace{1cm} (1.8)

where $\delta_s$ is the average discount rate of customer type ($s$) while $a_{t,s}$ denotes new reservations of customer type ($s$) on day $t$ which is actually a function of ($p_t, \rho_t$).

By multiplying $n_{t-1}$ with $\bar{p}_{t-1}$, bygone revenue of hotel 0 by day $t - 1$ is derived.\textsuperscript{5}

As for cancellations, the amount of refund is assumed to be the same with $p_t$. Let $\bar{p}_{t-1} = \lambda(n_t, c_t, a_{t,1}, \cdots, a_{t,s}, \bar{p}_t, p_t)$ denote the function of Eq. (1.8).

Now I suggest the Bellman equation for dynamic optimal hotel pricing model.

I start with the value function of the day after occupancy day expressed by time subscript $t = -1$. For convenience sake, I drop the time subscript in parenthesis for the value function. Let $V_{-1}(n, \bar{p}, p)$ be the confirmed profits of hotel 0 the day after occupancy, which is defined as below,

$$ V_{-1}(n, \bar{p}, \rho) = n \cdot (\bar{p} - \omega), $$  \hspace{1cm} (1.9)

where $\omega$ is the marginal cost of room maintenance by hotel 0 and includes a commission fee given to the online travel agency. Since $\omega$ is imposed on each room, marginal cost does not affect the choice of optimal price in this model. A more accurate marginal cost may reflect a fixed maintenance fee and invisible costs, such as loss of reputation.

\textsuperscript{4}See Appendix (E) for numerical evidence that the optimal price rule is not affected by average daily rate (ADR)

\textsuperscript{5}To clarify, $(n_{t-1} \times \bar{p}_{t-1})$ denotes the revenue which is confirmed in the morning of day $t - 1$. This note may be helpful to figure out the revenue of occupancy day.
Given the terminal value $V_{-1}$ in Eq. (1.9), the Bellman equation defines the expected profit functions $\{V_0, V_1, \ldots, V_T\}$ recursively as below,

$$V_t(n, p, \rho) = \max_p \left[ \int_{\rho'} \sum_{a-g} \sum_a \sum_c V_{t-1}(n', \bar{p}', \rho') e_t(c|n) f_t(a-g|p, \rho) g_t(a_g|p, \rho) h_t(\rho'|p, \rho) \right]$$

s.t.  \[ n' = n - c + a_g + a_g \]

\[ \bar{p}' = \lambda(n, c, \bar{a}, \bar{p}, p) \]

\[ n' = \eta(a_{-g}, a_g, c, n, \bar{p}) \].

(1.10)

The value function consists of four-state variables $t$, $n$, $\bar{p}$, and $\rho$. The goal of this value function is to find $p$, which maximizes the profit of hotel 0, on the day after occupancy, so $p_t^*(n, \bar{p}, \rho)$ is called optimal price. $p_t$ is a parsimoniously parameterized model, therefore the optimal price is contingent on the parameter values that are supposed to be estimated. Yu [35] proposes a demand estimation with the assumption of optimality by combining a dynamic optimal pricing model with an MSM method. The results show that his methods consistently estimate the demand parameters under the optimality assumption. Both price and demand estimate are closed to observed price and demand of hotel 0, therefore his method is useful in generating simulated data that is close to the actual data. I will provide a comparison between his estimate results with the estimate based on a two-step semi-parametric in the result section.

For both Yu [35] or this study, a hotel manager must first figure out the current status represented by the number of rooms occupied, the days in advance to arrival, the average price of reserved bookings, and competing hotels’ price in the morning, and then set the price in accordance with them. Since the final goal of optimal price is profit maximization of occupancy day, the model takes into account all possible combinations that will occur before arrival day. By backward induction, the optimal
price is computed from the last day, $t = 0$, to the first day, $t = 45$. By using the benchmarking price, $\rho_t$, the model can find a reasonable level of price more efficiently.

1.4 **Demand Estimation Without the Assumption of Optimality**

I examine demand estimation without optimality in this section. According to Cho et al. [9], the demand parameters are identified with a two-step semi-parametric estimation when a conditional independence assumption holds. Intuitively, the conditional independence assumption can be interpreted as follows: prices may be affected by unobserved shock or information which is independent of the distribution of demand, but conditions on observed factors for price. The unobserved factors are key for identification as they can be used for randomized price experiments and serve a similar role as traditional linear instrumental variables. However, the two-step semi-parametric estimation approach does not require to find this unobserved price shock. The new approach estimates market demand under weaker assumptions than the optimality assumptions. This method allows to estimate parameter even when only a few instrumental variables are available and demand is a non-linear function of price.

Therefore, this approach does not necessitate the assumption that hotel 0 sets its price

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6According to Cho et al. [9], following assumptions are necessary for parameter identifications:

**A0 (Correct Specification)** The true distribution of occupancy $a$ given $(r, p, \rho, x)$ is given by $f(a|r, p, \rho, x, \theta^*)$ and the true distribution of $r$ given $x$ is given by $\pi(r|x, \theta^*)$.

**A1 (Exclusion Restriction)** Hotels set prices prior to arrival of customers $r$ so prices are independent of $r$.

**A2 (Conditional Independence)** Hotel prices depend on a vector of unobservables $z$ as well as the observed demand shifter $x$, so we have $p = h(z, x)$ for some measurable function $h$. Demand $r$ is conditionally independent of $z$, i.e. the conditional distribution of demand satisfies $f(a|r, p, \rho, z, x, \theta^*) = f(a|r, p, \rho, x, \theta^*)$ and $\pi(r|z, x, \theta^*) = \pi(r|x, \theta^*)$.

A0 is also assumed in the demand estimation with the assumption of optimality. And A1 and A2 can be viewed either as exclusions, or equivalently as conditional independence assumptions.
optimally based on its own experiences and through the RMS system. Instead, this method estimates demand parameters with given prices of hotel 0 as it is.

1.4.1 Process of Two-step Semi-parametric Estimation

I classify arrival days of the data set into 4 quartiles of market occupancy and divide them by weekend, which is defined as Thursday, Friday and Saturday. So, I implement a total of $K = 8$ separate estimations with an assumption that parameters are the same within subsample $k$, where $k \in \{K_1, K_2, \cdots, K_8\}$. Based on this grouping, I collect a data set $D_k = \{\vec{n}_0, \vec{a}_0, \vec{c}_0, \vec{p}, \vec{\rho}, n_0^1\}_i$ where $i \in k$. I use the vector notation in $D_k$ to detail the history of each information of hotel 0. Therefore, $\vec{n}_0 = (n_0^0, n_0^1, \cdots, n_0^T)$ is the occupancy history of hotel 0 leading up to day $T$, $\vec{a}_0 = (a_0^0, a_0^1, \cdots, a_0^T)$ is the number of reservation history of hotel 0, $\vec{c}_0 = (c_0^0, c_0^1, \cdots, c_0^T)$ is the number of cancellation history of hotel 0, $\vec{p} = (p_0^0, p_1^0, \cdots, p_T^0)$ is history of BAR price set by hotel 0, and $\vec{\rho} = (\rho_0^0, \rho_1^0, \cdots, \rho_T^0)$ is the average of competing hotels’ price history where $T = 45$ in this estimation and subscript “0” indicates hotel 0 contrasted to “1” of competitors. Lastly, $n_1^0$ denotes the total occupancy of competitors with subscript “1” on arrival day ($t = 0$).

With the data set, I introduce a procedure of a two-step semi-parametric estimation for each subsample $k$ as follows,

1) Using semi-parametric or linear regression estimation, uncover a status quo price policy of hotel 0 given the state variables. So, price policy function is defined as $p_{f_i}(n, \bar{p}, \bar{\rho})$ connoting “i” for the first stage.

2) Given data set $D_k$, compute moment function $M_k$, which is defined by

$$M_k = \frac{1}{|k|} \sum_{i \in k} m(\{\vec{n}_0, \vec{a}_0, \vec{c}_0, \vec{p}, \vec{\rho}, n_1^0\}_i),$$

where $m(\cdot)$ be a $J \times 1$ vector which consists of functions in a given data set.
3) Using probability distribution mentioned in 1.3.2 and \( pf_t(n, \bar{p}, \rho) \), generate \( S \) simulation data set given a conjectured \( \theta \), where \( \theta \) contains parameters of demand, choice probability and so on. Let \( D_k^S(\theta) \) be a simulation set for sub-sample \( k \).

4) Given data set \( D_k^S(\theta) \), compute moment function \( M_k^S(\theta) \), which is defined by

\[
M_k^S(\theta) = \frac{1}{S} \sum_{i \in k} \sum_{j=1}^{S} m(\{\vec{n}_s^i, \vec{a}_s^i, \vec{c}_s^i, \vec{pf}^i, \vec{\rho}^i, n_{0,s}^i\}),
\]

(1.12)

where \( \vec{pf} \) is the price history of hotel 0 which is derived from policy function of 1).

5) Repeat 3) and 4) until an estimator of \( \hat{\theta} \) is obtained by method of simulated moments (MSM).

In step 3), there is a full trajectory of information in the simulation data set including competitors’ occupancy (\( \vec{n}_s^i \)) and reservation (\( \vec{a}_s^i \)). However, I limit them in the same way for observed data set \( D_k \) and keep only the occupancy of competitors on arrival day, \( n_{0,s}^i \), for MSM.

In step 1), I first gather the actual data set of hotel 0 and learn the observed pricing rule through an econometric analysis. Whenever the data set is updated and there is more data available, the latest data set can be re-analyzed to acquire a more accurate decision rule of hotel 0. The goal of this step is to find the relation between price and other variables instead of seeking the structural mechanism of price making it relatively less limited in the model specification. I suggest several price rule models using regression learning and their results in the next section.

The price rule \( pf_t \) found in the first stage is used in the second stage, namely step 2) - 5), to generate a simulated data set of stochastic demand and arrival process. It may be questionable a reason that I use \( pf_t \) instead of actual price data, \( p \). The
answer is very simple. If I use \( p_t \) in the second stage to generate a demand and arrival simulation such as \( n_t \) or \( a_t \), it works on the all simulation data on day \( t \). However, it raises the issue in the next day, \( t - 1 \). Since all data including \( n_t \) and \( a_t \), but \( p_t \), are simulated one, there does not exist an actual \( p_{t-1} \) corresponding them. Then the simulation tool does not make a further progress since day \( t - 1 \). In order to combine price rule with simulated demand data set which is built on underlying structural demand model, it is essential to use the estimated pricing policy \((p_{f, t})\) based on the those input variables.

In the second stage, by changing the parameters, MSM enables demand response against the estimated price rule \( p_{f, t} \) to be close to the demand in the real data, \( D_k \). Through all time periods, \((T = 45)\), the simulated data set \( D_k^S(\theta) \) is generated to be close to the data and then it ends until the distance between two moments sets converges enough. As a result of the second stage, the estimated demand parameters are given and it enables me to calculate the optimal pricing of the hotel as solving an optimal pricing rule problem, described in Yu [35].

1.4.2 Method of Simulated Moments

It is known that the maximum likelihood estimator (MLE) is an efficient method in the parametric model. However, Yu [35] and this study use the method of simulated moments MSM due to several reasons, such as data censoring and endogeneity problems, which make it harder to estimate parameters by MLE. As mentioned before, I do not observe the actual number of potential customers who are looking for their accommodations within seven hotels. The available partial information I observe is the number of market reservations at occupancy day, so it requires me to solve a high dimensional integration problem by integrating out a full likelihood function. For censoring problems, I may be able to apply MLE to solve the estimation if I am able
to overcome computational burdens to do it. By using MSM, I generate a simulated data set which is based on some probability distribution assumptions, so it does not require an integration problem anymore. This method matches the simulation data with the observed one and finally returns parameters which bring out a reasonable simulation data set.

The endogeneity problems are more severe than censoring problems in both the usual demand model and this hotel model. There is a high correlation between the price of hotel 0 and its occupancy level. To deal with endogeneity problems, the instrumental variables estimation is standard by finding one variable which is orthogonal to demand shifters, but affects the shifter of hotel 0’s price. However, this method works when demand function is assumed to be linear and there is a relevant instrumental variable. I assume that hotel demand of this market is stochastic probability function of price, in fact, it is a non-linear function. To come to be concrete, I make the assumption that hotel demand is an aggregate expected demand of individual based on the discrete choice model. Due to this reason, the IV method is not applicable to the hotel demand model and I use a different view related to demand estimation. The data set includes a final market demand, i.e. total market reservations at occupancy day, and competitors’ prices are given at any time points.\textsuperscript{7} Based on the assumption that hotel 0’s relative price to competitors is a key factor on consumer choice, MSM indirectly looks for demand function which can be matched with total market demand at occupancy day.

Compared to the generalized method of moments (GMM), MSM is useful when it is hard to evaluate the theoretical moments. By using the features of data statistics, especially moments of the data, the method makes the moments of simulations

\textsuperscript{7}Although competitors’ price seems to be an instrumental variable in the demand model, it could not be. Since all hotels share potential customers, competitors’ price must affect hotel 0’s demand, so it is not satisfied with IV condition.
closer to the actual moments. In this indirect way, I attempt to solve the econometric problems.

Let $\theta$ be a $M \times 1$ vector of parameters for the stochastic demand, choice probability and so on. A vector of $J \geq M$ moments are used in MSM. The estimator $\hat{\theta}$ minimizes the distance between $M_k$ and $M_k^S(\theta)$ by using the following quadratic equation,

$$\hat{\theta} = \arg\min_{\theta} [M_k - M_k^S(\theta)]'W(J)[M_k - M_k^S(\theta)]$$

(1.13)

where $W(J)$ is positive definite $J \times J$ weighting matrix.

With a Law of large numbers for $i.i.d$ observations which consist of each moment and suitable regularity conditions, I assume that $M^S_k(\theta)$ converges with probability 1 to $M_k$, the expectation of individual random vectors of demand and other factors. The result of the next section is based on the estimator $\hat{\theta}$ from this MSM process.

1.5 Result

1.5.1 Two-step Semi-parametric Approach

The first step of the estimation process mentioned in 1.4.1 begins with a semi-parametric price rule, namely $pf_t(n, \bar{p}, \rho)$. The goal of the first step is to find the most accurate price rule no matter which model is used for price prediction. I use regression learning to uncover the price rule.\(^8\) This method provides various price rules for each subsample $k$, and it follows that the best fit model is normally gaussian process regression (GPR) which returns the lowest level of RMSE and the highest $R^2$ for most subsamples. However, this semi-parametric model has an underlying problem of overfitting and is quite sensitive to noise. The linear regression results produce a more clear picture and display the influence of input variables, making it easier to

\(^8\)See Appendix (F) for details of regression results for Root mean squared error (RMSE), Mean Absolute Error (MAE), and Mean Squared Error (MSE).
interpret the estimations, however, this regression also tends to have a huge bias with regards to $R^2$ and RMSE. A more accurate price prediction is only feasible in demand estimation. However, when computing the optimal price with the dynamic pricing model, the model considers all the possible combinations of variables, even if they have not occurred in the actual data. To take care of the sensitive noise out of the actual variable range, I exclude the over-fitting model computed in the regression learner. With concern of interpretation and prudent stance, the remaining results of the second step are computed from the simplest price rule, linear model, though its average $R^2$ is relatively low, only 0.55, compared to the other model. However, future work for this study can apply more flexible models that originate from machine learning algorithm. Table F.1 in Appendix (F) shows several semi-parametric models which can be used to uncover the existing price rule of hotel 0.

![Figure 1.3: Demand simulation by sub-optimal and optimal (busy weekday)](image)

Once I get the price rule in the first step, the demand parameters are estimated in the second step. The model fit in the second step is quite good and illustrates the demand pattern on a busy weekday in the left panel of Figure 1.3. The upper one
shows the average demand of hotel 0 over DBA while the lower one displays competitors’ demand. The round-solid line in the upper panel shows the average daily bookings from the observed data set, which exhibits a rising curve with a periodic pattern. This means that more new reservations are made as occupancy day approaches. The average of simulated data set, indicated by a broken line in the upper panel, also exhibits similar movement. In other subsamples, the simulation data set follows demand patterns and cancellations very well.

Appendix (A) contains the estimate result from the second step of a two-step semi-parametric estimation. I estimate $M = 46$ parameters for $K = 8$ different subsamples, and Table A.2, A.4, A.6, A.8 and A.10 are the detail results of those subsample estimations. According to Eq. (2’), the demand elasticity $\frac{dQ}{dP} \cdot \frac{P}{Q}$ is expressed by $\beta_s \sigma_s P_t$. Using Table A.2, we see the demand elasticity of each customer type as a business customer is less sensitive to price for all subsamples, which are consistent with a common intuition about price elasticity. This result is consistent with the result of Yu [35], so both estimation methods return reasonable demand estimates.

The sign of parameter $\beta_s$ also implies a downward sloping demand curve. Due to the high co-movement between price and demand in the market, linear regression result of demand on price is upward sloping, which is counter-intuitive. However, the estimated $\beta_s$ has a negative sign and therefore, it shows that potential customers are less likely to choose hotel 0 when its price is relatively high compared to that of the competitors’.

Furthermore, I use zero-inflated negative binomial (ZINB) distribution instead of negative binomial to take into account of a potential number of customers in the market with excessive zeros, which are used for over-dispersed demand. Considering the estimated parameter values for constant term in Table A.4 and A.8, there is a partial effect to the demand of leisure and business customers. However, I did not find
any evidence that zero-demand patterns are time-varying features since the estimated value for $t$, $t^2$ and $t^3$ show their effects are close to zero.

While parameters for group customer demand is erratic with unpredictable values in each time $t$ varying term, cancellation rates are quite stable across all subsamples. Since I do not allow strategic cancellation in the hotel model, cancellations are not affected by the change in hotel 0's price. Therefore, the estimated parameters related to cancellations remain close to its initial value calculated before the second step estimation for the aggregate sample. The slight changes in cancel parameters are caused by subsample characteristics and I conclude that the hypothesis of exogenous cancellations with $e_t(c|n)$ gives a good approximation for the current cancellation status.

1.5.2 Optimal Price Without Optimality Assumption

![Figure 1.4: Optimal price trajectory (busy and busiest weekday)](image)

I now turn to the optimal price mentioned in section 3 with the outcome of the two-step semi-parametric estimation. I apply the estimated parameters to the pricing function and generate a simulated data based on them. Figure 1.4 shows the graphical
comparisons of optimal price and observed price. The left panel is a price trajectory for busy weekdays while the right one is for the busiest weekdays. In both figures, the prices of hotel 0, denoted by the star-dotted or the star-solid line: the solid one is the actual price and the dotted one is the optimal price trajectory. The two prices of hotel 0 are almost always located below the competitors’ prices - denoted by the round-dotted (actual data) or the round-solid line (simulated data). In the left panel of Figure 1.4, busy weekdays, the optimal price begins with at a higher level than the actual price, which is just below the competitors’ price. The computed optimal price strategy suggests that hotel 0 keeps the on average higher price for earlier days than 25 days out and lowers the price after that. When this occurs, the optimal price crosses the actual price around 17 days out. The slope of the optimal price is slightly flatter after the cross point, but the optimal price keeps the low price strategy until occupancy day.

The changed price for busy weekdays affects the demand for hotel 0 differently compared to the previous price, i.e. price data. In the right panel of Figure 1.3, I find different demand patterns under an existing price and the optimal price computed through a dynamic programming framework. While the same observed demand data are given in both panels, the demand pattern by the computed optimal pricing rule (right panel) shows a slower pace at the beginning. Around 25 days before occupancy, the slope of demand becomes steeper corresponding to the low price strategy shown in the left panel of Figure 1.4. Therefore, the optimal price of busy weekdays suggests that hotel 0 focus on the customers who make a reservation later rather than earlier. Since hotel 0 expects many potential customers to look for their accommodations just before occupancy, it is better to hold hotel rooms for those customers and give up on early booking customers whose willingness to pay are relatively low.
The optimal price strategies are different when it comes to different seasons. The right panel of Figure 1.4 displays the optimal price for the busiest weekday season. Compared to the left panel of a busy weekday, the suggested price of hotel 0 looks flatter and consistent over days before arrival (DBA). The observed price of competitors, denoted by the round-solid line, has an increasing curve and the simulation of competitors' price also follows the increasing trend of the actual data. Due to the increase in competitors' price, the actual price of hotel 0, denoted by star-dotted line, increases dramatically as it approaches occupancy day ($t = 0$). This shows that hotel 0 uses *pricing-following* strategy in the actual data and it is found in both panels of Figure 1.4. However, the optimal price suggests that the precise price-following strategy is not always optimal, and is flexible on using the price-following strategy. The price strategies in Figure 1.4 are the average of the optimal prices in corresponding seasons, and the price of each day varies by the given condition of hotel 0, i.e. occupancy rate, competitors’ price and so on.

I discuss the revenue of hotel 0, which is based on a new price strategy, i.e. the computed optimal price. I compare the optimal price with status quo price and quantify the gains from switching to the computed optimal pricing policy. This comparison is to see the validity of the optimal price in practice. If I find improvement in revenue of hotel 0 by using the optimal price, the model can suggest a better price option for hotel 0 and other hotels in the real world.

Table 1.3 is a summary of the whole estimation results. Once the estimation is complete, there are three different data sets, “actual data set (A)”, “simulated data set (B)” of the first step semi-parametric sub-optimal policy, and “simulated data set (C)” of the second step dynamic optimal policy. In both simulated data sets, I use the same demand and choice parameters, yet I have different simulated data sets since the mechanism of those price policies are different. Thus, the customer response against
Table 1.3: Expected simulation result of two-step semi-parametric estimation

<table>
<thead>
<tr>
<th></th>
<th>1st-step Semi-parametric policy (B)</th>
<th>2nd-step Dynamic Optimal policy (C)</th>
<th>Rate of change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Revenue ($)</td>
<td>Occupancy (%)</td>
<td>Avg. Price ($)</td>
</tr>
<tr>
<td>Weekday</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>calm</td>
<td>34,496.00</td>
<td>54.9%</td>
<td>185.30</td>
</tr>
<tr>
<td>normal</td>
<td>52,726.00</td>
<td>76.5%</td>
<td>207.46</td>
</tr>
<tr>
<td>busy</td>
<td>71,713.00</td>
<td>87.7%</td>
<td>247.21</td>
</tr>
<tr>
<td>busiest</td>
<td>86,074.00</td>
<td>98.6%</td>
<td>288.34</td>
</tr>
<tr>
<td>Weekend</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>calm</td>
<td>35,620.00</td>
<td>60.9%</td>
<td>170.29</td>
</tr>
<tr>
<td>normal</td>
<td>52,981.00</td>
<td>83.1%</td>
<td>190.38</td>
</tr>
<tr>
<td>busy</td>
<td>72,000.00</td>
<td>92.6%</td>
<td>234.55</td>
</tr>
<tr>
<td>busiest</td>
<td>73,188.00</td>
<td>93.6%</td>
<td>236.00</td>
</tr>
</tbody>
</table>

Avg. rate of Change (%) 8.7% 9.9% 0.5%

By comparing the simulation data sets with the actual data set, it finds how much the price policy improves the revenue of hotel 0. However, there is difficulty in comparing the simulation data with the actual data. The actual hotel reservations consist of various individuals and group segments, which have their own discount rate. Including complimentary reservations and group reservations which are bound in mutual contracts, the prediction of daily average rate (ADR) for each check-in day is not tractable in some points. In an attempt to match the discount rate, I use the average discount rate, but I find that they are more diverse in the range of discount rates and it prevents the estimation from predicting an accurate ADR. Therefore, I assume that the price rule and demand estimation by the first-step of the two-step method is a good approximation of the actual data. In Table 1.3, I present the two simulation results (B) and (C). Since they use the same discount rates by market segments (‘leisure’, ‘business’, ‘group’), they are more comparable and show enhancement by optimal price.

All results in Table 1.3 are calculated by eight subsamples and it shows the expected revenue, occupancy rate, and average daily rate for two simulation data sets. According to the table, I find that the total revenues have increased with the
optimal price policy for all subsamples, by an average of 8.7%. In the subsample of the calmest weekend, it shows that there is an increase in revenue by 31.3% with a lower average price (-6.4%) and higher occupancy rate (+45.0%). The optimal dynamic price suggests that the current price is rather expensive, so hotel 0 is able to succeed in reaching higher revenue by lowering the price. Under the first step price policy, the occupancy rate of hotel 0 is predicted at 60.9% while the optimal price enables the hotel to have an 88.3% occupancy rate. This is a remarkable result, especially when I compare this result with a calm season weekday subsample. Both are calm seasons, but there are more leisure customers during the weekend.

According to the estimation results, business customer is less price elastic than leisure customers. So, a lower-price is not very attractive to business customers during a weekday (Table A.2). This explains why the lower price strategy is more successful during the weekends. Still, the optimal price suggests that on average, a lower price on in calm weekday, but it is not as profitable as a weekend season. In summary, the dynamic optimal price implies a more aggressive price differentiation strategy between seasons is helpful to raise revenue. While the rate of change in the average daily rate is blended, the optimal price is expected to bring a higher occupancy rate compared to the first step price policy, or approximation of observed data. I see a significant increase in revenue regardless of each subsample’s change in occupancy and average price. This is because demand estimation without the assumption of optimality is not bound to the existing price rule of hotel 0.

