ESSAYS ON FINANCIAL INTERMEDIATION AND MONETARY POLICY
IN EMERGING MARKET ECONOMIES

A Dissertation
submitted to the Faculty of the
Graduate School of Arts and Sciences
of Georgetown University
in partial fulfillment of the requirements for the
degree of
Doctor of Philosophy
in Economics

By

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Washington, DC
April 16, 2020
ESSAYS ON FINANCIAL INTERMEDIATION AND MONETARY POLICY
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ABSTRACT

The dissertation examines the effects of capital inflows in models of monetary policy. In the first chapter, we develop a two-sector, two-agent New-Keynesian model of a small open economy with financial frictions and foreign currency debt in balance sheets. Focusing on an adverse foreign interest rate shock, the distributional consequences of alternative monetary-policy rules are analyzed to account for the exchange-rate stabilization motive in emerging market economies (EMEs). Under an inflation targeting regime, the depreciation of the domestic currency associated with the shock has an expansionary impact on the tradable sector via the expenditure switching channel. However, in the presence of nominal wage rigidities, the ensuing higher inflation associated with the depreciation reduces real wages, which makes households in the non-tradable sector worse off. Partially stabilizing the exchange rate, however, reverses the distributional consequences. Managed exchange rate regimes improve welfare of households in the non-tradable sector at the expense of households in the tradable sector; moreover, these policy regimes can improve aggregate welfare as long as the response to the exchange rate is not too strong. Strongly stabilizing (or fixing) the exchange rate reduces the welfare of both types of households. In an inflation-targeting regime, the use of capital controls or sterilized foreign-exchange interventions is considered as a second instrument. Solving for (Ramsey) optimal policy, we find that capital controls are effective in enhancing macroeconomic stability, while sterilized interventions are nearly ineffective in this environment.
In the second chapter, we offer an evaluation of claims by policymakers in the EMEs regarding adverse effects of the capital inflows that resulted from US monetary policy during the Great Recession. Our two-country model with financial frictions allows us to consider the welfare effects of contractionary shocks to capital quality under a passive US monetary policy. We compare these effects to the effects of the same shocks when US monetary policy responds with quantitative easing. We find that emerging-market complaints regarding the real exchange rate and current account are mostly due to the shock itself, and not to the US monetary policy. US monetary policy reacting to the shock brings welfare gains for both the US and the EMEs. The gains for the US are an order of magnitude larger than the welfare gains of the EMEs, reflecting the fact that a capital quality shock that originated in the US damages the US the most.

INDEX WORDS: Capital Inflows, Financial Frictions, Monetary Policy
I would like to acknowledge my indebtedness and offer my sincere thanks to my advisor, Professor Behzad Diba. His generous guidance and insightful advice have been invaluable throughout all stages of my research, and his continuous support and patience made this work possible. I am also extremely grateful to Professor Matthew Canzoneri and Professor Robert Cumby for their tremendous support and helpful suggestions, which have greatly improved my dissertation.
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2.1 Baseline Parameters for the Two-Country Model .............................. 81  
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1.1 Introduction

There is a conventional view that monetary policy in open economies should pursue domestic objectives and allow the exchange rate to freely fluctuate; fluctuations of the exchange rate should be taken into account only to the extent that the fluctuations have an impact on expected inflation.\(^1\) This prescription, however, is at odds with actual practice in emerging market economies (EMEs). Many of the central banks in EMEs have claimed to adopt inflation targeting under flexible exchange rate arrangements; however, in practice they have often attached a considerable weight to the stabilization of the exchange rate by using their policy rates (Ghosh et al. (2016)). Moreover, they have increasingly relied on capital controls and sterilized foreign exchange interventions (Blanchard et al. (2015)).\(^2\) What are the incentives that drive monetary policy in EMEs to stabilize the exchange rate? To what extent does the use of capital controls or sterilized foreign exchange interventions complement monetary policy in EMEs?

In this paper, based on the standard New-Keynesian open economy models with financial frictions, I develop a framework that can be used to analyze the distributional

---

\(^1\)See, for example, Clarida et al. (2001), Taylor (2001), Obstfeld (2002), and Gali and Monacelli (2005).