1.5.3 Consistency of a Two-step Semi-parametric Estimation

This section presents the validity of a two-step semi-parametric estimation with simulation data generated by a new parameter, $\theta_0$. This test is to see whether a two-step semi-parametric estimation uncovers a true parameter consistently. While the new
Table 1.4: True parameters ($\theta_0$) for validity test

<table>
<thead>
<tr>
<th>Parameter</th>
<th>note</th>
<th>Value</th>
<th>Parameter</th>
<th>note</th>
<th>Value</th>
<th>Parameter</th>
<th>note</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_s$</td>
<td>Leisure</td>
<td>1.720</td>
<td>$\mu_0$</td>
<td>Binomial (Business)</td>
<td>t</td>
<td>8.7E-6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>Leisure</td>
<td>0.000</td>
<td>$\gamma_t$</td>
<td>Zero</td>
<td>$t^2$</td>
<td>-1.0E-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_b$</td>
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<td>2.720</td>
<td>$\phi_t$</td>
<td>Negative</td>
<td>$t^2$</td>
<td>-0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_b$</td>
<td>Business</td>
<td>0.009</td>
<td></td>
<td>Inflation</td>
<td>$t$</td>
<td>0.010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_t$</td>
<td>Negative</td>
<td>$t^3$</td>
<td>$\gamma_t$</td>
<td>Zero</td>
<td>$t^3$</td>
<td>-0.105</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Binomial (Business)</td>
<td>1</td>
<td></td>
<td>Inflation</td>
<td>$t$</td>
<td>-0.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_t$</td>
<td>Leisure</td>
<td>0.009</td>
<td></td>
<td>(Group)</td>
<td>1</td>
<td>-1.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_t$</td>
<td>Group</td>
<td>0.005</td>
<td></td>
<td>Mean</td>
<td>$t^3$</td>
<td>3.9E-5</td>
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<td></td>
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<tr>
<td>$\mu_b$</td>
<td>Group</td>
<td>1.523</td>
<td></td>
<td>Arrivals</td>
<td>$t^2$</td>
<td>-0.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_t$</td>
<td>Leisure</td>
<td>$t^3$</td>
<td></td>
<td>(if arrival &gt;0)</td>
<td>$t$</td>
<td>0.208</td>
<td></td>
<td></td>
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<tr>
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<td>Negative</td>
<td>$t^2$</td>
<td></td>
<td></td>
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<td>0.0037</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Binomial (Business)</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>0.077</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Leisure</td>
<td>$t^3$</td>
<td></td>
<td>Probability</td>
<td>$t^3$</td>
<td>7.8E-5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td>$t^2$</td>
<td></td>
<td>of cancel &gt;0</td>
<td></td>
<td>-0.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Binomial (Business)</td>
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<td></td>
<td></td>
<td>1</td>
<td>20.723</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>Leisure</td>
<td>$t^3$</td>
<td></td>
<td>Probability</td>
<td>$t^3$</td>
<td>3.9E-5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td>$t^2$</td>
<td></td>
<td>of cancel &gt;0</td>
<td></td>
<td>-0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Binomial (Business)</td>
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<td></td>
<td>1</td>
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<td></td>
</tr>
<tr>
<td>$\beta_t$</td>
<td>Leisure</td>
<td>2.0E-15</td>
<td></td>
<td>Probability</td>
<td>$t^3$</td>
<td>3.9E-5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Business</td>
<td>0.077</td>
<td></td>
<td>of cancel &gt;0</td>
<td></td>
<td>0.077</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_b$</td>
<td>Group</td>
<td>1</td>
<td></td>
<td>Rate</td>
<td>$t^3$</td>
<td>4.093</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_b$</td>
<td>Group</td>
<td>0.005</td>
<td></td>
<td></td>
<td>1</td>
<td>0.077</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_b$</td>
<td>Group</td>
<td>1.523</td>
<td></td>
<td></td>
<td>1</td>
<td>0.077</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_t$</td>
<td>Leisure</td>
<td>$t^3$</td>
<td></td>
<td></td>
<td>1</td>
<td>0.077</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td>$t^2$</td>
<td></td>
<td></td>
<td>1</td>
<td>0.077</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Binomial (Business)</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>0.077</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Demand parameters generate a simulation data set, the test pretends not to know the change in demand. Therefore, I assume that an econometrician or hotel manager keeps a one price policy, which they believe to be optimal, making it unreasonable with a new demand distribution. The validity test is motivated from a real world issue, where it is possible that there is a demand shift in the market without the knowledge of the hotel manager, therefore the manager keeping the original optimal policy based on previous beliefs on demand.

For the validity, I generate a simulation data set with $\theta_0$ as shown in Table 1.4 and one price policy $\hat{p}w$ I use in section 5.2. It is also allowed to use a randomly allocated price rule to verify a two-step semi-parametric estimation. For a set of sampled days $i = 1, \ldots, N$, the new data set consists of $D_k^*(\theta_0) = \{\tilde{n}_0^s, \tilde{a}_0^s, \tilde{c}_0^s, \hat{p}w, \hat{p}, n_1^s\}$. Although the simulation data contains all information occurred in the local hotel market, I only use select data that can be observed in the actual data. Now that I have a simulation
dataset which has the same shape of the actual data, the next step is to apply a two-step semi-parametric estimation to find the true parameter $\theta_0$.

Note: $E = 34$

Figure 1.5: Parameters estimates ($\beta_s$) with suboptimal price policy

I apply the two-step methods for total $E = 34$ repeatedly, which resulted in different estimated parameter values each time. Figure 1.5 shows the kernel density of estimated parameters for price elasticity. Through the two-step semi-parametric estimation, I find that the true parameters are successfully estimated as seen in Figure 1.5. If the two-step semi-parametric method succeeds in uncovering the true parameters of demand, which are derived from the suboptimal price rule, it shows that this method provides a consistent estimator regardless of the optimality condition of policy and censoring problems. This indicates feasibility that combining the optimal dynamic pricing model with a two-step semi-parametric estimation would have practical value in the real world.

1.5.4 The Validity of the Optimizing Assumption: Hausman Test

In addition to evaluating the demand estimation effect on the optimal price rule, I test the consistency of the two estimation approaches for the validity of the optimality
assumption. This test helps decide which estimators should be used for the demand estimation. Comparing the estimate result of Yu [35] with the result of two-step approach shows in Appendix A, this test will proceed. When the conditional independence assumption suggested in Cho et al. [9] holds, the two-step semi-parametric estimator is consistent. However, it is less efficient under the null hypothesis that the existing hotel pricing is optimal. This is where the Hausman test provides a rigorous test to compare the two estimators, demand estimator with optimality and without optimality.

Hausman [19] suggests the Hausman specification test be used to evaluate the consistency of an efficient estimator compared to an alternative estimator which is known to be consistent, but less efficient. The estimated parameters under the optimality assumption is denoted by $\hat{\theta}_1$ while the estimated parameters with the two-step semi-parametric estimator is denoted by $\hat{\theta}_2$. Since $\hat{\theta}_1$ is an efficient estimator of the true parameter $\theta$, the Hausman test check whether $\hat{\theta}_1$ is consistent. The test is straightforward when we have variance-covariance matrix for each estimator, $\hat{\theta}_1$ and $\hat{\theta}_2$. Wu-Hausman statistic is defined as

$$H = (\hat{\theta}_1 - \hat{\theta}_2)\hat{\Omega}_2^{-1}(\hat{\theta}_2)\hat{\Omega}_1^{-1}(\hat{\theta}_1)\hat{\theta}_1 - \hat{\theta}_2)$$  \hspace{1cm} (1.14) $$

where $\hat{\Omega}_1$ denotes the Moore-Penrose pseudoinverse, $\hat{\Omega}_1(\hat{\theta}_1)$ is a variance-covariance matrix for $\hat{\theta}_1$, and $\hat{\Omega}_2(\hat{\theta}_2)$ is one for $\hat{\theta}_2$. Under the null hypothesis, $\hat{\theta}_1$ and $\hat{\theta}_2$ are both consistent and Wu-Hausman statistic $H$ asymptotically follows the chi-square distribution with the number of degrees of freedom as the rank of matrix $\hat{\Omega}_2(\hat{\theta}_2) - \hat{\Omega}_1(\hat{\theta}_1)$.

If the test rejects the null hypothesis, $\hat{\theta}_2$ remains consistent while $\hat{\theta}_1$ is inconsistent. According to Table 1.5, I find the Wu-Hausman statistics by subsamples and all of the weekday subsamples reject the null hypothesis. For weekends, only the calm season
Table 1.5: Wu-Hausman statistic

<table>
<thead>
<tr>
<th></th>
<th>Calm (0-25%)</th>
<th>Normal (25-50%)</th>
<th>Busy (50-75%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekday</td>
<td>2.49E+3</td>
<td>2.51E+3</td>
<td>968.26</td>
</tr>
<tr>
<td>Weekend</td>
<td>149.78</td>
<td>56.33</td>
<td>33.37</td>
</tr>
</tbody>
</table>

Note: \( M = 46 \) parameters are estimated, so the degree of freedom \((M - 1)\) is 45. \( \chi^2_{0.90}(45) = 57.51 \), \( \chi^2_{0.95}(45) = 61.66 \), and \( \chi^2_{0.99}(45) = 69.96 \).

demand estimation rejects the null hypothesis, while the results of other two seasons are accepted. However, the computed Wu-Hausman statistics for the busiest season are actually negative when they should be positive since \( \hat{\theta}_1 \) is assumed to be an efficient estimator.\(^9\) With an efficiency assumption of \( \hat{\theta}_1 \), matrix \( \hat{\Omega}_2(\hat{\theta}_2) - \hat{\Omega}_1(\hat{\theta}_1) \) is positive definite and its inverse matrix is as well. The failure of finding a positive definite matrix in the Wu-Hausman test might be caused by a finite-sample problem or incorrect assumption used in the estimation process.

When comparing the critical value of the chi-square shown in the note of Table 5, the statistics for weekday season is quite large. This can be interpreted that the two-step semi-parametric estimation is reasonable for the weekday sample since it seems that hotel 0 does not set its price optimally. However, it is may also indicate that the underlying assumptions I use for this test are either incorrect or due to a numerical error that might have occurred while computing the variance-covariance matrix. It is challenging to get an accurate gradient vector when using a variance-covariance matrix computation, and I am working on it to figure out the result of the Hausman test.

\(^9\)The Wu-Hausman statistics for busiest season have a negative value. The statistics of busiest weekday is \(-358.57\) and one of busiest weekend is \(-22.74\). It may be caused by numerical computation error or the hypothesis of consistency of \( \hat{\theta}_2 \) does not hold in the busiest season.
The current Wu-Hausman statistics for weekdays show that the demand estimation with the assumption of optimality is no longer consistent. When conditional independence holds and the semi-parametric price estimation is consistent, this statement is valid. This statement is valid when the demand estimation with a two-step semi-parametric is consistent under the assumption that conditional independence holds and semi-parametric price estimation in the first step of the two-step approach is also consistent. To verify those conditions, more studies are necessary, and at this stage, I cannot confidently evaluate the results of the Hausman test. The same can be said for the test outcome for weekends.

1.6 Conclusion

I have shown how it is possible to relax the assumption of optimality to estimate demand without any instrumental variables in presence of endogeneity. Using an empirical example with data from a luxury hotel market, both two approaches deal with the endogeneity and censoring problem, but use different identifying assumptions. One approach assumes that hotels set prices optimally and uses this assumption as a powerful identifying restriction in demand estimation while the other approach estimates demand without this optimality assumption. Instead, the second approach introduces a conditional independence assumption on how unobservable variables may affect hotel prices. This study shows how the two approaches result in difference in estimation results, implementing both approaches using the hotel data.

For the demand estimation without the assumption of optimality, I applied a two-step semi-parametric estimation to obtain consistent parameters of hotel demand. In this approach, a pricing rule of a hotel is discovered using a semi-parametric regression, which I use to simulate pricing data for estimation of demand parameters by
MSM. According to the estimation results, the price elasticity of a business customer is smaller than that of a leisure customer, which is to be expected. Lastly, the computation of dynamic programming for hotel pricing is implemented with demand parameters obtained from the two-step semi-parametric estimation. This new pricing rule results in an increase in hotel profit by 9% and raises the occupancy rate by 10% on average, compared to the optimal pricing rule when the assumption of optimality is imposed. In effect, the optimality assumption distorts the estimated demand parameters, so that the MSM estimator can fit observed prices as optimal prices.

While the first approach forces the optimality assumption of status quo policy rule to deal with identification, the two-step method enables the demand model to relax that assumption. The approach with optimality assumption may be more reasonable only when there is an existing price regarded as optimal like the RMS system. However, a two-step semi-parametric estimation is more useful in the sense that it is applicable when there is no optimal policy rule since the demand response against any pricing rule can be estimated.

For the validity of the two-step approach, I generate a simulated data with known parameter values. The two-step semi-parametric estimation successfully estimates consistent parameter values in the simulation. This means that the two-step method is a consistent estimator when model specification is correct. Furthermore, I test the validity of demand estimation with optimality by the Hausman test. Since two-step semi-parametric estimation is consistent with the simulation test, this test is applicable. Depending on the result of the test, I can check the consistency of the demand estimation with optimality and finally, decide which approach is better to use for demand estimation. However, the Hausman test result is difficult to interpret and demonstrates that more studies are necessary to decide between the two.
My outcomes depend on many simulations instead of field experiments, making it hard to verify how hotel revenue can be improved by the suggested optimal pricing rule. Since the parameters estimated by any demand estimator is difficult to identify even with a strong assumption, experiments in the real world are one of few ways to validate the pricing model. However, it is hard to implement the pricing rule suggested by the model since it deals with actual revenues and expenditures. But, in the future work, I hope I have opportunity to conduct actual field experiment with an actual hotel to test whether counter-factual pricing rule really do increase profits of the hotel as my calculation suggested.

Despite these concerns, there is huge potential to develop within this framework and apply in the real world. The demand estimation without the optimality assumption, i.e. two-step semi-parametric approach, can take an improvable machine learning algorithm and produce more accurate information parameters that can be used in the structural mechanism, thus bringing higher returns. I believe this could be a great stepping stone into using economics that can be combined with best practices and result in practical use.
Chapter 2

Semi-parametric Instrument-free Demand Estimation: Relaxing Optimality and Equilibrium Assumptions

2.1 Introduction

In most markets consumer demand results from a compound arrival/choice process: consumers arrive to a market over time stochastically and make independent discrete choices over which item to purchase, including the choice not to purchase (or to purchase an item outside of this market which economists refer to as the “choice of the outside good”). Market demand results from an aggregation of individual consumer choices, and is more appropriately modeled as a nonlinear price and state-dependent stochastic process rather than a linear demand curve that is traditionally studied in the literature on demand estimation. We consider the problem of identification of consumer preferences and arrivals when market prices are endogeneously determined but the implied stochastic process for demand is nonlinear in prices and there are no relevant instrumental variables to deal with price endogeneity. In addition, most data are censored: we typically do not observe the number of arriving customers or the subset choosing the outside good.

The motivation for our paper is an empirical analysis of hotel pricing in a specific luxury hotel market in a major US city, see Cho et al. [8]. Demand shocks in the face of the fixed room capacity for the 7 hotels in this local market lead to strong positive correlation between hotel prices and room occupancy: the hotels raise their prices to
ration their available capacity on days where the demand for rooms in this market is high, but lower their prices significantly to compete for market share on days when demand is low and occupancy rates are well below 100%. This leads to strong co-movement in prices and occupancy rates in the hotels in this market. OLS estimation of room occupancy on hotel prices results in positively sloped “demand curves” for hotel rooms due to the endogeneity in hotel pricing in response to fluctuating demand.

Price endogeneity also arises as a result of unobserved characteristics of the hotels in this market: although all seven hotels are classified as “luxury hotels” and we can control for their star rating, there are unobserved characteristics that make customers willing to pay more to stay in the top tier hotels in this local market, and their higher willingness to pay enables these hotels to charge more.

Though it has long been recognized that it is possible to deal with the latter form of endogeneity in certain types of nonlinear models using market share data only without the benefit of observing the discrete choices of individual customers (see e.g. Berry et al. [5]), these approaches depend on the ability to invert market shares to obtain transformed equations that are linear in prices, to which instrumental variable approaches can be applied. However it is unclear how to apply this approach in our hotel market example, where we typically observe only observe the occupancy of a single hotel in the market, but not occupancies at competing hotels. As a result, we do not observe market shares that the BLP estimator inverts to form the regression equations to which instrumental variables can be applied.

Even in situations where we can observe the joint occupancies of all of the hotels, we still almost never observe the total population of who “arrived” and considered whether to book a room in this market. Thus, we do not observe the share of consumers who chose the “outside good” (i.e. to not stay in any of the hotels in this market). Without information on the outside good, we cannot construct the market
shares necessary to apply BLP. We argue that this is a typical situation in many if not most markets.

On top of this, there are many situations where there may not be any relevant instrumental variables. Our hotel data set is one such example. A good instrument is an observed variable that causes “exogenous shifts” in a hotel’s price but does not enter the hotel’s pricing strategy. It is hard to think of such variables in the hotel market example. A typically used instrumental variable, the so-called “Hausman instrument”, is the price of competing hotels. However the validity of instrumental variables depends on an “exclusion restriction” that the prices of competing hotels are not a determinant of any given hotel’s pricing decision in this market. For the hotel we study, the price of competing hotels is probably the most important determinant of its own pricing decisions, and thus can hardly be plausibly assumed to be an excluded variable from the hotel’s pricing equation (which is a reduced-form approximation to its pricing decision).

These problems motivate a search for new approaches to demand estimation that can also handle econometric problems such as censoring. A recent paper by MacKay and Miller [24] introduces a novel “instrument-free” approach to demand estimation: “Our main result is that price endogeneity can be resolved by interpreting an OLS estimate through the lens of a theoretical model. With a covariance restriction, the demand system is point identified, and weaker assumptions generate bounds on the structural parameters. Thus, causal demand parameters can be recovered without the availability of exogenous price variation.” (p. 32). However their approach depends on the assumption that demand or market share data, after a suitable transformation, is a linear (or semi-linear) function of price and it requires a prior restriction on the covariance between cost shocks and an additively separable unobserved demand shock or unobserved product characteristic. In our hotel example, there is no transformation
of occupancy rates or market shares that results in the linear equations needed to implement the MacKay and Miller \cite{24} estimator. Further, since the supply of rooms is inelastic in the short run, we are not aware of covariance restrictions on cost and demand shocks that could help identify the stochastic process for demand for hotel rooms and enable us to predict how consumers react to hotel prices.

Structural models of demand and firm price setting provide an attractive alternative to traditional linear instrumental variables approach to demand estimation because they enable us to more directly model the dynamics of demand in real world markets, such as the hotel market we study in this paper. Identification of arrival probabilities and heterogeneous consumer preferences can be obtained without the use of instrumental variables or the use of covariance restrictions as in MacKay and Miller \cite{24}. However the structural approach is heavily dependent on three key assumptions: 1) parametric functional forms for consumer preferences and the stochastic process governing arrival probabilities, 2) firms set prices optimally, and 3) prices are in equilibrium, i.e. their prices mutually satisfy conditions for a Nash (or Markov Perfect equilibrium or some related solution concept in dynamic models).

Together these three assumptions are often powerful enough to secure identification of the unknown parameters of consumer preferences and the stochastic process for arrival of customers to the market. The structural approach requires modeling the entire market and incorporating observed and unobserved variables that capture the demand shocks and unobserved product characteristics that result in the endogeneity in the prices we observe. By explicitly modeling the endogenous determination of prices under the assumptions of optimality and equilibrium, dynamic structural models are able to bring to bear “cross equation” identifying restrictions that imply that equilibrium prices are an implicit function of firms’ beliefs about the stochastic process of customer arrivals and preferences, as well as each others’ price-
setting and strategic behavior. Demand is “downward sloping” in a well formulated structural model, since upward sloping demand would result in price dynamics that are at odds with what we actually observe.

However a big drawback of structural models is the computational demands of solving and simulating a dynamic Markov Perfect Equilibrium for an entire market. This is a daunting task even for a relatively small local hotel market consisting of 7 luxury hotels that we analyze in this paper. Fortunately, recent work has shown that it is often possible to identify demand in the presence of endogeneity in a framework that relaxes the equilibrium assumption so long as the optimality assumption is still imposed. The idea is that the behavior of competing firms or agents can be flexibly modeled using semi-parametric estimators by treating the pricing strategies of competing firms as infinite-dimensional “nuisance parameters.”

For example, Merlo et al. [27] studied optimal dynamic strategies of home sellers in the London housing market. Endogeneity arises in this market due to the presence of unobservable characteristics of houses that make some more attractive to most buyers. Homes that are superior in unobservable dimensions (even after controlling for a large set of observed hedonic neighborhood and home characteristics) experience a high rate of arrival of offers and sell for more. Thus observed housing demand appears to be “upward sloping” in the list price of a home if we fail to control for price endogeneity. Yet there were few instrumental variables the authors could find to do this. The alternative of estimating a dynamic structural model that imposes the assumption of a full dynamic equilibrium in the London housing with thousands of competing buyers and sellers is computationally infeasible.¹ Merlo et al. [27] were able to flexibly

¹If there is twos-sided incomplete information, then even the bargaining “subgame” between a seller and an buyer can have have a huge multiplicity of equilibria. To our knowledge nobody has succeeded in solving the overall twos-sided search, matching and bargaining game that would be the most realistic way to model real housing markets.
model the arrival of buyers and the dynamic bargaining process they employed. These can be regarded as the infinite-dimensional nuisance parameters in their estimation problem. However by assuming home sellers follow an optimal dynamic pricing and bargaining strategy in response to these beliefs, the authors were able to structurally estimate their model and obtain a sensible “downward sloping” demand for housing. The hypothesis of optimality restricts demand to be downward sloping in price, since if it were upward sloping, then it would be optimal for sellers to set far higher list prices than we actually observe.

We wish to take this approach one step further, to see if it is possible to identify demand by relaxing the hypothesis of optimality as well as equilibrium in the hotel market. Though the assumption of optimality is a powerful identifying assumption, it is also a potentially dubious one that could distort our estimates of demand if firms do not behave optimally. Herbert Simon introduced the concept of bounded rationality as a key reason why organizations and firms fail to optimize in complex environments.\textsuperscript{2} Cho et al. [8] discuss a multi-billion revenue management industry that is experiencing very rapid growth by helping hotels, airlines, and other firms in the hospitality industry set better prices. If all of the hotels, airlines and other firms were already optimizing (the typical default assumption in most economic models), there would be little need and value-added for the revenue management industry. Yet, Phillips [28] notes that despite the fact that pricing decisions “are usually critical determinants of profitability” “pricing decisions are often badly managed (or even unmanaged).” (p. 38). If this is true, it calls into question the standard structural assumptions that individual firms price optimally, and even more so the stronger

\textsuperscript{2}See Rust [29] for further discussion and evidence in support of Simon’s view that many firms satisfy rather than optimize.
assumption that firm prices collectively are determined as the outcome of a Bayesian-Nash equilibrium.

In this paper we adopt a structural semi-parametric approach that explicitly models and attempts to identify the probability distribution governing the arrival of potential customers that constitutes the fundamental “demand shock” that leads to the co-movement of prices and occupancy in this market. We also identify consumer preferences for the different hotels and their willingness to pay to stay in them. Via microaggregation of the individual discrete choices of arriving consumers (which we do not observe) we derive a mixed and censored multinomial distribution for the joint occupancies of the hotels in this market. The censoring arises because of the hotel capacity constraints: when a hotel is fully booked, some consumers who would have liked to stay there are turned away and will either book at a competing hotel or choose the outside good. We can identify both the probability distribution of arriving customers and the fraction of them choosing the outside good, even though we only observe joint occupancies of the hotels and not the total number of arriving customers, nor the number choosing the outside good.

We are able to do this without the imposition of equilibrium or optimality assumptions, or the use of instrumental variables. Our key identifying assumption is a conditional independence assumption on firm prices that we refer to as conditional exogeneity. In essence, we assume there is an observed demand shifter $x$ that hotels in this market observe and use in their price setting decisions that constitutes a sufficient statistic for their beliefs about “demand shocks” that result in the positive correlation between demand (arrivals) and the hotels’ prices. In our application, $x$ is the expected market occupancy rate i.e. expected market demand for rooms in this market divided by the total room capacity. Though we do not directly observe $x$, we show how we can proxy for this using predicted occupancy where we treat $x$ as a latent variable.
Firm prices are a function of this demand shifter $x$ and other variables $z$ which we do not observe. We interpret these other $z$ variables as “pricing shocks” that arise due to potential mistakes or other idiosyncratic factors on the part of the hotels in this market. The conditional independence assumption states that once we condition on $x$ and the hotels’ prices, the $z$ variables do not affect the distribution of hotel demand. In essence, once we condition on $x$ any residual variation in hotel prices can be considered to be “virtual random price experiments” that enables us to identify consumer prices and the crucial price parameters in their utility functions.\footnote{Our conditional independence assumption is similar to the “unconfoundedness” assumption in the treatment effects literature, where conditional on $x$ the distribution of potential outcomes of a binary treatment are independent of the treatment assignment. That is, conditional on $x$ the “treatment assignment” $T$ is akin to a “virtual random experiment” which enables analysts to identify the average treatment effect in situations where treatments are actually “endogenous” and not actually determined by randomized assignment. In our application we can regard the hotels’ prices $p$ as a “continuous treatment” and the “average treatment effect” we are attempting to identify is the effect of prices on demand for hotels in this market, i.e. the slope of the demand curve.} Following the literature, we will also refer to our assumption as conditional exogeneity: i.e. conditional on $x$ the realized prices $p$ the idiosyncratic random variables $z$ affecting prices are independent of other random factors affecting the realized demand for hotels.

We discuss the non-parametric identification of our model demand in the presence of endogeneity and censoring when we relax the assumptions of equilibrium and optimality. We treat the price setting strategies of firms as infinite-dimensional nuisance parameters and discuss semi-parametric estimation strategies including the method of sieves that are capable of consistently estimating these infinite-dimensional nuisance parameters while still estimating the parameters defining consumer preferences at the usual $\sqrt{N}$ rates (where $N$ is the sample size). There is a cost to relaxing any maintained assumption, and relaxing the optimality and equilibrium assumptions has the consequence that the estimated pricing strategies are no longer implicit.
functions of firms’ beliefs about consumer preferences and arrival rates. That is, the semi-parametric estimators we propose no longer benefit from the “cross equation restrictions” that link consumer preferences and arrival rates to the pricing strategies of firms. Even though actual pricing strategies undoubtedly do depend on firms’ beliefs about their customer preferences and arrival rates, the use of semi-parametric methods to estimate our econometric model comes at the cost of a loss of information from failing to impose the optimality and equilibrium restrictions. At a minimum this loss of information is reflected in larger asymptotic standard errors for the parameters, but in the worst case the demand model parameters may not be identifiable, and thus cannot be consistently estimated.

On the other hand, structural estimators that impose optimality and equilibrium restrictions will generally result in inconsistent demand estimates if actual firm behavior violates these assumptions. Thus, it is desirable to develop estimators that can identify demand without imposing these strong assumptions. We consider identification of the model under two scenarios: 1) we observe only the occupancy at a single hotel, but not its competitors; 2) we observe the occupancy of all hotels in the market. We show that it is not possible to identify the demand for competing hotels and the outside good when we only have occupancy data for a single hotel in the market: this model is only partially identified. Yet data on a single hotel is sufficient to identify its own demand function when we also have data on the prices charged by its competitor, and this is sufficient to conduct a number of interesting counterfactuals such as calculating the optimal pricing strategy for this hotel.

However in order to test more advanced hypotheses, such as whether the overall pricing by the firms in the market is best described as a Bayesian-Nash equilibrium, or to conduct certain counterfactuals, such as predicting the effect of hotel mergers or collusion by the hotels in the market, it is crucial to identify the joint demand function
for all of the hotels as well as the demand for the outside good. If this is possible, it opens up a potentially powerful new avenue for econometrics to be useful for policy making, by enabling us to test hypotheses about firm behavior without imposing them a priori. For example, Harrington [18] and Ezrachi and Stucke [13] raise the specter of “algorithmic collusion” by sophisticated revenue management systems (RMS). The nature of “deep learning” algorithms from the artificial intelligence and reinforcement learning literatures makes it difficult to actually inspect the computer code used by commercial RMS to determine if it been explicitly designed to collude, or whether the algorithms “learn to collude” through repeated interaction. As we noted earlier, there is strong co-movement of prices and occupancy rates in the hotel market we analyzed, and some analyses might interpret such co-movement as a telltale sign of algorithmic collusion. Further, a structural model that imposed the hypothesis of collusion may result in distorted estimates of demand to help “rationalize” the maintained assumption of collusion. If it is possible to estimate demand without imposing strong assumptions about the type of equilibrium in this market, we can use the estimated demand model to solve for equilibrium under different equilibrium concepts, such as collusive pricing or Bertrand (competitive, non-collusive) pricing, and compare the predicted behavior to non-parametric estimates of the pricing strategies firms are actually using in this market. This may allow us to reject the hypothesis of algorithmic collusion in favor of a model of competitive, Bertrand pricing where the price and occupancy co-movements are a natural response of a competitive market with inelastic supply of rooms that is subject to variable demand shocks.

Revenue management systems are proprietary so we do not know what sort of optimization principles they use and what types of data and econometric methods they employ. McAfee and te Veld [25] note that “At this point, the mechanism determining airline prices is mysterious and merits continuing investigation because airlines
engage in the most computationally intensive pricing of any industry." (p. 437). For reasons that are unclear, the RMS industry seems to have largely ignored econometric modeling and the substantial literature on demand estimation in the industrial organization literature. Phillips [28] notes that “The tools that pricers use day to day are far more likely to be drawn from the fields of statistics or operations research than from economics.” (p. 68) and he credits marketing (which he regards as a subfield of operations research and management science) from bring “some science to what was previously viewed as a ‘black art’” (p. 70). Yet “there remains a gap between marketing science models and their use in practice. The reasons for this gap are numerous. Many marketing models have been build on unrealistically stylized views of consumer behavior. Other models have been build to ‘determine if what we see in practice can happen in theory.’ Other models seem limited by unrealistically simplistic assumptions.” (p. 70).

Our study can be viewed as an attempt to show that econometric literature on demand estimation may have value to the RMS industry and may shed light on the optimality of the price recommendations from these systems. Our empirical analysis of hotel pricing demonstrates that it is possible to identify a realistic stochastic model of hotel demand that relaxes equilibrium and optimality assumptions, and therefore provides a way to test rather than assume them. We focus on a single hotel that uses the IdeaS™ RMS, a subsidiary of the SAS statistical software company. This hotel, which we refer to as “hotel 0” due to a non-disclosure agreement that prevents us from revealing its identity, follows the price recommendations of the IdeaS RMS approximately 60% of the time. On other occasions the revenue manager at hotel 0 deviates and chooses her own price. Though we are unable to observe which prices are the IdeaS recommended prices and which are set by the human revenue manager, we do know the information hotel 0 uses to set its prices, including the
information in its own reservation database and real time information on the prices of its competitors from the Market Vision\textsuperscript{TM} pricing service. Hotel 0 cannot access the reservation databases of its competitors, and it generally does not know their occupancy rates, either \textit{ex ante} (i.e. the number of rooms booked so far) or \textit{ex post} (the actual or realized occupancy on a day by day basis). Though we show that it is only possible to partially identify the demand model parameters when we do not observe occupancy of competing hotels, our ability to identify overall demand, the distribution of arrivals, and the fraction of consumers choosing the outside good was greatly assisted by auxiliary data we obtained from the company STR Global which collects price and occupancy data on over 63,000 hotels worldwide. The augmented data set enables us to identify consumer preferences and the distribution of arrivals and thus to fully construct the probability distribution of demand in this market.

Using the estimated demand model parameters, we calculate counterfactual optimal and equilibrium dynamic hotel pricing strategies. In essence, our econometric demand model and optimization algorithm constitute our own “RMS” and prediction of optimal recommended prices. We find that the optimal prices from our model deviate significantly from the prices that hotel 0 actually charged. As a result, we are also able to strongly reject the hypothesis that pricing of the hotels in this market is consistent with Bertrand-Nash equilibrium. Given our relatively inelastic demand estimates and relatively low substitution to the outside good, we predict that all hotels in this market should be pricing significantly higher. Overall, we conclude that far from engaging in “algorithmic collusion” we can reject the hypothesis that hotel 0 is even using an optimal pricing strategy (i.e. a best response to its competitors). Thus we can also reject the hypothesis that the firms in this market are setting prices in accordance with a dynamic Bertrand-Nash equilibrium. This conclusion is broadly consistent with Herbert Simon’s work on satisficing behavior by firms. Since...
we do not observe which prices charged by hotel 0 are those recommended by the IdeaS RMS and which were chosen by its revenue manager, we cannot determine whether the source of hotel 0’s suboptimality is due to its RMS or decisions by its human revenue manager.

There have been a number of claims that commercial RMS (known as “yield management systems” in the airline industry) lead to significant improvements in profitability. Gallego and van Ryzin [16] claim that “The benefits of yield management are often staggering; American Airlines reports a five-percent increase in revenue worth approximately $1.4 billion dollars over a three-year period, attributable to effective yield management.” (p. 1000). However, we are unaware of studies that provide scientific validation (say via controlled experiments or other means) of the claims that commercial RMS have resulted in significant increases in hotel revenues and profits. Our study can be regarded as providing one of the first independent evaluations of the pricing strategy of a particular hotel, though further testing including field experiments would be required to confirm that the gains we calculated from stochastic simulations of our econometric model could be realized in practice.

The general approach developed in this paper, i.e. of using a semi-parametric estimator to estimate demand parameters while relaxing the assumption of optimality, has been used previously. Hall and Rust [17] studied the pricing and inventory investment decisions by a firm that trades (“speculates”) in the steel market. In their application, they had to confront the econometric problem of “endogenous sampling” due to censoring in the wholesale prices of steel that the firm buys steel at. That is, the firm only records the price of steel on days it purchases steel. They compared a dynamic structural model of optimal steel price speculation with an “unrestricted” model that relaxes the assumption of optimality. Using the Method of Simulated Moments (MSM, McFadden [26]), they showed that it is possible to consistently
estimate the parameters of the wholesale price process by censoring simulated data for the firm in the same way that actual data are censored. Using a Hausman test, they were able to test and reject the assumption that the firm’s steel purchases were governed by an optimal \((S, s)\) inventory investment strategy. Using stochastic simulations, they also showed that if the firm had adopted an optimal \((S, s)\) strategy it would have earned significantly higher trading profits over the sample period.

Section 2 summarizes the key features of our data set on hotel pricing, providing a concrete illustration of the price endogeneity and censoring problems we face. Section 3 discusses the identification of demand in a simplified static setting where the key ideas underlying our approach to identification can be explained more clearly. Section 4 applies our model to the hotel market and provides estimates of demand both for the mixed censored multinomial model introduced in section 3 but also for a linear demand model that does not attempt to identity the probability distribution of arrivals, the fraction of consumers choosing the outside good, or derive demand from a microaggregation of individual discrete choices. While we show that in-sample, the estimated expected demand functions from this simpler linear model and our structural mixed censored multinomial demand model are quite similar to each other, the out-of-sample predictions of the model, particularly under collusive scenarios are quite different. We believe this is due to the inability of the linear model to adequately capture substitution to the outside good as firms collectively raise their prices. Section 5 provides some conclusions and suggested directions for future research.

### 2.2 Hotel Data

This section describes the hotel market that motivates the questions about demand estimation and identification that we attempt to answer in this paper, and in par-

57
ticular the simple static model of hotel demand that we introduce in section 3. As we noted in the introduction, due to a non-disclosure agreement with the hotel that provided the data for our study, we are unable to provide too much detail about the local market in which hotel 0 operates to guarantee the anonymity of the hotel and the owner. We can say that it is a luxury hotel located in a highly desirable downtown location of a major US city.

Hotel 0 is one of seven luxury hotels operating in a well defined local market that is recognized by online travel agencies (OTAs) and other travel agents. Though customers can book at other luxury hotels in other parts of this city, the locations of these other luxury hotels are sufficiently far from this particular desirable area that they are not regarded as relevant substitutes for customers who wish to stay in this specific area of the city. We consider any choice of another hotel outside the seven hotels in this local hotel market, including the decision not to stay in any hotel, as a choice of the outside good.\(^4\)

Table 2.1 lists some summary information about the seven hotels: all are 4-star or higher rated luxury hotels. To avoid identifying the hotels we show only the relative capacity, where we normalize the capacity of the largest hotel to 1. However our model uses all relevant information including the actual capacity, which we will show is an important factor in hotel pricing. The customers of the hotel are both business/government customers who mainly stay in the hotel on weekdays and tourists who typically stay on weekends. Since business customers and government customers are reimbursed for their travel expenses, we can expect them to be more price inelastic than tourists. On the other hand, many government agencies and large corporations

\(^4\)There is also limited capacity of private residences in this area, so alternatives such as AirBnB is a minor factor in this market, and so we also lump this option into the catch-all category, “outside good.”
that do frequent business in this city have negotiated government and corporate discounted rates with this hotel. These discounted rates are typically a fixed percentage, often 15 to 20%, off the currently quoted price that is called the best available rate (BAR). The revenue manager of hotel 0 is in charge of updating an array of BARs for different room classes and different future arrival dates and posting these prices to the web via the Global Distribution System (GDS, a network of computer connections that give travel agents access to a hotel’s reservation database to check availability and reserve rooms) and via its own website.

Table 2.1: Hotels in the local market in our study

<table>
<thead>
<tr>
<th>Property</th>
<th>Avg. BAR</th>
<th>Star</th>
<th>Class</th>
<th>Chained Brand</th>
<th>Rate</th>
<th>Relative Capacity</th>
<th>Distance to mass transit</th>
<th>Cancel Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>hotel 0</td>
<td>$293.26</td>
<td>4</td>
<td>Luxury</td>
<td>No</td>
<td>4.4</td>
<td>79%</td>
<td>3 min</td>
<td>1 day before</td>
</tr>
<tr>
<td>hotel 1</td>
<td>$282.64</td>
<td>4.5</td>
<td>Upper Up</td>
<td>No</td>
<td>4.4</td>
<td>81%</td>
<td>5 min</td>
<td>3 day before</td>
</tr>
<tr>
<td>hotel 2</td>
<td>$285.16</td>
<td>4</td>
<td>Upper Up</td>
<td>No</td>
<td>4.4</td>
<td>63%</td>
<td>3 min</td>
<td>1 day before</td>
</tr>
<tr>
<td>hotel 3</td>
<td>$338.29</td>
<td>4</td>
<td>Upper Up</td>
<td>Yes</td>
<td>4.2</td>
<td>99%</td>
<td>8 min</td>
<td>2 day before</td>
</tr>
<tr>
<td>hotel 4</td>
<td>$397.09</td>
<td>4</td>
<td>Luxury</td>
<td>No</td>
<td>4.6</td>
<td>100%</td>
<td>10 min</td>
<td>Strict</td>
</tr>
<tr>
<td>hotel 5</td>
<td>$253.51</td>
<td>4</td>
<td>Upper Up</td>
<td>No</td>
<td>4.2</td>
<td>47%</td>
<td>8 min</td>
<td>3 day before</td>
</tr>
<tr>
<td>hotel 6</td>
<td>$454.30</td>
<td>5</td>
<td>Luxury</td>
<td>Yes</td>
<td>4.7</td>
<td>52%</td>
<td>10 min</td>
<td>1 day before</td>
</tr>
</tbody>
</table>

Customers generally book hotel rooms in advance, and generally only a small fraction of customers (approximately 8%) book their rooms on the same day that they intend to occupy them. Hotel 0’s reservation database provides information on when each hotel room was booked and through which channel. A hotel room can be booked over the phone, via hotel 0’s website, via a traditional travel agency, or via an online travel agency (OTA) such as Priceline or Expedia. The decentralized nature by which consumers search for hotels and book rooms implies that there is no single site or source that observes all consumers who “arrive” to book a room in a particular market during a particular point in time. A hotel will know how many consumers have booked one of its own rooms at every possible future arrival date, but there is
no entity that observes all consumers searching for rooms or the number of bookings made in all of the competing hotels in a given market for arrival at any given future date. So this is the sense in which *arrivals are unobserved*.

Similarly, no single entity will know how many of the customers who arrive to book a room in a market will choose the outside good, i.e. to either choose to stay at a hotel outside this market or other form of accommodation such as AirBnB. Thus, from hotel 0’s perspective, suppose that it received 10 bookings on a particular day. Hotel 0 cannot distinguish between situations a) and b):

a) 100 customers arrived and 10 of these customers chose hotel 0, 50 of them chose one of its competitors, and the remaining 40 chose the outside good

b) 70 customers arrived, 10 booked at hotel 0, 50 booked at one of its competitors and only 10 chose the outside good.

In both cases a) and b) hotel 0 observes only the 10 customers who booked one of its rooms, but it has no information on the number of customers arriving in total, the number choosing the outside good, or even the number of customers who book a room at one of its competitors. In view of this, it is very difficult for a hotel to determine its market share on any particular day. Hotels do, of course, observe each others capacities and they can obtain (at a cost) historical data on occupancies of their competitors from firms such as STR, but hotel 0 does not have real time information on the bookings and occupancies of its competitors.

Hotel 0’s revenue manager uses a uniform price strategy and does not sell blocks of rooms to wholesalers under contracts that give wholesalers discretion to set their own prices for the blocks of rooms they purchase. Thus, there is no ability to “arbitrage” prices of rooms for hotel 0 by searching different OTAs. However hotel 0 does pay a significant commission, ranging from 15 to 25%, for reservations that are made via
OTAs such as Expedia. The GDS that hotel 0 uses allows the revenue manager to change prices as frequently as she desires, though there is a short lag before the prices are propagated everywhere on the Internet including the leading OTAs. However for hotel 0’s own website and reservation system, price changes take place instantaneously, and hotel 0 has its own loyalty program that provides discounts to customers who are members of the program. There are other groups that include weddings that involve a larger group of guests that are typically individually negotiated with the hotel revenue manager, but the discounts to these groups are typically quoted as a percentage discount off the BAR similar to corporate and government contract rates.

As we noted above, hotel 0 subscribes to the IdeaS RMS that provides recommended prices. The hotel revenue manager uses her own discretion to select a relatively small number of different possible BARs (effectively, she discretizes the pricing space) which are treated as a predefined choice set that is entered into the RMS. Based on a proprietary algorithm that considers remaining availability, seasonal effects, cancellation rates and competitors’ prices, the RMS communicates a recommended BAR to the revenue manager at the start of each business day. Even though the revenue manager has some control over the prices the RMS can recommend via her choice of a predefined finite set of possible BARs, she often ignores the recommended price from the RMS and instead sets her own BARs based on her own experience, judgement and intuition. Unfortunately, our data do not specify which prices were ones recommended by IdeaS and which are ones she set herself, but she told us that she estimated she used the recommended prices approximately 60% of the time.

Thus, we do not know to what extent the IdeaS RMS is able to observe and adapt to the knowledge that the revenue manager is disregarding their recommended prices. This would seem to be important information that any RMS would want to collect, including the revenue manager’s feedback about the overall quality of the
recommended prices from the system. We can imagine that manual “price overrides” are common for newly launched hotels where the RMS may initially not have enough data to form good predictions about demand, or when there are unexpected changes to demand or entry/exit of other hotels in the local market. In these cases we might expect that the recommended prices from the RMS would be less trustworthy until sufficient data are accumulated to enable the RMS to provide an updated model of customer demand that provides accurate predictions for the local market in question.

Hotel 0 provided us information from its reservation database that enabled us to track all bookings, cancellations, and prices for a 37 month period between September 2010 and October 2013. In addition, we were provided aggregate daily reports and their competitive daily rates of hotel 0’s six competitors from a service called Market Vision provides quotes from hotel 0’s six competitors for several room rate categories several times per day. While Market Vision provides excellent data on prices, as we noted above, it provides no information on the number of bookings or occupancies at hotel 0’s competitors. This information does not seem to be readily available, but we were able to obtain data on the occupancy of hotel 0’s competitors on a daily basis thanks to data provided by STR. Table 2.2 summarizes the data sources we used for our study.

Table 2.2: Data sources used in this study

<table>
<thead>
<tr>
<th>Data</th>
<th>The first day of occupancy</th>
<th>The last day of occupancy</th>
<th>Observations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>market vision</td>
<td>2010-09-21</td>
<td>2014-08-13</td>
<td>609,181</td>
<td>competitors’ price</td>
</tr>
<tr>
<td>reservation raw</td>
<td>2009-09-01</td>
<td>2013-10-31</td>
<td>201,176</td>
<td>reservations detail information</td>
</tr>
<tr>
<td>cancellation raw</td>
<td>2009-09-01</td>
<td>2013-10-31</td>
<td>29,241</td>
<td>cancel detail information</td>
</tr>
<tr>
<td>daily pick-up report</td>
<td>2010-09-16</td>
<td>2014-05-21</td>
<td>475,187</td>
<td>daily revenue report</td>
</tr>
<tr>
<td>STR market data</td>
<td>2010-01-01</td>
<td>2014-12-31</td>
<td>1,731</td>
<td>competitors’ occupancy</td>
</tr>
</tbody>
</table>

| Data range      | 2010-10-01                 | 2013-10-31                 | 37 months    |
Our data are unique in the level of detail we have on reservations and cancellations. Our reservation database contains the full history of each individual booking, including the channel through which the booking was made. Each booking is identified with a unique reservation identification number that is created when the reservation is initiated and becomes the permanent identifier for each reservation along with time stamps and dates of arrival and departure and amounts actually paid including incidental charges.

Hotel 0 has two basic categories of rooms: regular rooms and higher tier rooms such as luxury suites, but 95% of the rooms in the hotel are regular rooms. Thus we focus on the demand and prices of regular rooms. We rarely observe overbooking of the regular rooms, though on the few occasions where this happens the overflow customers are automatically upgraded to a room in one of the the higher tiers.

There are around 200 rate codes which can be broken into 14 categories. To simplify our analysis, we divided the codes into two; transient and group bookings. Transients are individual travelers who pay the BAR or discounted BAR. Although the net of commission price that hotel 0 receives differs depending on which channel was used to do the booking (i.e. an OTA versus hotel 0’s own website), transient customers themselves pay the same price regardless of channel, namely the BAR in effect at the time they booked. Group bookings are also generally based on the BAR in effect when they booked, however it will vary by pre-negotiated contract discount rate that differs from different groups (rate codes).

Another commonly analyzed price in the hotel industry is the average daily rate ADR. This is simply the ratio of the total gross revenue collected each day divided by the total number of rooms booked (including no-shows who are generally charged for their rooms). Although the data we have on hotel 0 provides an incredible level of detail, as we show in the next section, our model requires more data about the reser-
vation/cancellation quantity dynamics of hotel 0’s competitors that are not provided in the Market Vision data, which provide only competitors’ prices. As we shall see, information on the total number of consumers who “arrive” and book rooms at one of the seven hotels in this local market is critical for our inferences about customer demand, and especially how customers respond to daily fluctuations in the relative BARs of the seven competing hotels. Unfortunately we do not have access to the reservation databases of hotel 0’s competitors, so we are unable to observe the total number of new reservations that are made in at all the hotels and at which prices (including group, corporate discounts, etc) besides hotel 0.

However as we show in the next section, it is possible to make inferences on the booking and reservation/cancellation dynamics of hotel 0’s competitors given their prices if we can at least observe the total final occupancy rates of its competitors. Fortunately we were able to obtain this information from STR via an academic research contract it has with Georgetown University. In addition to total occupancy at each competing hotel on a daily basis, the STR data provide information on the competitors’ ADRs and total revenue. The STR data turn out to be crucial for our ability to estimate a credible demand model. Unfortunately, the STR data do not identify the individual occupancy and ADRs of hotel 0’s competitors: it only provides the aggregate occupancy at the 6 competing hotels on a daily basis. Therefore, we will simplify our analysis by treating these 6 competing hotels as a single competitor, which we will refer to as “hotel c.”

2.2.1 Data Summary

As we noted in the introduction, there is strong co-movement in prices and occupancy rates in this market. Figure 2.1 illustrates the cyclical pattern of occupancy and prices, both over a given week and over the year, reflecting seasonal variations in
the demand for hotels. The bars in the left hand panel of figure 2.1 show a typical
weekly cycle of occupancy for hotel 0 where the lowest occupancy is on Sunday, but
a peak occupancy on Saturday, and a midweek peak occupancy on Tuesdays and
Wednesdays. The ADR peaks on Tuesday, and the higher rates during the weekdays
reflects price discrimination for less price elastic business guests, whereas the lower
rates on Fridays and Saturdays are designed to attract more price elastic tourists.
Occupancy is lowest on Sundays when tourists are checking out to return home for
work on Monday, whereas a typical business guest checks in during the middle of the
week and departs before the weekend. The right hand panel of figure 2.1 shows the
price and occupancy dynamics over the year. Occupancy rates are the highest in the
spring and early fall, and are lowest around holidays such as Thanksgiving, Christmas
and New Year’s. The black line in the figure plots hotel 0’s ADR and total revenues,
and we see that both of these move in sync with the ups and downs in occupancy
rates. This suggests that prices and revenues at hotel 0 are highly “demand driven”.

![Figure 2.1: Booking and price dynamics over the week and year](image)

Figure 2.2 compares the price dynamics for hotel 0 to those of its six competitors
over the year. It plots the weekly average BAR from October 2010 to October 2013 for
same-day reservations using the Market vision data, though we would obtain similar
results if we plot a time series of ADRs using the STR data. The bold line plots the average BAR of hotel 0 while the other lines are the BARs of its six competitors. We see strong co-movement in the prices of the seven hotels, and that they follow similar cyclical fluctuations, and hotel 0 generally underprices its competitors with the exception of hotel 5. There is strong seasonality in prices which are highest in the spring and the fall with peaks in early May and mid-September and October. Prices are lowest at the key holidays: Thanksgiving, Christmas, New Year’s, as well as early

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\(^5\)Since we have BARs for each of the 6 competing hotels, we prefer to plot these more detailed data: as we noted above, the STR data does not allow us to determine hotel-specific ADRs, only an average ADR for all of hotel 0’s 6 competitors.
July and August. During peak periods the average BAR of hotel 0 can be over $350 per night, whereas in the lowest periods it averages about $200.

The pattern of co-movement in the prices in this market might be described as “price following” and given the fact that most hotels use RMSs and have extensive knowledge of their competitors’ prices from services such as Market Vision, it could raise concerns about the possibility that RMSs enable these hotels to engage in algorithmic collusion. The price troughs following price peaks might be interpreted as “price wars” that are designed to punish hotels that deviate from the recommended prices that are highest when prices are peaking. However, as we will see, we do not think this is the correct conclusion to draw from these price patterns.

Figure 2.3 plots the time series of ADRs and occupancy rates for all seven hotels in this market for the first half of 2010 using the STR data. The top left panel plots the occupancy rate for hotel 0 versus the occupancy rate of its competitors, where the competitor occupancy rate is defined as the total occupancy at the six competing hotels divided by the total room capacity of those hotels. With few exceptions, we see that occupancy follows the same weekly cycle at all of the hotels that we illustrated in the left panel of figure 2.1 for hotel 0, as well as the seasonal fluctuations (i.e. higher in the spring but lower at end of June) that we observed in the right panel of figure 2.1. The top right panel of figure 2.3 shows that all seven hotels also have strong weekly cycles in their ADRs and the reasons are likely to be much the same as we conjecture for hotel 0: higher mid-week prices to discriminate against less price elastic business guests and lower weekend rates to try to attract the more price elastic tourists.

The lower two panels of Figure 2.3 plot the cycles in occupancy rates (red lines) and ADRs (blue lines) for hotel 0 (right hand panel) versus its competitors (left hand panel). The data suggests that the weekly price cycles are driven not only by different
compositions of guests (business versus tourists) but also to ration scarce capacity, since these hotels tend to be fully booked midweek but not on weekends. Both hotel 0 and its competitors follow similar weekly occupancy and price cycles, as well as similar seasonal price/occupancy cycles. For example we see that ADRs for both hotel 0 and its competitors peaked in mid April 2010, during a period where occupancy was close to 100% both mid-week and on the weekends.

It is natural to ask the question: which motive is more important for hotel 0? That is, does the revenue manager increase prices mainly to ration scarce capacity, or to try to exploit the more inelastic demand of business travelers who are more likely to be staying in the hotel midweek? Or, is hotel 0 simply following the prices of its competitors? If so, is this price following behavior a sign that all of the hotels
are following the recommended prices from their RMS, and could this be evidence of tacit collusion mediated by the RMS?

Table 2.3 provides some insight into this question by presenting the results of a simple OLS regression of the logarithm of hotel 0’s ADR on the average ADR of its six competitors and on its own and competitors’ occupancy rates. This simple model results in an $R^2$ of 86% when we also add dummies for different days of the week and months of the year to capture the weekly and seasonal price cycles.

Note that the occupancy also affects hotel 0’s pricing but in a counterintuitive fashion: hotel 0’s occupancy rate has a negative coefficient, but the occupancy rate of its competitors has a much larger positive coefficient. We may suspect that the co-movement in occupancy rates leads to a collinearity issue but hotel 0’s own occupancy has a negative coefficient even after we remove the occupancy of the competing hotels from the regression. The coefficient estimate for Hotel 0’s own occupancy rate only turns positive when we remove the ADR of the competing hotels, but then the fit of the model drops precipitously, to an $R^2$ of 0.17.

Table 2.3: Ordinary least squares regression with dependent variable ADR$_0$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>27.93</td>
<td>2.24</td>
</tr>
<tr>
<td>ADR$_c$</td>
<td>0.73</td>
<td>0.01</td>
</tr>
<tr>
<td>OCC$_0$</td>
<td>−0.09</td>
<td>0.027</td>
</tr>
<tr>
<td>OCC$_c$</td>
<td>0.273</td>
<td>0.044</td>
</tr>
</tbody>
</table>

$N = 1277, R^2 = 0.83$

The regression findings suggest that the effect of occupancy on hotel 0’s pricing decisions are second order relative to the dominant effect of the prices set by its competitors. To a first approximation, hotel 0 sets its prices at 70% of the average of its competitors’ prices. The $R^2$ drops to 0.69 when we remove ADR$_c$ from the
regression but retain occupancy variables and daily and seasonal dummies. Overall, the regression results suggest that the revenue manager is setting prices in accordance with a “price following” strategy, and that knowledge of her competitors’ prices is the most important piece of information besides the day of the week and the season of the year that she uses to set her own prices. The fact that hotel 0’s own occupancy appears to have only a second order effect on its price setting once we condition on the prices of competitors suggests that raising prices to ration scarce capacity is not an important motive for hotel 0.

On the other hand it is not clear whether the fact that hotel 0’s behavior is well approximated by “price following strategy” is evidence in favor of “algorithmic collusion” that Ezrachi and Stucke [13] and Harrington [18] discuss. Even if demand for rooms cycles in a systematic way during the week versus weekends, it is not clear that collusive prices would necessarily follow the same cyclical pattern that we observe in this market. In particular, we would expect that if the hotels in this market operated as a cartel, their prices would rise sufficiently high that there would be excess capacity even during the peak weekday periods, and the excess capacity would serve in part as a credible threat to engage in a price war that would deter any of the hotels that contemplated deviating from the collusive recommended prices, see Benoît and Krishna [3] and Davidson and Deneckere [11].