\(^2\)See also Ghosh et al. (2015) and Ilzetzki et al. (2017) for reviews of exchange rate regimes for the central banks in EMEs.
consequences of monetary policy in EMEs and show that it can account for the exchange rate stabilization motive in EMEs.\textsuperscript{3} Specifically, I extend the standard small open economy framework developed in Gali and Monacelli (2005) in four dimensions. First, I incorporate a banking sector into the framework that allows for financial frictions as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), and features the "original sin" of having foreign borrowings denominated in foreign currency.\textsuperscript{4} Second, I introduce two-sectors that produce tradable and non-tradable goods where intersectoral labor mobility is limited.\textsuperscript{5} Third, I assume two-types of households who differ in three aspects: (i) their working sector, (ii) their firm ownership, and (iii) their access to financial markets. In particular, the first type of households consists of workers in the tradable sector and bankers; they own all of financial and non-financial firms, and have unfettered access to domestic financial markets. The second type of households consists of only workers in the non-tradable sector; they have some restricted access to domestic financial markets. Fourth, I introduce nominal wage rigidities as in Calvo (1983).\textsuperscript{6}

In this environment, conditional on foreign interest rate shocks, I analyze the distributional and aggregate consequences of three alternative monetary policy rules: i) an inflation targeting regime, characterized by a simple Taylor rule that targets

\textsuperscript{3}This paper builds upon the literature that studies the effects of monetary policy in EMEs using New Keynesian open economy models with financial frictions: for example, Gertler et al. (2007), and Aoki et al. (2018).

\textsuperscript{4}Chui et al. (2016) documents that aggregate foreign currency mismatches of the private sector in EMEs have rapidly increased since 2010. In the non-government sector, foreign currency debt represents on average about 25 percent of total debt over the period 2006-2014.

\textsuperscript{5}Artuc et al. (2015) show that there are large costs in intersectoral labor mobility in EMEs, thereby restricting households to have diversified income across sectors.

\textsuperscript{6}Christiano et al. (2005) describes how nominal wage rigidities represent a key transmission channel through which monetary policy affects the real economy. Also, there is a growing evidence that supports the role of nominal wage rigidities in exacerbating the downturn during financial crises (see, e.g., Schmitt-Grohe and Uribe (2016)).
domestic inflation under flexible exchange rates, ii) a managed exchange rate regime, in which the nominal interest rate is set according to an augmented Taylor rule that responds to fluctuations of the nominal exchange rate in addition to inflation, and iii) a fixed exchange rate regime, where the central bank sets the nominal interest rate to maintain the nominal exchange rate at a constant value. Also, I consider that the central bank has two-types of second instruments of monetary policy: i) capital controls, as levying taxes on foreign currency borrowings of banks, and ii) sterilized foreign exchange interventions, as changing the quantities of domestic and foreign currency bonds in the portfolio of the central bank. Focusing on the Ramsey problem, under which the central bank sets capital controls or sterilized foreign exchange interventions to maximize aggregate welfare subject to all the equations characterizing the decentralized equilibrium, I analyze the effects of optimal use of alternative second instruments with an inflation targeting regime.

In this setup, an adverse foreign interest rate shock leads to a recession. The rise in the foreign interest rate induces a depreciation of the exchange rate, which deteriorates the net worth of banks exposed to foreign currency borrowing and therefore raises the cost of credit that non-financial borrowers face. This leads to a reduction in capital price and triggers an adverse feedback effect between the exchange rate depreciation, capital price decline, and the tightening of balance sheet constraints. Additionally, the depreciation raises the price of imports, leading to higher inflation. Under an inflation targeting regime, the central bank raises its nominal interest rate in response to the increased inflation, which further degrades the balance sheets of banks. These effects together generate a sharp contraction in real activity. The depreciation affects not only aggregate variables, but distributional as well. The depreciation, on the one hand, improves the trade competitiveness, and thus, restores some of the demand for domestically-produced tradable goods. As a result, output and employment in
the tradable sector do not fall as much; therefore, households in the tradable sector benefit from the depreciation. On the other hand, in the presence of nominal wage rigidities, the ensuing higher inflation associated with the depreciation reduces real wages, which in turn implies procyclical movements of real wages. Therefore, the rise in the volatility of real wages associated with the depreciation makes both types of households worse off, but to a greater degree in the non-tradable sector.