An alternative hypothesis is that this market is best approximated by a dynamic competitive equilibrium in a market characterized by strong Bertrand price competition subject to fixed capacity constraints. Stochastic shocks to demand lead to the price cycles we observe, with prices peaking to ration the available capacity in periods where demand exceeds available supply, but prices falling significantly as predicted by Bertrand price competition in periods of low demand where there is excess capacity. In this paper we will argue that the latter explanation is more likely to be closer to
the truth, especially given what we have already reported about hotel 0’s disinclination to follow the recommended prices of its RMS, combined with the fact that the revenue manager believes that the recommended prices are too low.

Regardless of the interpretation, the strong co-movement of hotel 0’s prices with the prices of its competitors creates real difficulties for demand estimation. We can see the problem in figure 2.4, which is a scatterplot of hotel 0’s own occupancy against its ADR over the period of our sample. This figure shows an upward sloping relationship between price and occupancy, which encapsulates the endogeneity problem we discussed in the introduction. We believe the endogeneity is emblematic of the classic Cowles Commission simultaneous equations type of endogeneity between prices and quantities. If the market is well approximated as a Bertrand equilibrium but subject to large stochastic demand shocks, then we would expect to see high prices that ration demand given the finite hotel capacity when demand is high but we observe low prices when the hotels compete for the available demand in periods where there is excess

Figure 2.4: Price scatterplots for hotel 0 and its competitors
supply of rooms. This will generate a positively sloped scatterplot of prices similar to what we observe in figure 2.4 and generally a positively sloped relationship between ADRs and occupancy for each hotel individually. Thus, simple OLS regressions will result in positively sloped demand curves in this market.

There are no obvious instrumental variables that can solve this endogeneity problem. One possible instrumental variable is a decrease in capacity of the hotel. If we regard the hotel as setting prices to ration demand, then in periods where there is a reduction in available rooms for exogenous reasons (such as a bursted pipe or other problems that remove rooms from service temporarily, or planned upgrades to rooms that take rooms out of service for a period of time or permanently, such as when the hotel converted 23 of its standard rooms to deluxe rooms), then the decrease in supply of rooms may serve as exogenous supply shifter that might allow us to estimate a negatively sloped demand curve. Unfortunately when we tried to use available capacity as an instrument we find highly unreliable and generally insignificant results. There is not enough exogenous variation in hotel 0’s available capacity to make this a good instrument for estimating the effect of hotel 0’s price on demand.

2.3 Identification of a Static Model of Hotel Demand

We analyze the identification problem in the context of a simple static model of hotel demand, i.e. a model where there are no advance bookings of hotel rooms. The static setting allows us to illustrate our approach and key issues in a simpler model with less notational complexity. We focus on a particular local hotel market and assume that the $L$ hotels in this market do not take advance bookings but rather on each day $t$ the hotels set prices (simultaneously), then a random number of customers arrive and choose which hotel to stay at, after observing the vector of prices $p_t$ that the hotels
set that day. One of the options available to all consumers is the “outside good” i.e. to not stay in any of the hotels. We develop a model that is rich enough to reflect the censoring and endogeneity problems that we observe in our hotel data set, and thus provides a simple initial framework to convey the basic ideas underlying our approach.

The fundamental challenge is one of identification: the econometrician does not observe \( \tilde{A} \), the number of customers arriving to book rooms, nor does the econometrician observe the number of arriving customers who choose the outside good. In the statistics literature this is known as a problem of truncation. We also face a related problem of censoring: at each hotel we only observe the minimum of its actual demand for rooms and the hotel’s room capacity. A further data problem is that the econometrician may not observe the occupancy rates at competing hotels. We also consider the possibility of unobserved heterogeneity i.e. the econometrician does not observe an individual’s preferences for the various hotels or their degree of price sensitivity, which is crucial information necessary to determine the customer’s willingness to pay to stay in various hotels in this market.

The econometrician can observe the vector of prices \( p \) chosen each day by the hotels, and the realized occupancies, \( d \), (an \( L \times 1 \) vector) though as we noted, these occupancies are censored at the capacities \( C \) (also an \( L \times 1 \) vector) at each of the hotels. When a hotel reaches capacity it turns customers away. In reality when a hotel reaches capacity and has to turn customers away, they may choose the outside good or a competing hotel in this market that has availability. However to keep our analysis simpler, we will assume all rationed customers choose the outside good, and with probability 1 the hotel capacity constraint \( d \leq C \) is enforced. Let \( d_t \) be the total number of consumers who are booked in one of the hotels on day \( t \) where \( e \) is an \( L \times 1 \) vectors of 1’s. We have with probability 1, \( \tilde{A}_t \geq d_t e \), and the difference, \( \tilde{A}_t - d_t e \) is
the number of customers who end up with the outside good, either via a voluntary choice, or because the hotel they chose was full.

We assume the hotels (and the econometrician) observe a vector of demand shifters $x_t$ that helps them predict the number of arrivals $A$. There may also be other idiosyncratic variables $z_t$ that are specific to each hotel that each hotel observes, that affect their pricing decision. We assume the econometrician does not observe the vector $z_t$, which we assume for simplicity is an $L \times 1$ vector where the $l$th component $z_{tl}$ represents a scalar idiosyncratic shock that is observed only by hotel $l$ and reflects factors specific to hotel $l$ how sets its price that the other hotels (and the econometrician) do not observe. Thus $x_t$ is common knowledge among the hotels, but each shock $z_{tl}$ is private information observed only by hotel $l$, though we place no restriction on the correlation between $z_{tl}$ and $z_{t'l'}$ for $l \neq l'$.

The timing of decisions in each market day $t$ are as follows:

1. At the start of the day, all $L$ hotels observe the demand shifter $x_t$ that help them predict the number of customers who will arrive to book rooms later that day. Each hotel $l$ also observes idiosyncratic factors that affect its own pricing decision, $z_{tl}$, but not the idiosyncratic shocks observed by each of its competitors $\{z_{t'l'}\}$ for $l' \neq l$. Customers may observe $x_t$ but no customer nor the econometrician, observes the idiosyncratic shocks $z_{tl}$, $l \in \{0, \ldots, L - 1\}$.

2. Based on the information $(x_t, z_t)$ the firms set their prices, so we can write $p_t = p(x_t, z_t)$ is the price set at the start of day $t$ prior to the arrival of customers. The price set by hotel $l$ depends only on its own idiosyncratic realization $z_{tl}$, so the pricing rule for hotel $l$ is independent of $\{z_{t'l'}\}$, $l' \neq l$ and so can be written as

$$p_{tl} = p_t(x_t, z_{tl}), \quad l \in \{0, \ldots, L - 1\}.$$  \hfill (2.1)
The hotel pricing also reflects the common knowledge of an $L \times 1$ vector $\xi$ of characteristics of each of the hotels that constitute attributes of each of the hotels that affect customer preferences that the hotels and customers observe but the econometrician does not observe. We assume that customers observe $\xi$ and these characteristics affect their preferences for the hotels. Similarly the hotels’ perception of customer preferences in turn affects their pricing decisions. However we do not include the unobservable variables $\xi$ as an explicit argument of the pricing function (2.1), though our model does allow prices to be an implicit function of their perceptions of consumer preferences which may in turn depend on the time-invariant unobserved attribute vector $\xi$.

3. Customers observe the demand shifter $x_t$ and the characteristics of the competing hotels (and the outside good) $\xi$, but the number of customers arriving to book a room in hotel market is a random process that does not depend on prices $p_t$ given $x_t$. We will let $H(A|x)$ be the distribution of consumers who arrive when market conditions are summarized by the observed demand shifter $x$ but assume that the distribution of arrivals does not depend on $\xi$ or prices $p$ since customers only learn about $\xi$ and $p$ once they arrive at the market and consider the available alternatives.

4. We assume that each customer who arrives on day $t$ to book a room observes $\xi$ and the price vector $p_t$ and chooses to stay in one of the $L$ hotels based on a simple static utility maximization decision, where the hotel chosen by customer $a \in \{1, \ldots, \hat{A}\}$ can be any of the $L$ hotels or $l = \emptyset$ denotes the choice of the outside good (to not stay in any of the $L$ hotels and go to some other hotel outside of this local market). We assume there are a finite number of possible types of consumers indexed by $\tau$ and there are IID random utility shocks that
affect each consumer’s choice of which hotel to stay at. We assume that the choice of hotel by consumer \( j \) on day \( t \), \( l_{tj} \), is given by

\[
l_{tj} = \arg\max_{l \in \{\emptyset, 0, 1, \ldots, L-1\}} [u_\tau(l, p_t, x) + \epsilon_{tj}(l)]
\]  

The utility each consumer obtains from the different hotels is an implicit function of the hotels’ attributes \( \xi \), so in this sense, the set of consumer utility functions \( \{u_\tau\} \) is a sufficient statistic for the set of hotel attributes \( \xi \) and in order to set prices hotels should have a good knowledge of consumer preferences. Knowledge of how hotel attributes \( \xi \) affect customer preferences is only relevant for longer term decisions by hotels, such as investment in hotel upgrades, etc.

5. We assume the \((L+1) \times 1\) vectors of the random utility components \( \epsilon_{tj} \equiv \{\epsilon_{tj}(l) | l \in \{\emptyset, 0, 1, \ldots, L-1\} \) are continuously distributed with unbounded support over \( \mathbb{R}^{L+1} \) and are independently distributed across different customers who arrive in this market, the behavior of a consumer of type \( \tau \) can be represented by a conditional choice probability \( P_\tau(l|p, x) \) which provides the probability that the consumer will choose hotel \( l \) (or the outside good if \( l = \emptyset \)) given the price vector \( p \) when the observed demand shifter is \( x \). Since choices of different customers are made independently of each other, the total potential demand by the \( A \) customers who arrive on \( t \) given by a multinomial distribution with parameters \((A_t, \pi_\emptyset(p, x), \pi_0(p, x), \ldots, \pi_{L-1}(p, x)) \) where

\[
\pi_l(p, x) = \sum_{\tau=1}^{T} P_\tau(l|p, x)g(\tau|x), \quad l \in \{\emptyset, 0, 1, \ldots, L-1\}
\]  

where \( g(\tau|x) \) is the fraction of consumers of are of type \( \tau \) on a day where the observed demand shifters equal \( x \).

6. Let \( f(d|A, p, x) \) be the conditional distribution of realized occupancy in the \( L \) hotels, where \( d = (d_0, d_1, \ldots, d_{L-1})' \) is an \( L \times 1 \) vector of occupancies in the
$L$ hotels that satisfies each hotel’s capacity constraint, $d_l \leq C_l$, where $l \in \{0, \ldots, L-1\}$. This distribution is a censored multinomial distribution given by

$$f(d|A, p, x) = \begin{cases} M(d|A, p, x) & \text{if } d_l < C_l, \ l \in \{0, \ldots, L-1\} \\ \sum_{d' \in D^c(d)} M(d'|A, p, x) & \text{otherwise} \end{cases} \quad (2.4)$$

where

$$D^c(d) = \left\{ d'|d'_l \geq d_l \text{ if } d_l = C_l, \ d'_l = d_l \text{ if } d_l < C_l, \ \text{where } \sum_l d'_l \leq A\right\} \quad (2.5)$$

where $C_l$ is the room capacity of hotel $l$ and $M(d|A, p, x)$ is the uncensored multinomial density of potential demand

$$M(d|A, p, x) = \frac{A!}{(A - \sum_{l=0}^{L-1} d_l)! d_0! d_1! \cdots d_L!} \pi_\emptyset(p, x)^{A - \sum_{l=0}^{L-1} d_l} \pi_0(p, x)^{d_0} \pi_1(p, x)^{d_1} \cdots \pi_{L-1}(p, x)^{d_{L-1}} \quad (2.6)$$

where $d_l \in \{0, 1, \ldots, A\}$, $l \in \{0, \ldots, L-1\}$ and $\sum_l d_l \leq A$.

Figure 2.5 illustrates the rectangular support of the censored multinomial distribution in the case of $L = 2$ hotels, one with a capacity of $C_0 = 20$ rooms and the other with a capacity of $C_1 = 15$ rooms. The triangular region illustrates the support of the uncensored trinomial distribution when there are $A = 40$ arriving customers. Thus, each pair of potential demands $(d_0, d_1)$ satisfying $d_0 + d_1 \leq 40$ is in the support of the trinomial distribution, with the residual customers $d_\emptyset = 40 - d_0 - d_1$ taking the outside good. Realized occupancies $(d_0, d_1)$ must satisfy the capacity constraints $d_0 \leq C_0$ and $d_1 \leq C_1$ with probability 1. For example the demand $(d_0, d_1) = (20, 20)$ is not feasible since the demand for hotel 1 is $d_1 = 20$ which exceeds its capacity, $C_1 = 15$. In such a case we assume the hotels serve customers on a first come, first served basis until their capacity is reached and all “excess customers” choose the outside good. Thus the realized occupancies equal $(20, 15)$ in this case, and the excess
5 customers who could not be accommodated at hotel 2 are assumed to choose the outside good. When we calculate the probability of any realized occupancy where one or more hotels is sold out, we sum the multinomial probabilities over all possible potential demands $d'$ that are consistent with the specified hotels being at capacity, i.e. over all indices $d'_l \geq C_l$ for those hotels $l$ which are at capacity, $d_l = C_l$. For example the probability of the occupancy pair $d = (d_0, d_1) = (0, 15)$ is the sum over all potential demands $d' = (d'_0, d'_1)$ in the set $D^c(d)$ which is the set of all indices $d'$ where $d'_0 = 0$ and $d'_1 \geq 15 = C_1$ and $d'' = 40 - d'_1$, i.e. the sum of the probabilities of all integer coordinates on the $x$ axis of figure 5 from hotel 1’s capacity of $C_1 = 15$ to the total number of arrivals, $A = 40$.

We now state the key assumptions about the timing of decisions and price setting and demand in our stylized static model of the hotel market given above.

**Assumption 0 (Endogeneity)** Hotels set their prices prior to knowing the number of customers $A$ who arrive to book rooms in the market, however due to the common
dependence on $x$, prices $p$ and arrivals $A$ will generally be positively correlated, and thus also positively correlated with occupancy $d$. Hotel prices will also generally depend on the unobserved characteristics of hotels, $\xi$, which affect consumer preferences and willingness to pay for different hotels.

**Assumption 1 (Stationarity and Independence)** The pricing rule that hotels use to determine prices in equation (2.1) is time-invariant. The demand shifters \( \{x_t\} \) that enter the pricing rule may be serially correlated, but the process \( \{x_t, z_t\} \) is strictly stationary. There may be correlation between $x_t$ and $z_t$ and contemporaneous correlation between the components of $z_t$, i.e. if $l \neq l'$, then $z_{tl}$ and $z_{t'l'}$ may be dependent random variables for each $t$.

**Assumption 2 (Conditional Independence)** The censored multinomial conditional distribution of occupancy given in equation (2.4) is time-invariant and independent of $z$ given $(A, p, x)$. That is, we have:

$$f(d|A, p, x, z) = f(d|A, p, x), \quad (2.7)$$

where $f(d|A, p, x)$ is the censored multinomial conditional distribution of hotel occupancy given in equation (2.4). The distribution of arrivals is also independent of $p$ and $z$ given $x$

$$H(A|x, p, z) = H(A|x) \quad (2.8)$$

Assumption 2 is the key to our semi-parametric identification strategy. The unobserved “price shocks” $z$ create random variability in prices that enables us to identify the effect of price on demand for hotel rooms after controlling for $x$, which is the observable demand shifter that is the fundamental source of endogeneity in this model. We might also refer to Assumption 2 as *conditional exogeneity of prices* since
conditional on \( x \), the remaining variation in prices is random, so we have price variation that is akin to a randomized experiment that helps us to identify the effect of prices on consumer demand for hotels.

We also think of Assumption 2 as analogous to the conditional independence assumption in the treatment effects literature, where the assignment of a “treatment” is assumed to be independent of the potential outcomes, conditional on a vector of covariates \( x \). Thus, assignment of a treatment can be treated as a virtual random assignment, given \( x \) in the sense that prices can be regarded as “randomly assigned” given \( x \) due to the effect of unobserved idiosyncratic factors \( z \) affecting how the hotels set their prices. The conditional distribution \( G(p|x) \) captures the variability in prices due to the effect of the \( z \) shocks and the conditional distribution \( H(A|x) \) reflects the uncertainty about the number of arrivals given only knowledge of the observed demand shifter \( x \). We believe it is plausible that \( H(A|x) \) does not depend on \( p \) because customers are not able to learn about prices until they actually arrive at the market (e.g. visit a website to check prices). However this does not imply that arrivals are independent of \( p \); there will be positive correlation between \( p \) and \( A \) and thus between \( p \) and realized occupancies \( d \) via the common dependence on the demand shifter \( x \).

We make a final assumption about the variables the econometrician observes in this market.

**Assumption 3 (Observables and Unobservables)** The econometrician is able to observe joint occupancy, \( d \), the demand shifter \( x \), and the vector of prices charged by the hotels, \( p \). The econometrician does not observe arrivals, \( A \), or the vector of pricing shocks, \( z \).

Note that Assumptions 0 to 3 are completely agnostic about how hotels set prices in the market, and in particular there is no assumption about hotels setting prices optimally or in a manner consistent with a price equilibrium, e.g. Bertrand Nash
equilibrium. Let \( G(p|x) \) be the conditional distribution over the prices set by the hotels given the observed demand shifter \( x \) that are induced by the idiosyncratic price shocks \( z \). In effect, this distribution is an infinite dimensional “nuisance parameter” since our primary interest is to infer consumer preferences and the arrival probability \( H(A|x) \).

With enough data on a given market, it is possible to non-parametrically estimate the distribution \( G(p|x) \) and the conditional distribution of occupancy given \((x,p)\), \( f(d|p,x) \). For the purposes of a theoretical analysis of the identification of the model we will treat \( G \) and \( f \) as known conditional distributions. By Assumptions 0 to 3 above, we can write \( f \) and \( G \) as follows

\[
\begin{align*}
    f(d|p,x) & = \sum_A f(p|A,p,x)H(A|x) \\
    G(p|x) & = \int I\{p(x,z) \leq p\} \psi(dz|x)
\end{align*}
\]  

(2.9)

where \( I\{\cdot\} \) is the indicator function, \( p(x,z) \) is the joint pricing strategy, i.e. the function hotels use to set their prices as a function of their information \((x,z)\), \( \psi(z|x) \) is the conditional distribution of \( z \) given \( x \) and \( f(d|A,p,x) \) is the censored multinomial distribution given in equation (2.4). From our standpoint the pricing function \( p(x,z) \) and the conditional distribution \( \psi(z|x) \) are nuisance functions that are not of direct interest for estimation. Instead our interest is to identify and estimate the censored multinomial distribution \( f(d|A,p,x) \) and the distribution of arrivals \( H(A|x) \) which plays the role of a mixing distribution in our context. Still deeper, we are also interested in identifying the conditional choice probabilities \( \pi_l(p,x) \) and potentially from these, the distribution of consumer types and the type-specific choice probabilities given in equation (2.3).

Note that prices may reflect the effect of unobserved characteristics of hotels \( \xi \), and we will show that prices can reflect endogeneity due to classical simultaneous
equations bias. That is, variations in the number of arriving customers across different market days with different \textit{ex ante} values of $x$ that constitute observed demand shifters is enough in the face of the limited capacity of the hotels in this market to result a) a strong co-movement in prices among the various hotels in the market, and b) an upward sloping relationship between price and occupancy at individual hotels. Note, however, that since we can non-parametrically estimate $G(p|x)$ we do not have to take a stand on how individual hotels set their prices. We will now show that Assumptions 0-3 are sufficient to identify the effect of price on the demand for hotel rooms without requiring us to develop a detailed model of equilibrium in the hotel market, or even to assume anything about how individual hotels set their prices as a function of $x$ and $z$, such as the assumption that individual hotels set their prices optimally as a best response to their beliefs of the prices set by their competitors.

Figure 2.6 illustrates the censored multinomial occupancy distribution $f(d|A, p, x)$ for two hotels ("hotel 0" and "hotel c") whose capacities are $C_0 = 30$ and $C_1 = 50$ respectively. We assume that the number of arrivals is $A = 120$ and the two hotels have accurate signals of the number of arrivals and set their prices accordingly. Since hotel 0 has a smaller capacity of $C_0 = 30$, it sets a price of $p_0 = 169$ and hotel c with the larger capacity of $C_1 = 50$ sets a price of $p_1 = 181$. We see that there is significant probability that hotel 0 sells out, while the chance that hotel c sells out is close to zero due to its higher capacity.

To illustrate how our framework is consistent with a fully rational, Bayesian-Nash equilibrium model of price setting, consider a duopoly market with two hotels, 0 and c. Suppose the unobserved private information that these firms use and that affects that their pricing decisions is partitioned as $z = (z_0, z_c)$ and the hotels have knowledge of the joint distribution of $(x, z)$ and thus have well defined conditional probability measures $\Psi_0(z_c|x, z_0)$ (i.e. hotel 0’s belief about the distribution of private information
Figure 2.6: Occupancy distribution, $A = 120$

of hotel $c$) and conversely $\Psi_c(z_0|x, z_c)$ is hotel c’s belief about the private signal of hotel 0. A Bayesian-Nash equilibrium is a pair of functions $\{p_0(x, z_0), p_c(x, z_c)\}$ defined by

$$p_0(x, z_0) = \arg\max_{p_0} \left[ \int_{z_c} \sum_{A} \sum_{d_0} \sum_{d_c} \min(C_0, d_0)(p_0 - c_0)f(d_0, d_c|A, p_0, p_c(x, z_c), x)H(A|x)\Psi_0(dz_c|x, z_0) \right]$$

$$p_c(x, z_c) = \arg\max_{p_c} \left[ \int_{z_0} \sum_{A} \sum_{d_0} \sum_{d_c} \min(C_c, d_c)(p_0 - c_c)f(d_0, d_c|A, p_0(x, z_0), p_c, x)H(A|x)\Psi_c(dz_0|x, z_c) \right],$$

(2.10)

where $(C_0, C_c)$ are the capacities of the two hotels, $(c_0, c_c)$ are the marginal costs of servicing rooms (which could be negative, if hotel guests consume other hotel services such as minibar, restaurant and other hotel amenities). The main point here is that identification of the “structural objects” $\{f, H\}$ enables us to calculate counterfactuals and test whether the actual conditional distribution of prices, $G(p|x)$ is consistent with
the distribution induced by the assumption that firm behavior is given by Bayesian-Nash equilibrium in prices.

However the observed distribution of prices \( G(p|x) \) may be consistent with any number of other theories of firm price-setting, including theories based on bounded rationality and that incorporate the effects of “pricing mistakes” that are the source of the random shocks \( z \) affecting firm prices. Given assumptions 0-3, any one of these theories can generate data that reveals the endogeneity. We will now show that by controlling for the demand shift variable \( x \), Assumptions 0-3 can enable us to identify a downward sloping expected demand curve, where the expected demand (more precisely, expected occupancy) for hotel 0 is given by

\[
E\{d_0|p_0, p_c, x\} = \sum_A \sum_{d_0} \sum_{d_c} \min(C_0, d_0) f(d_0, d_c|A, p_0, p_c, x) H(A|x). \tag{2.11}
\]

**Lemma 1** Assume Assumptions 0-3 hold. If the choice probability for hotel 0, \( \pi_0(p_0, p_c, x) \) is downward sloping in \( p_0 \) and upward sloping in \( p_c \), then the expected demand function \( E\{d_0|p_0, p_c, x\} \) is also downward sloping in \( p_0 \) and upward sloping in \( p_c \).

**Proof** Accounting for the outside good, for each \( A \) we can express the expectation in equation (2.11) as the expectation with respect to the marginal binomial distribution for \( d_0 \) with parameters \( (A, \pi_0(p_0, p_c, x)) \), since given the number of arrivals \( A \) the marginal distribution of the trinomial distribution of outcomes for hotel 0 occupancy, hotel c occupancy and the outside good, respectively, \( (d_0, d_c, A-d_0-d_c) \), is a binomial distribution. It is well known that a binomial distribution satisfies the property of first order stochastic dominance with respect to the probability of occurrence, i.e. a binomial random variable \( d_0 \) with parameters \( (A, \pi) \) stochastically dominates a binomial random variable \( d'_0 \) with parameters \( (A, \pi') \) if and only if \( \pi \geq \pi' \). This in turn implies that the expectation of any monotone increasing function \( h(d_0) \) is a monotone
function of $\pi_0$, and since $h(d_0) = \min(C_0, d_0)$ is monotone increasing, we conclude that if $\pi_0$ is monotone decreasing in $p_0$ and increasing in $p_c$ than $E\{d_0|A, p_0, p_c, x\}$ is also monotone decreasing in $p_0$ and monotone increasing in $p_c$. Since $E\{d_0|p_0, p_c, x\}$ is a mixture of monotone functions, it is also monotone decreasing in $p_0$ and increasing in $p_c$.

The Lemma shows that if we were to estimate the demand model by nonlinear regression, then by controlling for the demand shifter $x$ we obtain a regression function that has the right slopes with respect to $p_0$ and $p_c$ under Assumptions 0-3, provided the individual choice probabilities also slope in the right way as a function of $(p_0, p_c)$. The latter will be true for a wide range of choice models such as where $\pi(p, x)$ are mixtures of logits, probits, etc. Further, the conditional exogeneity assumption 2 implies that once we control for $x$, the prices $(p_0, p_c)$ are econometrically exogenous and thus, we can recover the expected demand for hotel 0 by non-parametric regression using only occupancy data for hotel 0, the joint prices for all hotels (and the outside good) $p$, and the observed demand shifter $x$.

2.3.1 Non-parametric Identification

We now consider the problem of identification, to understand what objects can be identified under Assumptions 0-3. We are particularly interested in whether it is possible to identify consumer preferences and arrivals from potentially endogenously generated market price and occupancy data.

**Definition 1** The structure of the hotel pricing problem consists of the objects

$\Gamma = \{\{g(\tau|x)||\tau \in \{1, \ldots, T\}\}, \{P_\tau(ll|p, x)||\tau \in \{1, \ldots, T\}, \}

l \in \{0, 1, \ldots, L\}\}, H(A|x), x \in X$.  \hspace{1cm} (2.12)

We exclude the conditional distribution $G(p|x)$ from the elements of the structure from the problem since we wish to relax the assumptions of 1) equilibrium (the firms’
set prices in accordance with a Bertrand-Nash equilibrium), and 2) optimality (firms set optimal prices, given possibly non-equilibrium beliefs about the prices charged by their competitors). However we consider the choice probabilities $P_c(l|p, x)$, the distribution of consumer types $g(x|\tau)$ and the arrival probability $H(A|x)$ as structural objects that would not change under different assumptions about how the hotels set their prices, such as if they set prices optimally and in accordance with a Bertrand Nash equilibrium. Thus, if we can identify the structural objects, we can in principle solve for a Bertrand-Nash equilibrium and compare the distribution of prices $G^*(p|x)$ arising under a Bertrand-Nash equilibrium to the distribution $G(p|x)$ that could potentially be identified under the status quo. Thus, $G$ is not invariant, and depends on what assumptions we make about how the hotels set their prices.

**Definition 2** The identified objects for the hotel pricing problem are given by

$$\Lambda = \{f(d|x,p), G(p|x)\}, \quad x \in X. \quad (2.13)$$

We assume that we can observe the hotels over a sufficiently long period of time where the stationarity assumption holds to consistently estimate the conditional distribution of prices given $x$, $G(p|x)$ so we can take this conditional distribution as “known” for purposes of the analysis of identification. Since we can consistently estimate $G(p|x)$ without imposing the assumption of equilibrium or optimality, we can be agnostic about firm behavior. Similarly, given sufficient data, we assume we can estimate the conditional distribution of occupancy given $(x, p)$ via non-parametric methods, $f(d|x, p)$.

We start by making some easy observations about the model: Lemma 2 below shows that the expected demand curves are also identified, and the identification of these objects are already sufficient to enable to do to interesting counterfactual calculations and hypothesis tests. For example, we can use the demand functions to
test whether the hotels are profit-maximizing, or whether the prices in the market
are consistent with the Bayesian-Nash equilibrium outcomes.

**Lemma 2** Under Assumptions 0-3, the expected occupancy function, \( E\{d|p,x \} \) is non-parametrically identified.

**Proof** This follows immediately from the non-parametric identification of the joint occupancy density, \( f(d|p,x) \), since \( E\{d|p,x \} \) is simply the expectation with respect to this joint density.

Although elementary, there are a number of immediate implications of Lemma 2. First, it implies that we can make interesting counterfactual calculations under much weaker assumptions than are usually imposed in the structural estimation literature. For example, we can test the hypothesis that hotel 0 is using an optimal pricing strategy given the strategy of its competitors. To do this, we use the expected demand function to calculate the optimal pricing strategy \( p_0^*(x) \) for hotel 0 as follows

\[
p_0^*(x) = \arg\max_{p_0} \int_{p_c} (p_0 - c_0) E\{d_0|p_0, p_c, x \} G_c(dp_c|x).
\]

(2.14)

where \( G_c(p_c|x) \) is the marginal distribution of \( p_c \) calculated from the joint conditional distribution \( G(p|x) \). The pricing strategy in equation (2.14) does not depend on any private information shock \( z_0 \) that only hotel 0 can observe. This strategy will be optimal for hotel 0 provided that any private information it observes is independently distributed from any private information that hotel c observes, \( z_c \), but it will be suboptimal if hotel 0 does have private information \( z_0 \) (not observed by the econometrician) that is correlated with \( z_c \) and helps it to predict the prices charged by hotel c. If \( z_0 \) and \( z_c \) are independently distributed and Assumptions 0-3 hold, hotel 0’s optimal pricing strategy should only be a function of the demand shifter variable \( x \) and not on any other extraneous information \( z_0 \). This implies that \( G_0(p_0|x) \) is a
degenerate distribution that puts probability 1 on the price \( p_0^*(x) \) which is a testable restriction on the identified joint conditional distribution \( G(p|x) \).

Similarly, we can also compute a full-information Bertrand-Nash equilibrium for hotels 0 and \( c \) under the assumption that any private information they observe, \( z_0 \) and \( z_c \) respectively, are independently distributed. Then hotel \( c \) will also have an optimal pricing strategy \( p_c^*(x) \) that does not depend on its private information shock \( z_c \), and in a Bertrand-Nash equilibrium, the pricing strategies \( (p_0^*(x), p_c^*(x)) \) will satisfy the fixed-point condition

\[
\begin{align*}
p_0^*(x) &= \arg\max_{p_0} (p_0 - c_0) E\{d_0|p_0, p_c^*(x), x\} \\
p_c^*(x) &= \arg\max_{p_c} (p_c - c_c) E\{d_c|p_0^*(x), p_c, x\}
\end{align*}
\]

and this hypothesis has the testable implication that \( G(p|x) \) is degenerate distribution that puts probability 1 on the point \( (p_0^*(x), p_c^*(x)) \).

However there are other counterfactual calculations and hypotheses that require knowledge of the deeper structure of the problem, particularly to separately identify consumer preferences (both their preferences for different hotels and price sensitivity as well as the distribution of different types of consumers in the population), and the arrival probability distribution \( H(A|x) \). For example, we might be interested in predicting how the market would be affected if there was a shift in the probability distribution of arrivals, or what would happen to prices in the market if one of the hotels expanded its capacity, or one of the hotels exited the market, or there was a hotel merger. These counterfactuals cannot be calculated given knowledge of the distribution of occupancy \( f(d|p, x) \) under the status quo which posits a fixed number \( L \) of competing hotels, with fixed capacities \( C \) and a fixed arrival distribution \( H(A|x) \).

As we noted in the introduction, a counterfactual of particular interest is to calculate what hotel prices would be in this market if the hotels were to collude, possibly in
response to recommended prices if they were to all use a common RMS that was capable of calculating and recommending a collusive price vector. We would like to calculate a collusive price strategy and compare the implied distribution of prices to the distribution $G(p|x)$ that holds under the status quo $G(p|x)$ to provide a test for “algorithmic collusion.” However in order to do this, we need to know more about consumer preferences and particularly the rate of substitution to the outside good in the event collusive pricing would significantly raise prices in this hotel market relative to nearby markets. Thus, in the remainder of this section we provide some results on the non-parametric identification of the structure given in Definition 1.

The identification problem concerns the question as to whether there is an invertible mapping from the identified objects $\Lambda$ to the structure $\Gamma$. Define a mapping $\Psi(\Gamma)$ from the structure to the first component of the identified objects $\Lambda$ by

$$f(d|x,p) = \sum_A f(d|A,x,p)H(A|x) \equiv \Psi(\Gamma),$$

(2.16)

where $f(d|A,x,p)$ is the censored multinomial distribution of hotel occupancy given the number of arrivals $A$ that we introduced in equation (2.4). Note that equations (2.4) and (2.5) imply that $f(d|A,x,p)$ is itself a function of the other components for the structure $\Gamma$, i.e. the distributions over consumer types $g(\tau|x)$ and the consumer choice probabilities $P_{\tau}(l|p,x)$, for $\tau \in \{1, \ldots, T\}$ and $x \in X$.

**Definition 3** Two structures $\Gamma \neq \Gamma'$ are observationally equivalent if $\Psi(\Gamma) = \Psi(\Gamma')$.

**Definition 4** The hotel model with identified objects $\Lambda$ is identified if there is a structure $\Gamma$ satisfying $\Psi(\Gamma) = \Lambda$ and there is no other structure $\Gamma' \neq \Gamma$ that is observationally equivalent to $\Gamma$.

The identification problem reduces to a question on the identification of a mixture models, as can be seen in equation (2.16), where we are interested in “inverting” the distribution of occupancy given $(x,p)$ given by $f(d|x,p)$ to uniquely determine
the “component distributions” \( f(d|A, x, p) \) and the conditional distribution of arrivals \( H(A|x) \). Actually we have a problem of identification of a nested mixture model, since in addition to identifying the component distributions \( \{f(d|A, x, p)\} \) and the mixing distribution \( H(A|x) \), we are also interested in identifying the choice probabilities \( \pi_l(p, x) \) and from them, the mixing distribution representation of unobserved heterogeneity in consumers given in equation (2.3). This is also clearly a problem of identification of mixtures, but in this case we presume knowledge of the type-unconditional choice probabilities \( \pi_l(p, x) \), \( l \in \{0, 1, \ldots, L - 1\} \) and from these identify the component probabilities \( \{P_l(l|p, x)\} \) and the mixing distribution \( g(\tau|x) \) for all possible values of \( (p, x) \).

We will assume that maximum number of consumers arriving to the market is uniformly bounded with a known upper bound (even though the actual support of \( H(A|x) \) may be unknown):

**Assumption 4 (Finite support)** Let \( |A|(x) \) be the size of the support of the integer valued distribution \( H(A|x) \). We have \( |A| \equiv \max_{x \in X} |A|(x) < \infty \) where the upper bound \( |A| \) is known a priori.

Assume for the moment that we can identify the choice probabilities \( \pi_l(p, x) \) of the censored multinomial representation of \( f(d|A, p, x) \) given in equation (2.4) in the “upper level” mixture identification problem. To identify the distribution of types \( g(\tau|x) \) and type-specific choice probabilities \( \{P_l(l|p, x)\} \) in the “lower level” mixture identification problem in (2.3) we need to impose some additional structure and assumptions.

**Assumption 5 (Mixed logit)** The utility functions for consumers given in equation (2.2) are linear in parameters

\[
  u_\tau(l, p_l, x) = \alpha(l, x) - \beta_\tau(x)p_l, \tag{2.17}
\]
where for the outside good, \( l = 0 \) we impose the normalization \( u_\tau(0, p_0, x) = 0 \), and we assume the error terms \( \epsilon(l) \) are standardized Type 1 extreme value (i.e. have mean zero and scale parameter 1). This implies that the type-specific choice probabilities \( P_\tau(l|p, x) \) take the standard multinomial logit form

\[
P_\tau(l|p, x) = \frac{\exp\{\alpha(l, x) - \beta_\tau(x)p_l\}}{1 + \sum_{l'=1}^{L} \exp\{\alpha(l', x) - \beta_\tau p_{l'}\}}.
\]

(2.18)

Given this we can apply the non-parametric identification result of Fox et al. [14] to establish that both the number of unobserved types \( T \) and the conditional distribution of types \( g(\tau|x) \) as well as the “random coefficients” \( \{\beta_\tau(x)\} \) for \( \tau \) in the support of the discrete distribution \( g(\tau|x) \), as well as the (non-type-specific) intercept terms \( \{\alpha(l, x)\}, l \in \{0, 1, \ldots, L - 1\} \) for each \( x \in X \).

Thus, the non-parametric identification of the model hinges on whether it is possible to separately identify the component distributions \( f(d|A, x, p) \) and the conditional distribution of arrivals (mixing distribution) \( H(A|x) \) in equation (2.16). Promising recent progress on the identification of mixture models by Kitamura and Laage [20] suggests that this may be possible. However we cannot directly apply their key result, Proposition 6.1, since the structure of our problem is not nested within the class of mixture models that they consider. Specifically, they consider the identification of mixture models that can be written as a regression equation for an observed dependent variable \( y \) given covariates \( x \)

\[
y = f(x) + \epsilon
\]

(2.19)

where the actual observations are drawn from a mixture of regression models

\[
y_j = f_j(x) + \epsilon_j, \quad j \in \{1, \ldots, J\}
\]

(2.20)

with probability \( \lambda_j \geq 0 \) with \( \sum_{j=1}^{J} \lambda_j = 1 \). In this case the \( \{\lambda_j\} \) are the mixing distributions and the \( \{f_j(x)\} \) are the component distributions. Proposition 6.1 of Kitamura
and Laage [20] establishes the non-parametric identification of this mixture model, i.e., given knowledge of the regression \( f(x) \) and the distribution of \( \epsilon \), they establish that the number of mixture components \( J \) and the mixing probabilities \( \{\lambda_j\} \) and the component regression functions \( \{f_j(x)\} \) are identified. However their result requires the error terms \( \{\epsilon_j\} \) are univariate random variables that are assumed to be continuously distributed and independent of the regressor \( x \), and IID across the different types \( j \). In addition their result relies on additional assumptions that guarantee that the regression functions \( f_j(x) \) are “non-parallel” as well as other technical assumptions about the moment generating functions of the \( \{\epsilon_j\} \).

In our case we can write occupancies as a multivariate system of regressions

\[
d = E\{d|p, x\} + \epsilon \tag{2.21}
\]

where

\[
E\{d|p, x\} = \sum_A E\{d|A, p, x\} H(A|x) \tag{2.22}
\]

so it is tempting to try to apply Proposition 6.1 to our setting. However the component regressions in our case are

\[
d_A = E\{d|A, p, x\} + \epsilon_A \tag{2.23}
\]

(i.e. where the number of arrivals \( A \) index the mixture components) but the error terms in our case, \( \epsilon_A = d_A - E\{d|A, p, x\} \) are multivariate random variables that are not continuously distributed or IID when considered as indexed over different values of arrivals \( A \).\(^6\) Thus, we cannot directly apply Proposition 6.1 of Kitamura and Laage [20] to establish the non-parametric identification of our hotel model. Further, they provide counter-examples showing the mixture model (2.19) is non-identified when the error terms \( \{\epsilon_A\} \) are heteroscedastic, as they are in our case.

\(^6\)If the capacities of the hotels are not symmetric, e.g. if \( C_l \neq C_{l'} \) for \( l \neq l' \), then the different components of \( \epsilon_A \) will have different distributions, and the overall vector \( \epsilon_A \) will have different distributions for different values of the arrivals \( A \).
Instead we establish the identification of mixtures of censored multinomials via a direct argument. First, observe that if we fix the continuous price regressor $p$ and the demand shifter $x$, we can consider the key equation (2.16) as nonlinear system of equations. It is actually a polynomial system of equations in $(\pi_\emptyset, \pi_0, \ldots, \pi_{L-1})$ and a linear system in $H(A|\pi)$ given $\pi$ and $(p, x)$. Let $|D| = \prod_{i=1}^{L}(C_i + 1)$ be the size of the support of $f(d|x, p)$. Then $|D|$ indexes the number of left hand side “known values” in equation (2.16), whereas $\{f(d|A, x, p), H(A|x)\}$ on the right hand side are the “unknowns.” Let $|A|(x)$ be an a priori known upper bound on the support of the number of arrivals. Without imposing any further special structure on the system of equations (2.16) identification would seem to be hopeless since it constitutes a system of $|D|$ equations in at most $|A|(1 + |D|)$ unknowns, and thus in principle there could be far more unknowns than equations. However there is substantial special structure to the hotel problem in view of the fact that $f(d|A, x, p)$ takes the form of a censored multinomial distribution in (2.4). For fixed $(p, x)$, this special structure reduces the problem to a nonlinear system of equations with $|D|$ equations in $L + |A|(x) - 1$ unknowns. If $|D| > L + |A|(x) - 1$, then equation (2.16) will be an over-determined system, i.e. it will have more equations than unknowns. We can consider this to be a basic “rank condition” for identification.

Note that for fixed $(p, x)$, if we treat the component distributions $f(d|A, p, x)$ as known, equation (2.16) can be viewed as a system of linear equations $f = f_A \times H$ where $f_A$ is a matrix of dimension $|D| \times |A|$ formed with the densities $f(d|A, p, x)$ arrays as its columns. If the matrix $f_A$ has full rank, then there is a unique mixing distribution $H(A|x)$ that solves (2.16) when the component distributions $f(d|A, p, x)$ are fixed at their true values. However we note that (2.16) is a nonlinear system of equations when we consider $\{H(A|x), \{\pi_l(p, x)\}, l \in \{\emptyset, 0, 1, \ldots, L - 1\}\}$ as the full
set of unknowns. Therefore a different argument is required to establish identification of these functions.

Our identification results below will consider two possible information structures:

- **Full information** The econometrician can observe the number of customers choosing the outside good, or the number of arrivals, or both.

- **Limited information** The econometrician cannot observe the number of customers choosing the outside good, or the number of arrivals.

We will now provide sufficient conditions for the non-parametric identification of the model under both the full information and limited information structures. We start by providing a lemma that establishes that the model is partially identified under either information structure.

**Lemma** Under Assumptions 0, ..., 4 if \( C_l \geq 1, \ l \in \{0, \ldots, L\} \) then the ratios of the choice probabilities, \( r_l(p, x) = \frac{\pi_l(p, x)}{\pi_0(p, x)} \) are identified for \( l = 1, \ldots, L - 1 \) for any \((p, x)\) such that \( \pi_0(p, x) > 0 \).

**Proof** Note first that if \( C_l \geq 1 \) for \( l = 0, \ldots, L - 1 \), then \( |D| \geq 2^L > L \) so the rank condition for identification is satisfied. Let \( e_l \) be an \( L \times 1 \) vector whose elements equal 0 except for element \( l \) which equals 1. Then \( f(e_l|p, x) \) is the probability that hotel \( l \) has exactly 1 customer occupying one of its rooms. By assumption this probability is known and positive for each \( l \). We have

\[
f(e_l|p, x) = \pi_l(p, x) \sum_A A \pi_0^{-1}(A|x), \quad l \in \{0, 1, \ldots, L - 1\}\]

From equation (2.24) it immediately follows that the ratios \( r_l(p, x) \) given by

\[
r_l(p, x) = \frac{f(e_l|p, x)}{f(e_0|p, x)} = \frac{\pi_l(p, x)}{\pi_0(p, x)}, \quad l \in \{1, \ldots, L - 1\}
\]

are identified. \( \square \)
We can write the choice probabilities \( \pi_l(p, x) \) in terms of the identified ratios of choice probabilities, \( r_l(p, x) \) as \( \pi_l(p, x) = \pi_0(p, x) r_l(p, x) \) and use the fact that the choice probabilities sum to 1 to write the probability of the outside good, \( \pi_\emptyset(p, x) \) in terms of \( (\pi_0(p, x), \ldots, \pi_{L-1}(p, x)) \), to reduce the identification problem to the solution of a system of \(|D| - L\) equations in \(|A|\) unknowns, \( (\pi_0(p, x), \{H(A|x)\}) \).

**Theorem 0** Suppose we have full information on arrivals. Then if Assumptions 0-5 hold and \( C_l \geq 1, l = 0, \ldots, L - 1 \) the hotel model is non-parametrically identified.

**Proof** Under full information, the hotels (and the econometrician) can observe all arrivals and all consumers who choose the outside good (though observing one enables us to deduce the other via the identity

\[
A = d_\emptyset + \sum_{l=0}^{L-1} d_l.
\]  

(2.26)

Since arrivals are observed, it follows that \( H(A|x) \) is identified for each \( x \) (since we presume for the purposes of the analysis of identification we have infinitely many observations and thus can consistently estimate the discrete distribution \( H(A|x) \) from the empirical distribution). So the question of identification reduces to the identification of the probability \( \pi_0(p, x) \). Define \( r_0(p, x) = 1 \). Since the choice probabilities sum to 1 for all \( (p, x) \), we have

\[
\pi_\emptyset(p, x) = 1 - \sum_{l=0}^{L-1} \pi_l(p, x) = 1 - \pi_0(p, x) \sum_{l=0}^{L-1} r_l(p, x)
\]

(2.27)

where the \( r_l(p, x) \) are known functions of \( (p, x) \) by the partial identification lemma above. Let \( \mathbf{0} \) be an \( L \times 1 \) vector of 0’s, so \( f(\mathbf{0}|p, x) \) is the probability of zero occupancy in all \( L \) hotels given \( (p, x) \), which is also a known function given our assumption that \( f(d|p, x) \) is identified. We have

\[
f(\mathbf{0}|p, x) = \sum_A \pi_\emptyset(p, x)^A H(A|x) = \sum_A \left[ 1 - \pi_0(p, x) \sum_{l=0}^{L-1} r_l(p, x) \right]^A H(A|x)
\]

(2.28)
by equation (2.27). Note that equation (2.28) is a polynomial equation in \( \pi_0(p, x) \) and we know it has at least 1 solution in the unit interval, where at least one root is the true value \( \pi_0(p, x) \) that customers choose hotel 0. Define the polynomial \( P(y) : \mathbb{R} \rightarrow \mathbb{R} \) by

\[
P(y) = \sum_{A} \left[ 1 - y \sum_{l=0}^{L-1} r_l(p, x) \right] A H(A|x).
\]  

(2.29)

Notice that \( P(0) = 1 \) and furthermore, we have

\[
P'(y) = - \left( \sum_{l=0}^{L-1} r_l(p, x) \right) \left[ \sum_{A} A \left( 1 - y \sum_{l=0}^{L-1} r_l(p, x) \right) A^{-1} H(A|x) \right] < 0 \quad y \in [0, 1].
\]

(2.30)

Since \( f(0|p, x) \in (0, 1) \) and we know there is one solution of the equation \( P(y) = f(0|p, x) \) in the unit interval (i.e. the true probability \( \pi_0(p, x) \)), equations (2.29) and (2.30) imply that there is only one solution in the unit interval, i.e. \( \pi_0(p, x) \) is identified, and thus the entire model \( \{(\pi_0(p, x), \ldots, p_{L-1}(p, x)), H(A|x)\} \) is identified. □

In the limited information case, we do not observe the number of consumers who arrive in the hotel market, nor the consumers who choose the outside good. We can only observe the occupancy in each of the hotels, and with sufficient data, this enables us to consistently estimate \( f(d|p, x) \), the joint distribution of occupancy given \( (p, x) \). Identification is more difficult in this case since we cannot directly recover the distribution of arrivals, \( H(A|x) \), which was the first key step to the proof of Theorem 0 for the case where we have full information (e.g. we observe arrivals). However the intuition that when \(|D| > L + |A|(x) - 1\) we have more equations than unknowns and so the rank order for identification is satisfied does not automatically lead to a proof of identification. Though we conjecture that the model is identified when this rank condition holds, at this point we require additional conditional to prove identification, given in Theorem 1 below.
Theorem 1 Suppose Assumptions 0, \ldots, 5 hold, and \( C_l \geq 1, \ l = 0, 1, \ldots, L - 1 \). Further, suppose that for each \( x \in X \) that \( |C| > |A|(x) \) where where \( |C| = \sum_{l=1}^{L} C_l \) is the total room capacity in the market. Also suppose for each \( x \) there exists a \( p \) that satisfies \( \pi_{\varphi}(p, x) = \pi_0(p, x) \). Then the structure of the hotel model is identified.

Proof: The largest number of arrivals when the demand shifter is \( x \) is identified from \( f(d|p, x) \) as the largest occupancy vector in the support of \( f(d|p, x) \):

\[
|A|(x) = \sup_d \{|d| | f(d|p, x) > 0 \}
\]  

(2.31)

where \( |d| = \sum_{l=0}^{L-1} d_l \). Next, by the assumption that for each \( x \) there exists a \( p \) satisfying \( \pi_{\varphi}(p, x) = \pi_0(p, x) \), we can solve for \( \pi_0(p, x) \) via the equation

\[
1 - \pi_{\varphi}(p, x) = 1 - \pi_0(p, x) = \pi_0(p, x) \left[ \sum_{l=0}^{L-1} r_l(p, x) \right]
\]

(2.32)

or

\[
\pi_0(p, x) = \pi_{\varphi}(p, x) = \frac{1}{1 + \sum_{l=0}^{L-1} r_l(p, x)},
\]

(2.33)

and hence the choice probabilities \((\pi_{\varphi}(p, x), \ldots, \pi_L(p, x))\) are identified, for this particular \( p \). Now we show how to identify \( H(|A|(x)|x) \), i.e. the probability of the maximum number of arrivals \( |A|(x) \) when the the demand shifter is \( x \). First, the identification of the choice probabilities \((\pi_{\varphi}(p, x), \ldots, \pi_L(p, x))\) implies that for any \( A \geq 0 \), the censored multinomial distribution \( f(d|A, p, x) \) given in equation (2.4) is identified. Since \( |A|(x) \) is the maximal number of arrivals in state \( x \), then for any \( d \) satisfying \( f(d|p, x) > 0 \) and \( |d| = |A|(x) \) we have

\[
f(d|p, x) = H(|A|(x)|x) f(d|A, p, x)
\]

(2.34)

so \( H(|A|(x)|x) \), the probability of \( |A|(x) \) arrivals in state \( x \), is identified.

Next we show by induction that \( H(A|x) \) is identified for all \( A < |A|(x) \). Suppose the arrival probabilities \( \{H(|A|(x)|x), H(|A|(x)−1|x), \ldots, H(A|x)\} \) are identified. We
show that $H(A - 1|x)$ is identified as follows. Let $d_A$ be any joint occupancy in the support of $f(d|p, x)$ satisfying: a) $|d_A| = A$ and b) $f(d_A|p, x) > 0$. Let $d_{A-1}$ be an occupancy vector satisfying $d_{A-1,l} = d_{A,l}$ for $l = 1, \ldots, L - 1$ and $d_{A-1,0} = d_{A,0} - 1$. Then we have $|d_{A-1}| = A - 1$ and we have

$$f(d_{A-1}|p, x) = f(d_{A-1}|A - 1, p, x)H(A - 1|x) + \sum_{A' = A}^{d_{A}(x)} f(d_{A-1}|A', p, x)H(A'|x). \quad (2.35)$$

By our inductive hypothesis, the sum on the right hand side of equation (2.35) is identified. Since $f(d_{A-1}|A - 1, p, x) > 0$, it follows that we can solve this equation for $H(A - 1|x)$ and so it is identified as well. Thus we conclude for each $x \in X$ that $H(A|x)$ is identified.

To complete the proof, we need to show that the choice probabilities $(\pi_{\emptyset}(p', x), \pi_0(p', x), \ldots, \pi_{L-1}(p'x))$ are identified not only for the particular $p$ for which $\pi_{\emptyset}(p, x) = \pi_0(p, x)$, but also for any $p'$ in the support of $G(p|x)$ that may not satisfy the restriction that $\pi_{\emptyset}(p', x) = \pi_0(p', x)$. However by repeating the proof of Theorem 0, we see once $H(A|x)$ is identified, it follows that the choice probabilities $(\pi_{\emptyset}(p', x), \pi_0(p', x), \ldots, \pi_{L-1}(p'x))$ are identified for all $p'$ in the support of $G(p|x)$. \hfill \Box

We believe the hotel model is identified under weaker assumptions than those assumed in Theorem 1, but we have not yet succeeded in providing a proof of this. Theorem 2 shows that once we are able to identify the choice probabilities, $\pi(p, x)$, we can also identify the distribution of unobserved heterogeneity $g(\tau)$ and the random coefficients $\{\beta_{\tau}(x)\}$ in the multinomial logit type-specific choice probabilities in equation (2.18).

**Theorem 2** Under the assumptions of Theorem 0 with full information, or Theorem 1 with limited information, the distribution of types $g(\tau)$ and associated random coefficients $\{\beta_{\tau}(x)\}$ are identified.

**Proof** This result follows from the identification result of Fox et al. [14].
We conclude this section with a negative result that is quite intuitive: if we only observe occupancy for a single hotel in the market, say hotel 0, then we can only identify the expected demand for hotel 0, but we cannot separately identify the choice probabilities for the other hotels, \( \pi_l(p, x), l = 1, \ldots, L \) and the probability of choosing the outside good, \( \pi_\emptyset(p, x) \), nor can we generally fully identify the distribution of arrivals, \( H(A|x) \). To see why, note that when we only observe occupancy at hotel 0, we can only identify the marginal distribution of occupancy at hotel 0, \( f(d_0|p, x) \), which is a mixture of binomials

\[
f(d_0|p, x) = \sum_A \binom{A}{d} \pi_0(p, x)^d [1 - \pi_0(p, x)]^{A-d} H(A|x). \tag{2.36}
\]

Note that since the right hand side only depends on \( H(A|x) \) and \( \pi_0(p, x) \), it will not be possible to identify the probabilities \( \{\pi_l(p, x)\}, l = 1, \ldots, L \) and \( \pi_\emptyset(p, x) \). Secondly, since the capacity of hotel 0, \( C_0 \), will generally be far smaller than the total capacity of the market as a whole, it is not plausible to assume that the maximal number of arrivals, \( |A|(x) < C_0 \), so in general we will not be able to infer \( |A|(x) \) from knowledge of \( f(d_0|p, x) \). In general, the upper tail of the arrival distribution will only be partially identified. Note if we knew the entire distribution \( H(A|x) \) we could adapt the proof of Theorem 0 to establish identification of \( \pi_0(p, x) \) from knowledge of \( f(d_0|p, x) \) and \( H(A|x) \). However if \( H \) is only partially identified, it is no longer even clear that it is possible to identify \( \pi_0(p, x) \). We record this as

**Theorem 3** If we only observe occupancy at a single hotel, the choice probabilities and arrival distribution are only partially identified. In general the only fully identified objects in this case are the conditional distribution \( f(d_0|p, x) \) and its expectation \( E\{d_0|p, x\} \).
2.4 Empirical Application to the Hotel Market

The identification analysis in the previous section shows that in principle the endogeneity problem can be solved and the deeper underlying structure of demand — consumer preferences and the distribution of arrivals — can be identified in a situation where we do not have instrumental variables and where face censoring and truncation problems. The analysis shows that demand for hotels can be consistently estimated under weak assumptions that relax the traditional optimality and equilibrium assumptions imposed in empirical work. However we emphasize the words in principle since the theoretical analysis of identification presumes we have access to an infinite amount of data and thus can non-parametrically estimate the conditional distribution of joint occupancy \( f(d|p, x) \) and the conditional distribution of prices, \( G(p|x) \).

However even though we have a unique set of data on both prices and occupancies of the hotels in this market, our data has limitations. As we noted in section 2, we only have 1737 daily observations on occupancies and ADRs for the hotels in our market. However, even conditioning on a single \((p, x)\) pair, the total number of elements in the support of \( f(d|p, x) \) (i.e. the total number of possible elements of the joint distribution of occupancy) is \( 2 \times 10^{17} \), so it is clearly hopeless to estimate the entire conditional distribution \( f(d|p, x) \) non-parametrically from the limited data we have at hand. Further, as we noted in section 2, we only have the aggregated occupancies and ADRs of hotel 0’s competitors, an not the individual occupancies for each competitor on a daily basis. Therefore we have chosen to treat hotel 0’s competitors as a single “aggregate competitor” and model the market as if it were a duopoly with only two competing hotels: hotel 0 and hotel c (where the latter is the aggregate of the 6 competitors to hotel 0). However even when we do this, so \( f(d|p, x) \)
is reduced to a two-dimensional distribution over the joint occupancy of hotel 0 and hotel c, there are still nearly 600,000 possible \((d_0, d_c)\) values in the support of this conditional distribution.

Thus, it is clear that to proceed with an empirical analysis, additional parametric functional form assumptions are required. Our empirical work will be based on two different estimation strategies:

1. **Regression**, to uncover the demand curve only, \(E\{d|p, x\}\)

2. **Maximum likelihood**, to estimate the full structure of the model (i.e. preferences and the distribution of arrivals) using the likelihood function

\[
f(d|p, x, \theta, \gamma) = \sum_A f(d|A, p, \theta) H_N(A|x, \gamma) \tag{2.37}
\]

where \(f(d|A, p, \theta)\) is a censored trinomial distribution with parameters \((A, \pi)\) where \(\pi(p)\) is a \(3 \times 1\) vector of trinomial logit probabilities specified below and \(H(A|x, \gamma)\) is a multinomial logit probability distribution over a fixed set \(\{A_1, \ldots, A_N\}\) of \(N\) arrival support points discussed below.

We used both parametric and semi-parametric approaches for both estimation strategies. For the regression analysis, we estimated standard linear regression models for demand as well as non-parametric regressions (local linear regressions). Of course semi-parametric regression approaches can be employed as well such as a partially linear specification

\[
d_0 = g(x) + \sum_{k=1}^{K_0} \theta_0^k p_0^k + \sum_{k=1}^{K_c} \theta_c^k p_c^k + \epsilon_0 \tag{2.38}
\]

where the dependence on \(x\) is captured by the non-parametric component \(g(x)\) but the dependence of demand on prices is captured by a flexible polynomial specification. Other specifications could be tried that allow for interaction effects between \(x\) and
prices \( p \), allowing demand to be more inelastic when occupancy rates are close to 1 compared to days where there is substantial excess capacity.

For maximum likelihood estimation, we adopt a semi-parametric estimation strategy where the parametric part of the model consists of four preference parameters \( \theta = (\alpha_0, \alpha_c, \beta, \beta_1) \) for a trinomial logit model of hotel choice, where the three choices are 1) hotel 0, 2) hotel c, and 3) the outside good. The probability of choosing hotel 0 is given by \( \pi(p) = (\pi_0(p), \pi_c(p), \pi_\varnothing(p)) \) where

\[
\pi_0(p) = \frac{\exp\{\alpha_0 - \beta p_0\}}{\exp\{-\beta_1 p_c\} + \exp\{\alpha_0 - \beta p_0\} + \exp\{\alpha_c - \beta p_c\}}
\]

(2.39)

and the probabilities \( \pi_c(p) \) and \( \pi_\varnothing(p) \) are defined similarly. In equation (2.39) we have made a standard identification normalization, fixing the intercept term for the outside good to zero, \( \alpha_\varnothing = 0 \). We do not have data on the price of the outside good (e.g. prices on hotels outside this market) but we assume that \( p_\varnothing \) is a fixed multiple of \( p_c \), and we embed this into the price coefficient \( \beta_1 \). Thus the key parameters determining the slope of the implied demand curve are \( (\beta, \beta_1) \) and if \( p_\varnothing = p_c \), then we expect that \( \beta = \beta_1 \), and that \( \beta_1 \) will be higher or lower than \( \beta \) to the extent that the unobserved price of the outside good is higher or lower than \( p_c \). Notice under this specification, consumer preferences are assumed to be independent of \( x \), so the only way that \( x \) affects demand is by shifting the distribution of arrivals, \( H(A|x) \). It is possible that the distribution of consumer types who arrive in the market depend on \( x \), and if this is the case the mixing distribution would depend on price, resulting in mixed choice probabilities \( \pi(p, x) \) that do depend on \( x \). Or \( x \) could directly affect preference parameters if the set of consumers who arrive on a day known to be quite busy expect to pay more and are less price sensitive compared to consumers who may strategically choose to book rooms on days that are less busy (lower \( x \)). Both of these possibilities are allowed via our general specification of choice probabilities \( \pi(p, x) \) in equation
(2.3). However due to the limited number of observations, in our empirical analysis we opted to make an “exclusion restriction” that \( x \) does not enter consumer choice probabilities and only enters as a demand shifter in the arrival distribution \( H(A|x) \).

We avoid making strong assumptions on the conditional distribution of arrivals, \( H(A|x) \), and instead treat it as an unknown infinite dimensional parameter that we estimate via semi-parametric maximum likelihood. Since we cannot directly estimate \( H(A|x) \) as an infinite dimensional parameter, we approximate it via the method of sieves and rely on the consistency and asymptotic normality results of Wong and Severini [34] to establish the asymptotic distribution of the key parameters of interest, \( \theta_1 \). We consider a sieve consisting of a sequence of families of conditional distributions \( H_N(A|x) \) with a finite support over \( N \) integers \( \{A_1, \ldots, A_N\} \) and we index \( N \) to the sample size \( T \) which we denote by \( N(T) \). We allow \( N(T) \to \infty \) at the right rate as a function of the sample size \( T \) to ensure that \( H_N(A|x) \) can consistently estimate any conditional density \( H(A|x) \). In our empirical work we use a flexible family of multinomial logit models that depend on a vector of parameters \( \gamma \) of dimension \( 2N - 1 \) that provides a fully flexible distribution over an increasingly fine grid of \( N \) support points \( \{A_1, \ldots, A_N\} \) given by

\[
H_N(A_i|x, \gamma) = \frac{\exp\{\gamma_{1,i} + \gamma_{2,i}x\}}{\sum_{j=1}^{N} \exp\{\gamma_{1,j} + \gamma_{2,j}x\}}
\]  

(2.40)

where we impose an identifying normalization that \( \gamma_{1,1} = 0 \). In the empirical results we report below, we used a total of \( N = 12 \) support points given by \( \{A_1, \ldots, A_{12}\} = \{500, 1000, \ldots, 6500\} \), so the upper bound on the number of arrivals, \( |A| = 6500 \), is over 3 times larger than the total hotel capacity in this market.

A key part of the model is constructing a good variable for \( x \) the demand shifter. Contrary to the identification analysis in section 3, we do not actually observe \( x \) in this market. As noted in the introduction, we used the expected market occupancy
rate (i.e. the expected total occupancy for all 7 hotels in this market divided by their total capacity) as our proxy for the demand shifter $x$. Hotels do have a good expectation of what the market level occupancy will be on different days, as well as the likely occupancy for their own hotel. As we noted in section 2, hotels consider predictable seasonal factors, weekly variations in occupancy, and holidays, as well as less predictable events such as whether larger conventions or events will be taking place in the city or unusual weather that is likely to predict unusually high or low occupancy rates.

Via a linear regression, we used the STR data to regress daily level market occupancy rates on market occupancy rates on the same day one year in the future (i.e. by adding 364 days to the current date). The $R^2$ of this regression is 66% so this “predicted occupancy” $\hat{x}$ provides a proxy for whatever demand information $x$ the hotels’ are actually using when it comes to their price setting decisions. Note that the actual $x$ that hotels use is likely to reflect more information than is contained in our crude proxy for it, $\hat{x}$. If $\hat{x}$ is too poor of a proxy for the actual $x$, this could violate our key conditional independence assumption 2. That is, if actual demand and hotel prices depend on a value of $x$ that we do not actually observe, then conditioning on a coarse proxy for $\hat{x}$ may not satisfy the conditional independence assumption 2 because there is information contained in the true latent $x$ that leads to a correlation between hotel demand and the prices the hotels set, and this correlation (via the true latent $x$) is not adequately controlled for and eliminated when we condition on a poor proxy for $x$, $\hat{x}$.

To address this potential problem we use latent variable methods. That is, we parameterize a relationship between our proxy for the demand shifter $\hat{x}$ and the true demand shifter $x$ via a conditional density $g(x|\hat{x}, \delta)$ where $\delta$ is a vector of additional parameters to be estimated. We used the following simple linear specification for
\[ g(x|\hat{x}, \delta): \]
\[ x = \delta_0 + \delta_1 \hat{x} + \epsilon \quad \epsilon \sim N(0, \delta_2^2) \]  

(2.41)

However since \( x \) is unobserved, it is clear that scale and location normalizations are required. In our analysis, we normalize \( \delta_0 = 0 \) and \( \delta_1 = 1 \) and treat \( \hat{x} \) as an unbiased but noisy proxy for the actual demand shifter \( x \) that hotels use. Under this interpretation, \( x = \hat{x} + \epsilon \) and \( \epsilon \) represents the additional information hotels receive about the likely market occupancy on a particular day, above and beyond the \textit{ex ante} regression prediction \( \hat{x} \), which is an unbiased predictor of likely occupancy by construction. Thus, hotels have an initial expectation of market occupancy rates \( \hat{x}_t \) based on past experience, but just prior to setting their prices (in a simultaneous move game) at the start of each day \( t \) they collectively observe other information \( \epsilon_t \) and affects their expectation of what the realized market occupancy will be that day. Then based on the total information consisting of \( x_t = \hat{x}_t + \epsilon_t \) and the idiosyncratic pricing shocks \( z_t \) the hotels set their prices, \( p(x_t, z_t) \). Then \( A_t \) customers arrive, modeled as a draw from \( H(A|x_t) \). Finally, given the observed prices each customer independently chooses their preferred hotel, or the outside good.

Using the implied normal distribution for \( g(x|\hat{x}, \delta) \) we can “integrate out” the unobserved latent \( x \) and obtain likelihoods in terms of our observable proxy \( \hat{x} \) while still continuing to posit that the original model with the true but unobserved demand shifter \( x \) satisfies the conditional independence assumption. In the case of the regression estimation, we posit a “reduce-form” relationship for the pricing strategies of the two hotels \( (p_0(x, z_0), p_c(x, z_c)) \) given by simple linear models with normally distributed error terms

\[ p_0(x, z_0) = \eta_{0,0} + \eta_{0,1} x + \eta_{0,2} x^2 + z_0 \quad z_0 \sim N(0, \eta_{0,3}^2) \]
\[ p_c(x, z_c) = \eta_{c,0} + \eta_{c,1} x + \eta_{c,2} x^2 + z_c \quad z_c \sim N(0, \eta_{c,3}^2). \]  

(2.42)
and we assume independence between \( z_0 \) and \( z_c \): \( E\{z_0 z_c\} = 0 \). Thus, hotels 0 and \( c \) set their prices each day after observing the demand shifter \( x \) but only observing their own respective pricing shocks, \( z_0 \) and \( z_c \), resulting in the realized prices given in equation (2.42). Given these prices, realized demand is given by

\[
\begin{align*}
\hat{d}_0 &= \theta_{0,0} + \theta_{0,1} x + \theta_{0,2} x^2 + \theta_{0,3} p_0 + \theta_{0,4} p_c + \epsilon_0 \quad \epsilon_0 \sim N(0, \theta_{0,5}^2) \\
\hat{d}_c &= \theta_{c,0} + \theta_{c,1} x + \theta_{c,2} x^2 + \theta_{c,3} p_0 + \theta_{c,4} p_c + \epsilon_c \quad \epsilon_c \sim N(0, \theta_{c,5}^2)
\end{align*}
\]

(2.43)

where we also assume the demand residuals are independently distributed, \( E\{\epsilon_0 \epsilon_c\} = 0 \), though this restriction can easily be relaxed.

To form the likelihood for the latent \( x \) case, we first condition on the true, unobserved \( x \) and apply the conditional independence assumptions, and then we integrate out over \( x \) using the conditional density \( g(x|\hat{x}, \delta) \) to get the following likelihood for a single observation \((d_0, d_c, p_0, p_c, \hat{x})\)

\[
L(d_0, d_c, p_0, p_c, \hat{x}, \theta_0, \theta_c, \eta_0, \eta_c, \delta) = \int_x \phi(d_0|x, p_0, p_c, \theta_0) \phi(d_c|x, p_0, p_c, \theta_c) \phi(p_0|x, \eta_0) \phi(p_c|x, \eta_c) \phi(x|\hat{x}, \delta) dx
\]

(2.44)

For the case of maximum likelihood estimation of the mixed trinomial model for \( f(d|p, x) \) we can also apply the latent variable approach. We use the same linear relationship between \( x \) and \( \hat{x} \) given in equation (2.41) above, but also incorporate the reduce-form pricing relations in equation (2.42) to obtain the following likelihood \( f(d|p, \hat{x}, \theta, \gamma, \delta, \eta_0, \eta_c) \) given by

\[
f(d|p, \hat{x}, \theta, \gamma, \delta, \eta_0, \eta_c) = \int_x f(d|p_0, p_c, x, \theta, \gamma) \phi(p_0|x, \eta_0) \phi(p_c|x, \eta_c) \phi(x|\hat{x}, \delta) dx
\]

(2.45)

where \( f(d|p_0, p_c, x, \theta, \gamma) \) is the mixed trinomial probability density given in equation (2.39). In our empirical results below, we compare the implied estimated demand curves under the scenario where we assume that \( x = \hat{x} \) (observed demand shifter)
with the assumption of a latent demand shifter given in equation (2.41). Given the reasoning above, if \( \hat{x} \) is not a sufficiently good proxy for the true \( x \), we will expect to obtain more steeply sloped estimated demand curves from the model where \( x \) is latent compared to the model where we assume \( x \) is fully observed.

### 2.4.1 Imposing Optimality and Equilibrium Restrictions

Note that the estimation strategy discussed above is both instrument-free and it is also free of any assumptions about optimizing or equilibrium behavior of the firms. It is possible to perform estimation subject to these additional restrictions and use likelihood ratio tests to assess the validity of the assumptions of optimality and equilibrium. For example to impose the restriction of optimal pricing of hotel 0, we replace the reduced-form equation for \( p_0(x, z_0) \) in (2.42) with the equation

\[
p_0(x, z_0) = p_0^*(x) + z_0 \sim N(0, \theta_{0,5})
\]

(2.46)

where \( p_0^*(x) \) is the optimal price for hotel 0 given in (2.14) except we use \( \phi(p_c|x, \theta_c) \), the normal density implied by the reduced-form pricing equation for hotel \( c \) in (2.42) as the conditional distribution \( G_c(p_c|x) \) when calculating \( p_0^*(x) \) in equation (2.14). In the case where we estimate the linear specification for demand in equation (2.43) we simply use the estimated linear demand curve in place of the conditional expectation \( E\{d_0|p_0, p_c, x\} \) given in equation (2.11).

To impose equilibrium constraints in addition to optimality constraints, we need to solve for the Bertrand-Nash equilibrium functions \( (p_0^*(x), p_c^*(x)) \) as per equation (2.15) and use these instead of the unrestricted reduced-form equations for \( (p_0, p_c) \) in equation (2.42). In the case where demand is linear, (2.43), we can derive analytic expressions for \( (p_0^*(x), p_c^*(x)) \). However in the case where expected demand is calculated from the mixed trinomial model (2.37) there are no analytic closed-form
expressions for \((p^*_0(x), p^*_c(x))\) and they must be calculated numerically. In this case full structural estimation requires the use of a nested fixed point algorithm (e.g. Rust [30]) or the MPEC algorithm that estimates the model subject to the equilibrium constraints (e.g Su and Judd [32]).

Notice that imposing optimality or equilibrium constraints, assuming these assumptions are correct, leads to more efficient estimators. This is because of cross equation restrictions on the parameters. For example, when optimality is imposed, we no longer need to estimate the additional coefficients \((\eta_{0,0}, \ldots, \eta_{0,2})\) in the reduced-form equation for \(p_0(x, z_0)\) (2.42). Instead, the conditional mean of \(p_0(x, z_0)\) is equal to \(p^*_0(x)\) which depends only on the structural coefficients \((\theta, \gamma)\) and also on \(\delta\) if we assume the demand shifter \(x\) is latent. However if these assumptions are incorrect, they will generally lead to biased, inconsistent estimates of demand. Essentially, these assumptions coerce the estimated demand curves to be sufficiently price-elastic to “rationalize” the observed pricing of the hotels in the market.

2.4.2 Results

In section 2.2 we showed there are systematic and predictable differences in hotel prices depending on the day of the week. During weekends the hotels have a much greater share of more price elastic leisure customers, whereas on weekdays there are relatively more price inelastic business and group customers. For example over the period of our sample (January 1, 2010 to October 31, 2013) the average share of business and group customers at hotel 0 is 47%. However on the prime weekdays (Monday, Tuesday and Wednesday) the share is 58%. This difference in composition is reflected in the ADRs: the average over all days is $192, but for the prime weekdays the average ADR is $206. Therefore, we opted to estimate our model for a subsample
of the prime weekdays only, giving us a total of 575 daily observations on occupancy and ADR between January 1, 2010 and October 31, 2013.

We begin with the regression analysis of demand for hotel 0, illustrating how controlling for $x$ enables us to estimate downward sloping demand curves, without instrumental variables or imposing any optimality or equilibrium restrictions. Table G.1 shows the coefficient estimates for the linear model of demand and reduced-form pricing strategies given in equations (2.42) and (2.43) above. Table G.1 provides the estimated coefficients for the demand for hotel 0, $d_0$, under different scenarios and restrictions. The first column shows what happens when we do not include the demand shifter $x$: here the failure to control for endogeneity leads to positive and significant coefficient estimates on both $p_0$ and $p_c$. The next column shows that when we control for $x$ (assuming that $x = \hat{x}$, the observed demand shifter case, that we now obtain negative and significant coefficient estimates for $\theta_{0,3}$, the coefficient of $p_0$ in the demand curve for $d_0$. The next column shows the estimated parameters in the case where $x$ is treated as a latent variable. We see that as we expected, the estimated value of $\theta_{0,3}$ is more negative and the estimated standard deviation of the unobserved “information shocks” that firms receive about market occupancy, $\epsilon$, the $\delta_2$ parameter in equation (2.41), is large and highly significant. The total share of the variance of the latent demand shifter $x$ accounted for by unobserved shocks $\epsilon$ is 32%. However there is still significant uncertainty about $ex post$ occupancy rates: the total variance of the latent demand shifter $x$ relative to the variance of $ex post$ total market occupancy rates is estimated to be 72%. We can think of this as akin to the “$R^2$” for this predictor of market occupancy, and thus, the latent demand shifter $x$ can predict 72% of the variance in $ex post$ market occupancy rates, whereas the $ex ante$ regression predictor $\hat{x}$ only explains 54% of this variation.
Thus, we conclude that when we control for demand shifters, we can obtain downward sloping demand curves, but it is important to have a good proxy for what the demand shifter $x$ really is. If we have a poor proxy, there will still be residual information $\epsilon$ that affects arrivals and firm prices, in violation of our conditional independence assumption 2. Thus a sufficiently poor demand shifter proxy, $\hat{x}$, may not be a sufficient good control variable for prices to satisfy the conditional exogeneity assumption. However we have shown that by using latent variable methods, the latent $x$ can capture and control for the unobserved information $\epsilon$, resulting in more significantly downward sloping demand curves.

2.5 Conclusion

We have introduced a simplified static model of demand and showed that the demand parameters are identifiable and estimable under fairly weak assumptions even in the presence of econometric problems of endogeneity and censoring — and even when we relax standard strong maintained assumptions of optimality and equilibrium that are commonly imposed to help identify structural models.
Unfortunately, the conclusions the reader can take away from this analysis are rather mixed and inconclusive. On the positive side, we have established a new global non-parametric identification result for a demand model that can be regarded as a “nested mixture model” with an “upper level” mixture over an unobserved number of consumers arriving to the market, and an “lower level” mixture of different consumer types with different degrees of price sensitivity.

On the negative side, we have shown via illustrative calculations of the information matrix for parametric versions of the demand model that the degree of information decreases rapidly as the number of unobserved types in the model increases. Essentially, increasing the number of unobserved consumer types results in near collinearity in the information matrix, and this results in an exponential blow up in the asymptotic variance of coefficients that characterize the distribution of random coefficients. Thus, contrary to the conclusion of Fox et al. [14] who view their theoretical proof of identification as “comforting to empirical researchers” p. 210 we think there is a big gap between the “practical” aspect of identification that empirical researchers confront when estimating their models and theoretical analyses of identification that abstract from most of the practical problems that empirical researchers face.

In our view, the most relevant practical indicator of the strength of identification is the asymptotic variance of an estimator. Identification problems will naturally manifest themselves in unreasonably large estimated standard errors for parameters, which in turn is symptomatic of an estimation criterion that is virtually “flat” in the parameters, at least along certain directions of the parameter space. Though abstract theoretical analyses of identification such as provided in this paper or in Fox et al. [14] can serve as useful points of departure, ultimately the burden is on the empirical researcher to show that their parameter estimates are the unique global optimizer of the statistical objective function (e.g. likelihood function). This is often very difficult,
if not impossible to do. In most complex nonlinear models, about the best a researcher
can do is attempt a thorough search of the parameter space and show there is no other
parameter values that result in comparable fit. Short of that, empirical researchers are
justified in claiming their models are identified if: a) they can succeed in estimating
them, and b) they can calculate the standard errors accurately and show that they
are not unreasonably large.
### A.1 Parameters of Consumer’s Preference

#### Table A.1: Estimates of choice parameters \((a_\tau, b_\tau)\) with optimality \((\hat{\theta}_1)\)

<table>
<thead>
<tr>
<th>Segment</th>
<th>Parameter</th>
<th>Calm (0-25%)</th>
<th>Normal (25-50%)</th>
<th>Busy (50-75%)</th>
<th>Busiest (75-100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leisure</td>
<td>(a_\tau)</td>
<td>-1.301 (0.018)</td>
<td>-1.547 (0.013)</td>
<td>-1.329 (0.015)</td>
<td>-2.298 (0.073)</td>
</tr>
<tr>
<td></td>
<td>(b_\tau)</td>
<td>-0.008 (4.9E-5)</td>
<td>-0.007 (4.8E-5)</td>
<td>-0.010 (1.6E-4)</td>
<td>-0.074 (0.002)</td>
</tr>
<tr>
<td>Weekday</td>
<td>Business</td>
<td>(a_\tau)</td>
<td>-1.621 (0.032)</td>
<td>-1.905 (0.016)</td>
<td>-1.047 (0.010)</td>
</tr>
<tr>
<td></td>
<td>(b_\tau)</td>
<td>-0.006 (9.6E-5)</td>
<td>-0.006 (4.4E-5)</td>
<td>-0.006 (9.5E-5)</td>
<td>-0.000 (0.003)</td>
</tr>
<tr>
<td></td>
<td>Group</td>
<td>(a_\tau)</td>
<td>-0.540 (0.026)</td>
<td>-0.959 (0.011)</td>
<td>-1.167 (0.019)</td>
</tr>
<tr>
<td></td>
<td>(b_\tau)</td>
<td>-0.012 (2.6E-4)</td>
<td>-0.011 (2.1E-4)</td>
<td>-0.012 (2.2E-4)</td>
<td>-0.009 (0.004)</td>
</tr>
<tr>
<td>Weekend</td>
<td>Leisure</td>
<td>(a_\tau)</td>
<td>-1.577 (0.041)</td>
<td>-1.801 (0.050)</td>
<td>-0.296 (0.077)</td>
</tr>
<tr>
<td></td>
<td>(b_\tau)</td>
<td>-0.008 (2.9E-4)</td>
<td>-0.009 (2.8E-4)</td>
<td>-0.035 (0.003)</td>
<td>-0.128 (0.017)</td>
</tr>
<tr>
<td></td>
<td>Business</td>
<td>(a_\tau)</td>
<td>-1.354 (0.031)</td>
<td>-1.259 (0.053)</td>
<td>-2.203 (0.135)</td>
</tr>
<tr>
<td></td>
<td>(b_\tau)</td>
<td>-0.007 (4.1E-4)</td>
<td>-0.007 (4.2E-4)</td>
<td>-0.007 (5.5E-4)</td>
<td>-0.036 (0.026)</td>
</tr>
<tr>
<td></td>
<td>Group</td>
<td>(a_\tau)</td>
<td>-0.859 (0.091)</td>
<td>-0.986 (0.124)</td>
<td>-0.002 (0.008)</td>
</tr>
<tr>
<td></td>
<td>(b_\tau)</td>
<td>-0.012 (6.4E-4)</td>
<td>-0.017 (0.001)</td>
<td>-0.015 (0.004)</td>
<td>-0.134 (0.029)</td>
</tr>
</tbody>
</table>

*Note*: standard errors in parentheses.
Table A.2: Estimates of choice parameters \((a_r, b_r)\) without optimality \((\hat{\theta}_2)\)

<table>
<thead>
<tr>
<th>Segment</th>
<th>Parameter</th>
<th>Calm (0-25%)</th>
<th>Normal (25-50%)</th>
<th>Busy (50-75%)</th>
<th>Busiest (75-100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leisure</td>
<td>(a_r)</td>
<td>-1.570 (2.121)</td>
<td>-1.532 (5.336)</td>
<td>-1.706 (6.012)</td>
<td>-2.128 (1.948)</td>
</tr>
<tr>
<td></td>
<td>(b_r)</td>
<td>-0.044 (0.048)</td>
<td>-0.041 (0.211)</td>
<td>-0.038 (0.172)</td>
<td>-0.055 (0.073)</td>
</tr>
<tr>
<td>Business</td>
<td>(a_r)</td>
<td>-3.325 (3.644)</td>
<td>-3.492 (5.609)</td>
<td>-2.883 (3.621)</td>
<td>-2.936 (7.528)</td>
</tr>
<tr>
<td></td>
<td>(b_r)</td>
<td>-0.008 (0.021)</td>
<td>-0.005 (0.016)</td>
<td>-0.011 (0.032)</td>
<td>-0.010 (0.037)</td>
</tr>
<tr>
<td>Group</td>
<td>(a_r)</td>
<td>-1.540 (6.474)</td>
<td>-1.684 (6.938)</td>
<td>-1.563 (9.146)</td>
<td>-1.422 (4.713)</td>
</tr>
<tr>
<td></td>
<td>(b_r)</td>
<td>-0.715 (1.244)</td>
<td>-0.948 (4.268)</td>
<td>-0.617 (2.865)</td>
<td>-0.659 (1.881)</td>
</tr>
<tr>
<td>Leisure</td>
<td>(a_r)</td>
<td>-2.265 (3.929)</td>
<td>-1.886 (8.588)</td>
<td>-1.841 (6.210)</td>
<td>-1.816 (11.500)</td>
</tr>
<tr>
<td></td>
<td>(b_r)</td>
<td>-0.061 (0.233)</td>
<td>-0.044 (0.187)</td>
<td>-0.030 (0.149)</td>
<td>-0.036 (0.476)</td>
</tr>
<tr>
<td>Business</td>
<td>(a_r)</td>
<td>-3.315 (8.107)</td>
<td>-2.842 (10.154)</td>
<td>-3.010 (15.220)</td>
<td>-3.565 (22.849)</td>
</tr>
<tr>
<td></td>
<td>(b_r)</td>
<td>-0.008 (0.062)</td>
<td>-0.008 (0.099)</td>
<td>-0.009 (0.090)</td>
<td>-0.009 (0.124)</td>
</tr>
<tr>
<td>Group</td>
<td>(a_r)</td>
<td>-1.115 (9.325)</td>
<td>-1.468 (21.063)</td>
<td>-1.408 (17.352)</td>
<td>-1.436 (17.414)</td>
</tr>
<tr>
<td></td>
<td>(b_r)</td>
<td>-0.664 (5.564)</td>
<td>-0.575 (4.462)</td>
<td>-0.471 (4.891)</td>
<td>-0.607 (7.385)</td>
</tr>
</tbody>
</table>

*Note:* Standard errors in parentheses.
A.2 Weekdays Demand

Table A.3: Estimates of zero-inflated negative binomial (ZINB) demand parameters $(\gamma, \phi, \mu)$ with optimality $(\hat{\theta}_1)$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Polynomial Coefficient</th>
<th>Calm (0-25%)</th>
<th>Normal (25-50%)</th>
<th>Busy (50-75%)</th>
<th>Busiest (75-100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_t$</td>
<td>$t^3$</td>
<td>-2.6E-5 (1.3E-6)</td>
<td>1.3E-5 (2.2E-7)</td>
<td>-5.9E-5 (2.6E-6)</td>
<td>-2.5E-5 (9.2E-7)</td>
</tr>
<tr>
<td></td>
<td>$t^2$</td>
<td>0.002 (3.1E-5)</td>
<td>0.001 (1.8E-5)</td>
<td>0.002 (5.2E-5)</td>
<td>0.002 (9.2E-5)</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>-0.111 (0.001)</td>
<td>-0.110 (0.001)</td>
<td>-0.093 (0.001)</td>
<td>-0.088 (0.003)</td>
</tr>
<tr>
<td>Binomial</td>
<td>1</td>
<td>3.685 (0.019)</td>
<td>3.641 (0.029)</td>
<td>3.397 (0.028)</td>
<td>3.370 (0.009)</td>
</tr>
<tr>
<td>$\phi_t$</td>
<td>$t^3$</td>
<td>-2.8E-5 (1.6E-6)</td>
<td>-4.4E-5 (2.2E-6)</td>
<td>-3.4E-5 (2.0E-5)</td>
<td>-2.6E-5 (1.1E-5)</td>
</tr>
<tr>
<td></td>
<td>$t^2$</td>
<td>0.001 (1.8E-5)</td>
<td>0.002 (6.5E-5)</td>
<td>0.003 (2.5E-4)</td>
<td>0.002 (3.2E-5)</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>-0.006 (0.001)</td>
<td>-0.108 (0.001)</td>
<td>-0.013 (0.007)</td>
<td>-0.038 (0.002)</td>
</tr>
<tr>
<td>Binomial</td>
<td>1</td>
<td>0.544 (0.010)</td>
<td>0.291 (0.009)</td>
<td>0.430 (0.027)</td>
<td>-0.003 (0.004)</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>$t^3$</td>
<td>3.9E-5 (1.0E-6)</td>
<td>2.6E-5 (8.4E-7)</td>
<td>7.3E-6 (1.7E-7)</td>
<td>1.8E-5 (1.5E-6)</td>
</tr>
<tr>
<td></td>
<td>$t^2$</td>
<td>-0.003 (3.3E-5)</td>
<td>-0.002 (4.6E-5)</td>
<td>-0.002 (1.6E-5)</td>
<td>-0.003 (9.6E-5)</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>0.048 (0.001)</td>
<td>0.029 (4.6E-4)</td>
<td>0.034 (3.1E-4)</td>
<td>0.036 (0.002)</td>
</tr>
<tr>
<td>Binomial</td>
<td>1</td>
<td>0.333 (0.006)</td>
<td>2.751 (0.031)</td>
<td>2.246 (0.015)</td>
<td>3.002 (0.100)</td>
</tr>
<tr>
<td>$\phi_t$</td>
<td>$t^3$</td>
<td>3.8E-5 (8.4E-7)</td>
<td>5.8E-6 (4.7E-7)</td>
<td>2.2E-5 (5.4E-7)</td>
<td>-2.7E-5 (2.6E-6)</td>
</tr>
<tr>
<td></td>
<td>$t^2$</td>
<td>-0.003 (4.3E-5)</td>
<td>-8.4E-4 (2.5E-5)</td>
<td>-1.8E-3 (1.9E-5)</td>
<td>1.2E-3 (8.3E-5)</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>0.012 (2.8E-4)</td>
<td>-0.008 (5.0E-4)</td>
<td>0.009 (3.1E-4)</td>
<td>-0.022 (0.001)</td>
</tr>
<tr>
<td>Binomial</td>
<td>1</td>
<td>-1.099 (0.036)</td>
<td>-0.675 (0.017)</td>
<td>-0.373 (0.009)</td>
<td>-0.646 (0.032)</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>$t^3$</td>
<td>1.4E-18 (1.1E-18)</td>
<td>9.8E-19 (2.9E-18)</td>
<td>3.6E-18 (5.2E-18)</td>
<td>1.3E-18 (1.9E-18)</td>
</tr>
<tr>
<td></td>
<td>$t^2$</td>
<td>-1.1E-16 (9.1E-17)</td>
<td>-2.1E-17 (5.2E-17)</td>
<td>-1.5E-17 (2.7E-17)</td>
<td>-4.3E-17 (6.0E-17)</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>1.3E-15 (1.0E-16)</td>
<td>8.9E-16 (2.9E-15)</td>
<td>9.0E-16 (1.6E-15)</td>
<td>7.4E-16 (8.6E-16)</td>
</tr>
<tr>
<td>Inflation</td>
<td>1</td>
<td>18.658 (13.742)</td>
<td>26.126 (3.586)</td>
<td>32.977 (60.610)</td>
<td>18.989 (3.657)</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>$t^3$</td>
<td>1.1E-18 (0.9E-19)</td>
<td>1.2E-18 (3.0E-18)</td>
<td>1.6E-18 (1.8E-18)</td>
<td>7.5E-19 (10.0E-19)</td>
</tr>
<tr>
<td></td>
<td>$t^2$</td>
<td>-8.0E-17 (0.5E-17)</td>
<td>-1.7E-17 (1.4E-16)</td>
<td>-6.7E-17 (1.4E-16)</td>
<td>-4.7E-17 (7.8E-17)</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>2.5E-15 (1.8E-15)</td>
<td>1.1E-15 (3.1E-15)</td>
<td>7.7E-16 (2.1E-15)</td>
<td>7.1E-16 (8.4E-16)</td>
</tr>
<tr>
<td>Inflation</td>
<td>1</td>
<td>21.357 (17.786)</td>
<td>25.392 (79.062)</td>
<td>15.274 (30.321)</td>
<td>20.246 (2.074)</td>
</tr>
</tbody>
</table>

Note: $t$ denotes number of days before occupancy. Standard errors in parentheses.
Table A.4: Estimates of zero-inated negative binomial (ZINB) demand parameters $(\gamma, \phi, \mu)$ without optimality ($\hat{\theta}_2$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Polynomial Coefficient</th>
<th>Calm (0-25%)</th>
<th>Normal (25-50%)</th>
<th>Busy (50-75%)</th>
<th>Busiest (75-100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_t$, Leisure</td>
<td>$t^3$</td>
<td>-2.6E-5 (4.1E-5)</td>
<td>1.1E-5 (5.0E-5)</td>
<td>-3.7E-5 (2.1E-4)</td>
<td>-2.9E-5 (8.2E-5)</td>
</tr>
<tr>
<td>Negative</td>
<td>$t^2$</td>
<td>0.002 (0.001)</td>
<td>0.001 (0.003)</td>
<td>0.002 (0.007)</td>
<td>0.002 (0.005)</td>
</tr>
<tr>
<td>Binomial</td>
<td>$t$</td>
<td>-0.121 (0.062)</td>
<td>-0.102 (0.057)</td>
<td>-0.100 (0.205)</td>
<td>-0.093 (0.109)</td>
</tr>
<tr>
<td>(Leisure)</td>
<td>1</td>
<td>2.584 (0.788)</td>
<td>2.655 (2.380)</td>
<td>2.276 (2.567)</td>
<td>3.297 (1.445)</td>
</tr>
<tr>
<td>$\phi_t$, Leisure</td>
<td>$t^3$</td>
<td>-2.3E-5 (7.2E-5)</td>
<td>-3.1E-5 (8.5E-5)</td>
<td>-3.0E-5 (1.7E-4)</td>
<td>-2.4E-5 (7.5E-5)</td>
</tr>
<tr>
<td>Negative</td>
<td>$t^2$</td>
<td>0.002 (0.003)</td>
<td>0.002 (0.011)</td>
<td>0.003 (0.013)</td>
<td>0.001 (0.005)</td>
</tr>
<tr>
<td>Binomial</td>
<td>$t$</td>
<td>-0.067 (0.127)</td>
<td>-0.084 (0.323)</td>
<td>-0.080 (0.382)</td>
<td>-0.049 (0.118)</td>
</tr>
<tr>
<td>(Leisure)</td>
<td>1</td>
<td>0.384 (0.479)</td>
<td>0.332 (1.425)</td>
<td>0.329 (2.131)</td>
<td>-0.059 (0.447)</td>
</tr>
<tr>
<td>$\mu_t$, Business</td>
<td>$t^3$</td>
<td>3.5E-5 (9.4E-5)</td>
<td>1.6E-5 (7.8E-5)</td>
<td>9.8E-6 (4.4E-5)</td>
<td>2.1E-5 (7.6E-5)</td>
</tr>
<tr>
<td>Negative</td>
<td>$t^2$</td>
<td>-0.003 (0.005)</td>
<td>-0.002 (0.005)</td>
<td>-0.002 (0.004)</td>
<td>-0.003 (0.012)</td>
</tr>
<tr>
<td>Binomial</td>
<td>$t$</td>
<td>0.070 (0.156)</td>
<td>0.040 (0.211)</td>
<td>0.028 (0.072)</td>
<td>0.032 (0.119)</td>
</tr>
<tr>
<td>(Business)</td>
<td>1</td>
<td>2.243 (2.661)</td>
<td>2.745 (2.785)</td>
<td>3.225 (2.234)</td>
<td>2.013 (5.333)</td>
</tr>
<tr>
<td>$\phi_t$, Business</td>
<td>$t^3$</td>
<td>2.9E-5 (5.2E-5)</td>
<td>5.9E-6 (2.8E-5)</td>
<td>1.6E-5 (8.0E-5)</td>
<td>-2.5E-5 (9.4E-5)</td>
</tr>
<tr>
<td>Negative</td>
<td>$t^2$</td>
<td>-0.003 (0.003)</td>
<td>-0.001 (0.004)</td>
<td>-0.001 (0.004)</td>
<td>0.001 (0.009)</td>
</tr>
<tr>
<td>Binomial</td>
<td>$t$</td>
<td>0.014 (0.029)</td>
<td>-0.006 (0.021)</td>
<td>0.010 (0.074)</td>
<td>-0.023 (0.130)</td>
</tr>
<tr>
<td>(Business)</td>
<td>1</td>
<td>-1.219 (1.841)</td>
<td>-0.748 (2.774)</td>
<td>-0.523 (2.600)</td>
<td>-0.583 (2.050)</td>
</tr>
<tr>
<td>$\gamma_t$, Leisure</td>
<td>$t^3$</td>
<td>1.3E-18 (2.1E-18)</td>
<td>1.5E-18 (1.0E-17)</td>
<td>1.3E-18 (7.2E-18)</td>
<td>1.4E-18 (6.3E-18)</td>
</tr>
<tr>
<td>Zero</td>
<td>$t^2$</td>
<td>-1.1E-15 (3.5E-15)</td>
<td>-1.0E-16 (7.1E-16)</td>
<td>-1.0E-16 (5.9E-16)</td>
<td>-9.9E-17 (4.6E-16)</td>
</tr>
<tr>
<td>Inflation</td>
<td>$t$</td>
<td>1.6E-15 (3.1E-15)</td>
<td>2.1E-15 (1.0E-14)</td>
<td>2.1E-15 (1.0E-14)</td>
<td>2.0E-15 (1.0E-14)</td>
</tr>
<tr>
<td>(Leisure)</td>
<td>1</td>
<td>25.900 (61.871)</td>
<td>22.684 (144.392)</td>
<td>20.187 (112.676)</td>
<td>21.385 (75.003)</td>
</tr>
<tr>
<td>$\gamma_t$, Business</td>
<td>$t^3$</td>
<td>6.3E-19 (1.5E-18)</td>
<td>1.2E-18 (5.9E-18)</td>
<td>1.3E-18 (7.3E-18)</td>
<td>1.3E-18 (5.2E-18)</td>
</tr>
<tr>
<td>Zero</td>
<td>$t^2$</td>
<td>-2.2E-15 (5.7E-15)</td>
<td>1.9E-15 (1.2E-14)</td>
<td>1.9E-15 (1.0E-14)</td>
<td>1.9E-15 (8.2E-15)</td>
</tr>
<tr>
<td>Inflation</td>
<td>$t$</td>
<td>2.2E-15 (6.7E-15)</td>
<td>1.9E-15 (1.2E-14)</td>
<td>1.9E-15 (1.0E-14)</td>
<td>1.9E-15 (8.2E-15)</td>
</tr>
<tr>
<td>(Business)</td>
<td>1</td>
<td>22.991 (63.709)</td>
<td>17.720 (88.856)</td>
<td>21.002 (189.194)</td>
<td>21.877 (102.221)</td>
</tr>
</tbody>
</table>

*Note: t denotes number of days before occupancy. Standard errors in parentheses.*
### A.3 Weekdays Group Arrival and Cancellation

#### Table A.5: Estimates of group demand and cancellation with optimality ($\hat{\theta}_1$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Polynomial Coefficient</th>
<th>Calm (0-25%)</th>
<th>Normal (25-50%)</th>
<th>Busy (50-75%)</th>
<th>Busiest (75-100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_t$ Zero</td>
<td>$t^3$</td>
<td>-2.3E-5 (3.4E-6)</td>
<td>-1.2E-4 (7.6E-6)</td>
<td>-1.1E-4 (1.8E-5)</td>
<td>-1.1E-4 (9.8E-6)</td>
</tr>
<tr>
<td>Inflation (Group)</td>
<td>$t^2$</td>
<td>0.003 (1.6E-4)</td>
<td>0.006 (9.2E-5)</td>
<td>0.008 (6.3E-4)</td>
<td>0.007 (4.5E-4)</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>-0.019 (0.002)</td>
<td>-0.054 (0.002)</td>
<td>-0.027 (0.002)</td>
<td>-0.080 (0.012)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-0.977 (0.038)</td>
<td>0.401 (0.030)</td>
<td>0.636 (0.066)</td>
<td>0.280 (0.072)</td>
</tr>
<tr>
<td>Mean Arrivals</td>
<td>$t^3$</td>
<td>-4.6E-5 (2.6E-6)</td>
<td>4.7E-6 (1.1E-5)</td>
<td>-2.0E-5 (3.3E-5)</td>
<td>-1.8E-5 (2.2E-5)</td>
</tr>
<tr>
<td>(if arrival &gt;0)</td>
<td>$t^2$</td>
<td>0.003 (8.8E-5)</td>
<td>-0.004 (4.6E-5)</td>
<td>5.0E-4 (8.3E-4)</td>
<td>-7.3E-4 (6.3E-5)</td>
</tr>
<tr>
<td>(Group)</td>
<td>$t$</td>
<td>0.058 (0.002)</td>
<td>0.151 (0.002)</td>
<td>0.076 (7.8E-4)</td>
<td>0.099 (0.002)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.45 (0.019)</td>
<td>0.304 (0.088)</td>
<td>0.702 (0.017)</td>
<td>0.440 (0.022)</td>
</tr>
<tr>
<td>Probability of cancel &gt;0</td>
<td>$t^3$</td>
<td>2.4E-5 (2.6E-6)</td>
<td>3.8E-5 (1.1E-6)</td>
<td>4.7E-5 (2.1E-6)</td>
<td>6.6E-5 (1.2E-5)</td>
</tr>
<tr>
<td></td>
<td>$t^2$</td>
<td>-0.003 (2.3E-4)</td>
<td>-0.004 (7.0E-5)</td>
<td>-0.004 (9.6E-5)</td>
<td>-0.007 (0.001)</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>0.058 (0.002)</td>
<td>0.151 (0.002)</td>
<td>0.175 (0.004)</td>
<td>0.253 (0.028)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.45 (0.019)</td>
<td>0.304 (0.088)</td>
<td>0.702 (0.017)</td>
<td>0.440 (0.022)</td>
</tr>
<tr>
<td>Cancellation Rate</td>
<td>$t^3$</td>
<td>3.9E-5 (1.3E-6)</td>
<td>1.7E-5 (4.7E-5)</td>
<td>-2.1E-5 (2.1E-6)</td>
<td>1.1E-5 (4.2E-6)</td>
</tr>
<tr>
<td>(if cancel &gt;0)</td>
<td>$t^2$</td>
<td>-0.003 (5.1E-5)</td>
<td>-0.004 (1.2E-5)</td>
<td>4.1E-4 (4.2E-5)</td>
<td>-0.003 (3.7E-4)</td>
</tr>
<tr>
<td>(Group)</td>
<td>$t$</td>
<td>0.061 (0.002)</td>
<td>0.061 (0.001)</td>
<td>0.025 (0.001)</td>
<td>0.061 (0.009)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.06 (0.020)</td>
<td>3.06 (0.017)</td>
<td>3.43 (0.018)</td>
<td>4.63 (0.121)</td>
</tr>
</tbody>
</table>

Note: $t$ denotes number of days before occupancy. Standard errors in parentheses.

#### Table A.6: Estimates of group demand and cancellation without optimality ($\hat{\theta}_2$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Polynomial Coefficient</th>
<th>Calm (0-25%)</th>
<th>Normal (25-50%)</th>
<th>Busy (50-75%)</th>
<th>Busiest (75-100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_t$ Zero</td>
<td>$t^3$</td>
<td>-5.3E-5 (2.0E-5)</td>
<td>-8.6E-5 (3.7E-5)</td>
<td>-7.7E-5 (3.4E-5)</td>
<td>-9.7E-5 (1.9E-4)</td>
</tr>
<tr>
<td>Inflation (Group)</td>
<td>$t^2$</td>
<td>0.002 (0.001)</td>
<td>0.005 (0.003)</td>
<td>0.004 (0.002)</td>
<td>0.006 (0.015)</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>-0.023 (0.024)</td>
<td>-0.080 (0.073)</td>
<td>-0.037 (0.064)</td>
<td>-0.097 (0.382)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-1.387 (0.119)</td>
<td>-0.804 (0.53)</td>
<td>-0.907 (0.41)</td>
<td>-0.79 (3.367)</td>
</tr>
<tr>
<td>Mean Arrivals</td>
<td>$t^3$</td>
<td>-4.6E-5 (5.0E-6)</td>
<td>6.7E-6 (4.0E-5)</td>
<td>-2.1E-5 (1.9E-6)</td>
<td>2.1E-5 (8.3E-6)</td>
</tr>
<tr>
<td>(if arrival &gt;0)</td>
<td>$t^2$</td>
<td>0.002 (1.1E-4)</td>
<td>-0.004 (1.1E-4)</td>
<td>5.0E-4 (7.4E-7)</td>
<td>-1.3E-4 (2.1E-4)</td>
</tr>
<tr>
<td>(Group)</td>
<td>$t$</td>
<td>0.050 (0.004)</td>
<td>0.182 (0.006)</td>
<td>0.096 (0.003)</td>
<td>0.096 (0.006)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.447 (0.028)</td>
<td>0.380 (2.231)</td>
<td>0.834 (0.051)</td>
<td>0.496 (4.990)</td>
</tr>
<tr>
<td>Probability of cancel &gt;0</td>
<td>$t^3$</td>
<td>1.1E-5 (2.1E-5)</td>
<td>3.5E-5 (3.0E-5)</td>
<td>4.7E-5 (7.2E-5)</td>
<td>7.3E-5 (2.7E-4)</td>
</tr>
<tr>
<td></td>
<td>$t^2$</td>
<td>-0.003 (0.001)</td>
<td>-0.004 (0.002)</td>
<td>-0.004 (0.004)</td>
<td>-0.007 (0.023)</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>0.147 (0.024)</td>
<td>0.174 (0.030)</td>
<td>0.186 (0.081)</td>
<td>0.246 (0.402)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-1.182 (0.129)</td>
<td>-1.75 (0.189)</td>
<td>-2.052 (0.406)</td>
<td>-2.28 (2.102)</td>
</tr>
<tr>
<td>Cancellation Rate</td>
<td>$t^3$</td>
<td>2.3E-5 (3.9E-6)</td>
<td>4.8E-5 (5.7E-5)</td>
<td>3.9E-5 (6.8E-5)</td>
<td>1.9E-5 (1.8E-5)</td>
</tr>
<tr>
<td></td>
<td>$t^2$</td>
<td>-0.003 (3.1E-4)</td>
<td>-0.003 (0.003)</td>
<td>-0.004 (0.003)</td>
<td>-0.002 (0.003)</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>0.063 (0.012)</td>
<td>-0.012 (0.015)</td>
<td>0.073 (0.079)</td>
<td>0.055 (0.066)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4.280 (0.351)</td>
<td>4.632 (0.009)</td>
<td>4.709 (0.063)</td>
<td>5.25 (1.012)</td>
</tr>
</tbody>
</table>

Note: $t$ denotes number of days before occupancy. Standard errors in parentheses.
### A.4 Weekends Demand

Table A.7: Estimates of zero-inated negative binomial (ZINB) demand parameters ($\gamma, \phi, \mu$) with optimality ($\hat{\theta}_1$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Polynomial Coefficient</th>
<th>Calm (0-25%)</th>
<th>Normal (25-50%)</th>
<th>Busy (50-75%)</th>
<th>Busiest (75-100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_t$</td>
<td>$t^3$</td>
<td>-3.1E-5 (1.5E-5)</td>
<td>-2.1E-5 (1.7E-5)</td>
<td>-2.3E-5 (3.3E-5)</td>
<td>-4.3E-5 (3.3E-4)</td>
</tr>
<tr>
<td>Negative</td>
<td>$t^2$</td>
<td>2.3E-3 (2.7E-4)</td>
<td>0.002 (5.3E-4)</td>
<td>0.002 (0.005)</td>
<td>2.3E-3 (3.6E-3)</td>
</tr>
<tr>
<td>Binomial</td>
<td>$t$</td>
<td>-0.000 (0.022)</td>
<td>-1.017 (0.044)</td>
<td>-0.110 (0.098)</td>
<td>-0.146 (0.479)</td>
</tr>
<tr>
<td>Leisure</td>
<td>1</td>
<td>3.144 (0.547)</td>
<td>4.094 (0.926)</td>
<td>2.364 (2.501)</td>
<td>3.832 (6.629)</td>
</tr>
<tr>
<td>$\phi_t$</td>
<td>$t^3$</td>
<td>1.0E-5 (7.6E-6)</td>
<td>-3.4E-7 (1.2E-6)</td>
<td>-2.9E-5 (5.8E-5)</td>
<td>2.3E-5 (2.5E-4)</td>
</tr>
<tr>
<td>Negative</td>
<td>$t^2$</td>
<td>-4.4E-4 (2.9E-4)</td>
<td>3.5E-4 (1.8E-4)</td>
<td>2.8E-3 (3.1E-3)</td>
<td>-1.7E-3 (7.5E-3)</td>
</tr>
<tr>
<td>Binomial</td>
<td>$t$</td>
<td>-0.032 (0.029)</td>
<td>-0.010 (0.022)</td>
<td>-0.108 (0.086)</td>
<td>0.006 (0.020)</td>
</tr>
<tr>
<td>Leisure</td>
<td>1</td>
<td>0.666 (0.352)</td>
<td>0.009 (0.119)</td>
<td>0.739 (1.374)</td>
<td>0.191 (0.397)</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>$t^3$</td>
<td>2.6E-5 (1.3E-5)</td>
<td>3.5E-5 (7.6E-6)</td>
<td>2.7E-5 (3.5E-5)</td>
<td>8.7E-6 (6.6E-5)</td>
</tr>
<tr>
<td>Negative</td>
<td>$t^2$</td>
<td>-0.002 (3.4E-4)</td>
<td>-0.003 (0.001)</td>
<td>-0.002 (0.003)</td>
<td>-1.2E-3 (0.010)</td>
</tr>
<tr>
<td>Binomial</td>
<td>$t$</td>
<td>0.022 (0.003)</td>
<td>0.071 (0.003)</td>
<td>0.046 (0.020)</td>
<td>0.010 (0.190)</td>
</tr>
<tr>
<td>Business</td>
<td>1</td>
<td>2.021 (0.899)</td>
<td>1.361 (0.484)</td>
<td>2.313 (3.803)</td>
<td>1.661 (9.124)</td>
</tr>
<tr>
<td>$\phi_t$</td>
<td>$t^3$</td>
<td>2.1E-5 (9.2E-6)</td>
<td>2.2E-5 (6.0E-6)</td>
<td>-1.7E-6 (4.2E-6)</td>
<td>-6.2E-6 (7.7E-5)</td>
</tr>
<tr>
<td>Negative</td>
<td>$t^2$</td>
<td>-1.3E-3 (3.8E-4)</td>
<td>-1.4E-3 (1.1E-4)</td>
<td>2E-4 (4E-4)</td>
<td>2.8E-4 (1.8E-3)</td>
</tr>
<tr>
<td>Binomial</td>
<td>$t$</td>
<td>-0.009 (0.003)</td>
<td>-0.008 (0.004)</td>
<td>-0.037 (0.038)</td>
<td>-0.029 (0.116)</td>
</tr>
<tr>
<td>Business</td>
<td>1</td>
<td>-1.487 (0.408)</td>
<td>-0.871 (0.460)</td>
<td>-1.002 (2.436)</td>
<td>-0.744 (3.493)</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>$t^3$</td>
<td>7.9E-19 (1.1E-18)</td>
<td>6.9E-19 (2.6E-18)</td>
<td>9.1E-19 (6.6E-18)</td>
<td>8.1E-19 (1.5E-17)</td>
</tr>
<tr>
<td>Zero</td>
<td>$t^2$</td>
<td>-1.4E-17 (1.5E-16)</td>
<td>-1.4E-17 (6.3E-17)</td>
<td>-1.7E-17 (1.1E-16)</td>
<td>-5.1E-17 (8.5E-16)</td>
</tr>
<tr>
<td>Inflation</td>
<td>$t$</td>
<td>1.1E-15 (3.9E-15)</td>
<td>1.0E-15 (1.4E-15)</td>
<td>9.7E-16 (1.1E-15)</td>
<td>7.8E-16 (1.7E-13)</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>$t^3$</td>
<td>8.9E-19 (2.1E-18)</td>
<td>1.2E-18 (3.3E-18)</td>
<td>6.9E-19 (2.4E-18)</td>
<td>8.0E-19 (1.3E-16)</td>
</tr>
<tr>
<td>Zero</td>
<td>$t^2$</td>
<td>-5.3E-17 (1.2E-16)</td>
<td>-5.3E-17 (1.2E-16)</td>
<td>-3.3E-17 (8.0E-17)</td>
<td>-5.7E-17 (1.2E-14)</td>
</tr>
<tr>
<td>Inflation</td>
<td>$t$</td>
<td>7.5E-16 (8.5E-16)</td>
<td>9.1E-16 (6.9E-16)</td>
<td>7.5E-16 (1.9E-15)</td>
<td>7.9E-16 (7.7E-14)</td>
</tr>
<tr>
<td>Business</td>
<td>1</td>
<td>23.215 (89.462)</td>
<td>16.588 (26.847)</td>
<td>25.180 (44.460)</td>
<td>21.073 (403.918)</td>
</tr>
</tbody>
</table>

*Note: t denotes number of days before occupancy. Standard errors in parentheses.*
Table A.8: Estimates of zero-inated negative binomial (ZINB) demand parameters ($\gamma, \phi, \mu$) without optimality ($\hat{\theta}_2$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Polynomial Coefficient</th>
<th>Calm (0-25%)</th>
<th>Normal (25-50%)</th>
<th>Busy (50-75%)</th>
<th>Busiest (75-100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_t$</td>
<td>$t^3$</td>
<td>-3.5E-5  (2.0E-4)</td>
<td>-2.0E-5  (2.0E-4)</td>
<td>-2.3E-5  (2.1E-4)</td>
<td>-4.1E-4  (3.9E-4)</td>
</tr>
<tr>
<td>Negative</td>
<td>$t^2$</td>
<td>0.002 (0.009)</td>
<td>0.002 (0.010)</td>
<td>0.002 (0.008)</td>
<td>0.003 (0.025)</td>
</tr>
<tr>
<td>Binomial</td>
<td>$t$</td>
<td>-0.118 (0.142)</td>
<td>-0.091 (0.379)</td>
<td>-0.086 (0.279)</td>
<td>-0.121 (0.542)</td>
</tr>
<tr>
<td>Leisure</td>
<td>1</td>
<td>3.400 (3.332)</td>
<td>3.125 (7.640)</td>
<td>3.323 (5.218)</td>
<td>3.373 (6.901)</td>
</tr>
</tbody>
</table>

| $\phi_t$        | $t^3$                  | 9.0E-6  (8.9E-5) | -2.4E-7 (2.6E-6) | -4.6E-5 (5.3E-4) | 2.4E-5 (2.9E-4) |
| Negative        | $t^2$                  | -4.3E-4 (0.003) | -2.5E-4 (0.003) | 0.003 (0.028)  | -0.002 (0.023)  |
| Binomial        | $t$                    | -0.020 (0.126) | -0.017 (0.329) | -0.086 (0.175) | 0.006 (0.009)   |
| Leisure         | 1                      | 0.834  (7.932) | 0.569 (5.18)  | 0.759 (8.904)  | 0.303 (3.834)   |

| $\gamma_t$      | $t^3$                  | 2.7E-5  (2.0E-4) | 4.4E-5  (2.9E-4) | 1.9E-5  (1.4E-4) | 8.7E-6  (1.9E-4) |
| Negative        | $t^2$                  | -3.5E-4 (0.003) | -0.003 (0.018) | -0.002 (0.016) | -0.001 (0.016)  |
| Binomial        | $t$                    | 0.021 (0.143)  | 0.009 (0.920)  | 0.052 (0.320)  | 0.010 (0.172)   |
| Business        | 1                      | 1.814  (5.388) | 2.500 (13.338) | 2.170 (15.695) | 2.303 (10.099)  |

| $\gamma_t$      | $t^3$                  | 1.7E-5  (1.0E-4) | 3.4E-5  (2.1E-4) | -1.3E-6 (1.5E-5) | -5.8E-6 (1.1E-4) |
| Negative        | $t^2$                  | -0.001 (0.006)  | -0.002 (0.06)  | 2.4E-4 (0.003) | 2.8E-4 (0.005)  |
| Binomial        | $t$                    | -0.006 (0.006) | -0.010 (0.006) | -0.025 (0.003) | -0.026 (0.005)  |
| Business        | 1                      | -1.053 (0.06)  | -0.836 (0.104) | -0.798 (0.341) | -0.692 (0.458)  |

| $\gamma_t$      | $t^3$                  | 2.3E-18 (1.5E-17) | 1.3E-18 (1.6E-17) | 1.3E-18 (1.7E-17) | 1.1E-18 (1.9E-17) |
| Negative        | $t^2$                  | -9.2E-17 (1.4E-15) | -1.0E-16 (1.8E-15) | -1.2E-16 (1.4E-15) | -1.4E-16 (2.9E-15) |
| Inflation       | $t$                    | 1.9E-15 (1.6E-14) | 2.3E-15 (2.7E-14) | 1.9E-15 (1.9E-14) | 1.8E-15 (2.9E-14) |
| Leisure         | 1                      | 19.651 (180.93)  | 18.494 (187.13) | 19.611 (243.59) | 20.313 (403.33) |

| $\gamma_t$      | $t^3$                  | 1.4E-18 (1.3E-17) | 1.3E-18 (1.2E-17) | 1.2E-18 (1.4E-17) | 1.2E-18 (2.2E-17) |
| Negative        | $t^2$                  | -9.5E-16 (1.8E-16) | -1.1E-16 (1.8E-16) | -1.1E-16 (1.8E-16) | -1.1E-16 (1.4E-15) |
| Inflation       | $t$                    | 2.0E-15 (2.9E-14) | 2.3E-15 (1.9E-14) | 1.9E-15 (1.9E-14) | 1.9E-15 (3.9E-14) |

*Note*: $t$ denotes number of days before occupancy. Standard errors in parentheses.
### Table A.9: Estimates of group demand and cancellation with optimality ($\hat{\theta}_1$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Polynomial Coefficient</th>
<th>Calm (0-25%)</th>
<th>Normal (25-50%)</th>
<th>Busy (50-75%)</th>
<th>Busiest (75-100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_t$ Zero</td>
<td>$t^3$</td>
<td>-2.9E-5 (5.3E-5)</td>
<td>-1.3E-4 (4.1E-5)</td>
<td>1.1E-4 (5.8E-5)</td>
<td>-1.0E-4 (1.9E-4)</td>
</tr>
<tr>
<td>Inflation Group</td>
<td>$t$</td>
<td>0.007 (0.001)</td>
<td>0.007 (0.004)</td>
<td>0.007 (0.004)</td>
<td>0.007 (0.002)</td>
</tr>
<tr>
<td>Mean Arrivals (if arrival &gt;0) Group</td>
<td>$t^3$</td>
<td>-8.6E-5 (1.1E-4)</td>
<td>-1.2E-4 (2.1E-4)</td>
<td>-9.7E-5 (1.9E-4)</td>
<td>-1.2E-4 (1.4E-4)</td>
</tr>
<tr>
<td>Probability of cancel &gt;0 Group</td>
<td>$t$</td>
<td>0.005 (0.008)</td>
<td>0.008 (0.018)</td>
<td>0.007 (0.015)</td>
<td>0.007 (0.010)</td>
</tr>
<tr>
<td>Cancellation Rate (if cancel &gt;0) Group</td>
<td>$t$</td>
<td>-1.4E-5 (9.8E-4)</td>
<td>-1.0E-4 (2.1E-4)</td>
<td>-7.3E-5 (1.9E-4)</td>
<td>-8.4E-5 (4.2E-4)</td>
</tr>
</tbody>
</table>

Note: $t$ denotes number of days before occupancy. Standard errors in parentheses.

### Table A.10: Estimates of group demand and cancellation without optimality ($\hat{\theta}_2$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Polynomial Coefficient</th>
<th>Calm (0-25%)</th>
<th>Normal (25-50%)</th>
<th>Busy (50-75%)</th>
<th>Busiest (75-100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_t$ Zero</td>
<td>$t^3$</td>
<td>-3.8E-5 (1.9E-4)</td>
<td>-1.7E-5 (1.5E-5)</td>
<td>-1.7E-5 (1.4E-5)</td>
<td>-1.7E-5 (1.4E-5)</td>
</tr>
<tr>
<td>Inflation Group</td>
<td>$t$</td>
<td>0.007 (0.001)</td>
<td>0.007 (0.004)</td>
<td>0.007 (0.004)</td>
<td>0.007 (0.002)</td>
</tr>
<tr>
<td>Mean Arrivals (if arrival &gt;0) Group</td>
<td>$t^3$</td>
<td>-7.9E-5 (1.3E-4)</td>
<td>-6.9E-5 (0.9E-3)</td>
<td>-6.9E-5 (0.9E-3)</td>
<td>-6.9E-5 (0.9E-3)</td>
</tr>
<tr>
<td>Probability of cancel &gt;0 Group</td>
<td>$t$</td>
<td>0.005 (0.008)</td>
<td>0.008 (0.018)</td>
<td>0.007 (0.015)</td>
<td>0.007 (0.010)</td>
</tr>
<tr>
<td>Cancellation Rate (if cancel &gt;0) Group</td>
<td>$t$</td>
<td>-1.4E-5 (9.8E-4)</td>
<td>-1.0E-4 (2.1E-4)</td>
<td>-7.3E-5 (1.9E-4)</td>
<td>-8.4E-5 (4.2E-4)</td>
</tr>
</tbody>
</table>

Note: $t$ denotes number of days before occupancy. Standard errors in parentheses.
### The Law of Competitors’ Average Price

Table B.1: Regression result: Law of competitors’ price, $\rho_{t-1}$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_t$</td>
<td>0.846***</td>
<td>0.864***</td>
<td>0.848***</td>
<td>0.860***</td>
<td>0.844***</td>
<td>0.865***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$p_t$</td>
<td>0.125***</td>
<td>0.125***</td>
<td>0.121***</td>
<td>0.112***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta p_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.100***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>$p_{t-1}$</td>
<td>0.112***</td>
<td>0.110***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_t$</td>
<td>-0.00109</td>
<td>-0.00124</td>
<td>0.0128*</td>
<td>0.00523</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(-0.005)</td>
<td>(-0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>15.22***</td>
<td>13.15***</td>
<td>14.79***</td>
<td>12.42***</td>
<td>8.58</td>
<td>4.28</td>
</tr>
<tr>
<td></td>
<td>(-0.736)</td>
<td>(-0.715)</td>
<td>(-0.876)</td>
<td>(-0.843)</td>
<td>(-35.790)</td>
<td>(-32.923)</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. Significance: *** 0.1%, **1%, *5%.
This table reflects the feedback effect of hotel 0’s price ($p(t)$) on the competitors’ price ($\rho_{t-1}$) referred in Equation (1.3). Seen in the first column, $\rho_{t-1}$ is affected by both $\rho_t$ and $p_t$ where the effect of $\rho_t$ is 84.6% while the effect of $p_t$ is 12.5%. This result explains the relations pretty well with an adjusted R-sq of 0.90. Other columns also show the potential models of feedback effect on competitors’ price, but since the first column already explains the feedback effect well, I stick with the first column model in this paper. This model is used in the law of competitors’ price and it is important to find the optimal price rule using the DP framework since $\rho_{t-1}$ is an important state variable which I must forecast for the price rule.
APPENDICES C

IRRATIONAL PHENOMENON CAUSED BY COMPETITORS’ FEEDBACK EFFECT
Note: When changing hotel 0’s price (BAR), shown in the horizontal axis, the expected value (vertical axis) changes as well. The top two panels illustrate the highest expected value at a price level hovering $200 and drops precipitously until it reaches approximately $350. However, each panel displays an increase in expected value when the price increases with the top left panel growing at a slower rate than the top right panel. The bottom two panels show similar patterns, but the bottom left has the highest expected value at $200 and $1,000, while the bottom right actually has the highest expected value at $1,000. This is the irrational phenomenon caused by competitors’ feedback effect. This leads me to believe that there is no feedback effect where \( p_{t-1} \) is equal to or larger than \( \rho_{t-1} \). Without this assumption, regardless of what hotel 0 sets its price at, competitors are expected to follow that price. Therefore, as hotel 0 increases its price, other hotels’ also increase their price, resulting in a relatively high price for all hotels in the long run. This is shown in the bottom right panel which represents 8 days before arrival day. Finally, all hotels have a relatively high price, which leads to a high mark-up for hotel 0 and there is high incentive for hotel 0 to set the price as high as $1,000. However, this is difficult to observe, so I limit the feedback effect on price.

Figure C.1: Value function by various given occupancy is 0 and \( \rho \) is $300
Appendices D

Evidence of Strategic Cancellation

Table D.1: Strategic cancellation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pooled Logit</th>
<th>Random Logit</th>
<th>Fixed Logit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta p_t$</td>
<td>-0.000</td>
<td>-0.000***</td>
<td>-0.002***</td>
</tr>
<tr>
<td>constant</td>
<td>-4.713***</td>
<td>-5.883***</td>
<td></td>
</tr>
<tr>
<td>Log var.</td>
<td>1.278 ***</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Significance: *** 0.1%, **1%, *5% and $\Delta p_t = price$ after purchase − $price$ on purchasing day.

This table shows the test result for a cancellation binary choice model. I examine the existence of strategic cancellation by measuring the cancellation choice probability with changes in hotel 0’s price, $\Delta p_t$. All the coefficients of $\Delta p_t$ in the three tests are negative, resulting in a decrease in cancellation probability as the price for hotel 0 increases. There is evidence that hotel customers strategically cancel their booking depending on the price change. The Hausman test between random logit model and fixed logit model is more logical and shows the consistency of random effect estimator is rejected. Therefore, the fixed effect estimation is preferred in the cancellation binary choice model due to the consistency. However, both estimators show that there is strategic cancellation in the same direction and the strategic cancellation model is something that should be explored in the future.
Appendices E

Computational Evidence of a Reduced State Variable

Note: These plots illustrate the numerical results of theorem 2 in Yu [35]. According to the theorem, the optimal price is not affected by the average daily rate (ADR) for existing reservations. By reducing one state variable, a faster and more efficient dynamic programming computation is achieved. The two plots show the optimal pricing by different ADR given a fixed price of competitors and occupancy rate of hotel 0. It appears that the optimal price is not affected by ADR, though some discrepancy between the various ADR’s is present. Those discrepancies are caused by the interpolation of state variable during computation.

Figure E.1: Optimal price by different average daily rates (ADR)
APPENDICES F

FIRST-STEP SEMI-PARAMETRIC PRICE RULE
Table F.1: First-step pricing rule - regression learning

<table>
<thead>
<tr>
<th></th>
<th>weekday</th>
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<th>weekend</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>calm</td>
<td>normal</td>
<td>busy</td>
<td>busiest</td>
<td>calm</td>
<td>normal</td>
</tr>
<tr>
<td>Linear</td>
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<td></td>
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</tr>
<tr>
<td>RMSE</td>
<td>33.55</td>
<td>38.83</td>
<td>65.44</td>
<td>111.09</td>
<td>24.34</td>
<td>44.63</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.66</td>
<td>0.66</td>
<td>0.47</td>
<td>0.27</td>
<td>0.72</td>
<td>0.65</td>
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<tr>
<td>MSE</td>
<td>1125.80</td>
<td>1507.80</td>
<td>4282.70</td>
<td>12341.00</td>
<td>592.36</td>
<td>867.16</td>
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<td>MAE</td>
<td>25.30</td>
<td>29.73</td>
<td>48.25</td>
<td>78.29</td>
<td>16.29</td>
<td>21.49</td>
</tr>
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<td>Interactions Linear</td>
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<tr>
<td>RMSE</td>
<td>32.94</td>
<td>38.51</td>
<td>64.34</td>
<td>108.44</td>
<td>24.31</td>
<td>43.45</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.68</td>
<td>0.66</td>
<td>0.49</td>
<td>0.31</td>
<td>0.72</td>
<td>0.65</td>
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<tr>
<td>MSE</td>
<td>1084.80</td>
<td>1483.10</td>
<td>4139.00</td>
<td>11760.00</td>
<td>591.19</td>
<td>850.16</td>
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<td>MAE</td>
<td>24.99</td>
<td>29.23</td>
<td>47.83</td>
<td>76.78</td>
<td>16.27</td>
<td>21.06</td>
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<td>59.09</td>
<td>95.09</td>
<td>21.42</td>
<td>24.97</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.76</td>
<td>0.73</td>
<td>0.47</td>
<td>0.78</td>
<td>0.75</td>
<td>0.69</td>
</tr>
<tr>
<td>MSE</td>
<td>787.62</td>
<td>1190.20</td>
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<td>61.83</td>
<td>98.82</td>
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<td>0.69</td>
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<td>89.37</td>
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<td>0.53</td>
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<td>21.07</td>
<td>35.52</td>
<td>62.74</td>
<td>10.58</td>
<td>12.90</td>
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*Note*: Implemented by MATLAB regression learner app.
APPENDICES G

MAXIMUM LIKELIHOOD ESTIMATES OF HOTEL REGRESSION MODEL
Table G.1: Maximum likelihood estimates of hotel regression model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No (x) in (d_0) or (d_c)</th>
<th>Observed (x) ((x = \hat{x}))</th>
<th>Latent (x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_0) parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\theta_{0,0}) (constant)</td>
<td>41.41 (19.94)</td>
<td>-91.80 (45.80)</td>
<td>-39.94 (47.41)</td>
</tr>
<tr>
<td>(\theta_{0,1}) ((x))</td>
<td>4.14 (1.27)</td>
<td>3.57 (1.31)</td>
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</tr>
<tr>
<td>(\theta_{0,2}) ((x^2))</td>
<td>0.001 (0.009)</td>
<td>0.008 (0.009)</td>
<td></td>
</tr>
<tr>
<td>(\theta_{0,3}) (ADR(_0))</td>
<td>0.058 (0.223)</td>
<td>-0.174 (0.103)</td>
<td>-0.27 (0.11)</td>
</tr>
<tr>
<td>(\theta_{0,4}) (ADR(_c))</td>
<td>0.905 (0.207)</td>
<td>0.234 (0.096)</td>
<td>0.078 (0.108)</td>
</tr>
<tr>
<td>(\theta_{0,5}) (std((\epsilon_0)))</td>
<td>53.22 (2.11)</td>
<td>34.98 (0.88)</td>
<td>32.99 (0.95)</td>
</tr>
<tr>
<td>(d_c) parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\theta_{c,0}) (constant)</td>
<td>280.95 (110.82)</td>
<td>-327.37 (123.02)</td>
<td>-145.35 (120.87)</td>
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<tr>
<td>(\theta_{c,1}) ((x))</td>
<td>19.07 (3.33)</td>
<td>18.47 (3.35)</td>
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</tr>
<tr>
<td>(\theta_{c,2}) ((x^2))</td>
<td>0.019 (0.024)</td>
<td>0.041 (0.025)</td>
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</tr>
<tr>
<td>(\theta_{c,3}) (ADR(_c))</td>
<td>4.55 (1.05)</td>
<td>1.126 (0.244)</td>
<td>0.382 (0.328)</td>
</tr>
<tr>
<td>(\theta_{c,4}) (ADR(_0))</td>
<td>0.496 (1.201)</td>
<td>-0.669 (0.262)</td>
<td>-1.16 (0.310)</td>
</tr>
<tr>
<td>(\theta_{c,5}) (std((\epsilon_0)))</td>
<td>225.38 (11.86)</td>
<td>98.67 (2.78)</td>
<td>77.91 (4.99)</td>
</tr>
<tr>
<td>(p_0) parameters</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(\eta_{0,0}) (constant)</td>
<td>200.96 (48.60)</td>
<td>200.96 (48.60)</td>
<td>176.57 (38.83)</td>
</tr>
<tr>
<td>(\eta_{0,1}) ((x))</td>
<td>-2.89 (1.42)</td>
<td>-2.89 (1.42)</td>
<td>-2.27 (1.13)</td>
</tr>
<tr>
<td>(\eta_{0,2}) ((x^2))</td>
<td>0.035 (0.010)</td>
<td>0.035 (0.01)</td>
<td>0.031 (0.008)</td>
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<tr>
<td>(\eta_{0,3}) (std((z_0)))</td>
<td>28.67 (0.73)</td>
<td>28.67 (0.73)</td>
<td>26.02 (0.97)</td>
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<td>(p_c) parameters</td>
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<tr>
<td>(\eta_{c,0}) (constant)</td>
<td>265.37 (55.20)</td>
<td>265.37 (55.20)</td>
<td>228.27 (50.45)</td>
</tr>
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<td>(\eta_{c,1}) ((x))</td>
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<td>-4.56 (1.59)</td>
<td>-3.55 (1.43)</td>
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<td>(\eta_{c,2}) ((x^2))</td>
<td>0.051 (0.011)</td>
<td>0.051 (0.011)</td>
<td>0.044 (0.009)</td>
</tr>
<tr>
<td>(\eta_{c,3}) (std((z_0)))</td>
<td>31.55 (0.85)</td>
<td>31.55 (0.85)</td>
<td>27.99 (1.07)</td>
</tr>
<tr>
<td>(x) parameter</td>
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<tr>
<td>(\delta_2) (std((\epsilon)))</td>
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<td>0.00 (0.00)</td>
<td>3.22 (0.13)</td>
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<td>.740</td>
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<td>(R^2), (p_0)</td>
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<td>.502</td>
<td>.501</td>
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<tr>
<td>(R^2), (p_c)</td>
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<tr>
<td>Log-likelihood</td>
<td>-10465.1</td>
<td>-9748.9</td>
<td>-9718.5</td>
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Appendices H

Optimality and Equilibrium-constrained Maximum Likelihood Estimates of Hotel Regression Model
Table H.1: Optimality and equilibrium-constrained maximum likelihood estimates of hotel regression model

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<tr>
<th>Parameter</th>
<th>Optimality constrained</th>
<th>Equilibrium constrained</th>
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<td>$d_0$ parameters</td>
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<tr>
<td>$\theta_{0,0}$ (constant)</td>
<td>-10.48 (43.94)</td>
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<td>$\theta_{0,1}$ ($x$)</td>
<td>2.99 (1.16)</td>
<td>-2.86 (1.27)</td>
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<tr>
<td>$\theta_{0,2}$ ($x^2$)</td>
<td>0.007 (0.008)</td>
<td>0.056 (0.009)</td>
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<tr>
<td>$\theta_{0,3}$ (ADR$_0$)</td>
<td>-1.196 (0.108)</td>
<td>-0.952 (0.125)</td>
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<tr>
<td>$\theta_{0,4}$ (ADR$_c$)</td>
<td>0.988 (0.112)</td>
<td>-63.81 ($6.83 \times 10^{26}$)</td>
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<tr>
<td>$\theta_{0,5}$ (std($\epsilon_0$))</td>
<td>39.51 (1.25)</td>
<td>49.24 (2.30)</td>
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<td>$d_c$ parameters</td>
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<td>$\theta_{c,0}$ (constant)</td>
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<td>$\theta_{c,1}$ ($x$)</td>
<td>1.81 (4.69)</td>
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<td>$\theta_{c,2}$ ($x^2$)</td>
<td>0.156 (0.035)</td>
<td>0.173 (0.035)</td>
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<td>-4.36 (0.31)</td>
<td>-4.53 (0.32)</td>
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<td>$\theta_{c,4}$ (ADR$_0$)</td>
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<td>$\eta_{c,0}$ (constant)</td>
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<tr>
<td>$c_c$</td>
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Table I.1: Maximum likelihood estimates of mixed trinomial demand model

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<th>Latent $x$</th>
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<td>$\alpha_0$</td>
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<tr>
<td>$\alpha_c$</td>
<td>-0.91 (0.006)</td>
<td>-0.88 (0.007)</td>
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<tr>
<td>$\beta$</td>
<td>-0.261 (0.083)</td>
<td>-0.337 (0.085)</td>
</tr>
<tr>
<td>$\beta_\sigma$</td>
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<td>-0.257 (0.086)</td>
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<tr>
<td><strong>$x$ parameter</strong></td>
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<tr>
<td>$\delta_2$</td>
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<td><strong>Log-likelihood</strong></td>
<td>-7150.6</td>
<td>-11608</td>
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BIBLIOGRAPHY