Partially stabilizing the exchange rate, however, reverses the distributional consequences described above. The exchange rate stabilization leads to lower inflation; however, this also leads to a further rise in the cost of borrowing for banks, which reinforces the adverse feedback effect, resulting in a further contraction in real activity. Moreover, the exchange rate stabilization worsens the trade competitiveness, which in turn leads to a greater contraction of output in the tradable sector than in the non-tradable sector. And yet, the lower inflation associated with the stabilization raises real wages. Through this mechanism, with the effect of stabilizing inflation and real wages if the degree of stabilization is not too strong, managed exchange rate regimes can increase the welfare of households in the non-tradable sector at the expense of households in the tradable sector. This is the tradeoff that monetary policy faces. My quantitative analysis shows that as long as the response to the exchange rate is not too strong, managed exchange rate regimes can improve aggregate welfare. By contrast, strongly stabilizing (or fixing) the exchange rate reduces the welfare of both types of households; the welfare loss of households in the tradable sector is relatively large.

Turning to the analysis of second instruments of monetary policy under an inflation targeting regime, I find that optimal use of capital controls helps in stabilizing the real economy while sterilized foreign exchange interventions are nearly ineffective in this environment. In the face of an adverse foreign interest rate shock, by reducing
the tax rate on foreign borrowings of banks, the planner reduces the cost of foreign borrowing for banks and raises the share of foreign borrowing in the bank liability, which mitigates the exchange rate depreciation. This in turn enhances financial stability and therefore alleviates macroeconomic fluctuations caused by the shock. On the other hand, the sale of foreign bonds combined with purchasing domestic bonds of the central bank initially appreciates the exchange rate and leads to lower inflation. But the domestic demand for bank deposits rises as the supply of domestic bond falls and the decreased inflation associated with the appreciation causes a reduction in the nominal interest rate under an inflation targeting regime, which together result in a reduction in the cost of domestic borrowing for banks and a rise in the share of domestic deposits in the bank liability. This in turn causes capital outflows, which mitigate the initial appreciation and associated effects of relaxing the bank balance sheet constraints. Therefore, sterilized interventions would not be effective in dealing with capital outflows with associated financial instability; the effects of optimal use of sterilized interventions on macroeconomic stability are tiny in the face of foreign monetary tightening shock.

This paper is related to several strands of the literature. First, this paper is related to the literature that studies the credit channel of monetary policy, using the New-Keynesian models with financial frictions. From this literature, this paper is closely related to the recent works of Aoki, Benigno, and Kiyotaki (2018) and Akinci and

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7The financial accelerator mechanism has been studied in the context of closed economies and initially applied to non-financial firms (Bernanke, Gertler and Gilchrist (1999)) and households (Kiyotaki and Moore (1997)). In the context of open economies, Gertler, Gilchrist and Natalucci (2007) develops a small open economy model with financial frictions, while Faia (2007) develops a two-country version of this class of models. Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) develop the models that feature explicitly financial intermediaries in the context of closed economies. Aoki, Benigno, and Kiyotaki (2018) develop a small open economy model that features explicitly financial intermediaries with currency mismatches in balance sheets.
Queralto (2019). This paper develops a framework, based on the model of banking sector in Aoki, Benigno, and Kiyotaki (2018), and analyzes the effects of alternative monetary and exchange rate regimes in EMEs conditional on foreign interest rate shocks and how the degree of currency mismatches in balance sheets affects the desirability of exchange rate stabilization, as in Akinci and Queralto (2019).

But departing from their works, this paper introduces two-sectors and two-types of households with a limited intersectoral labor mobility and explores the distributional consequences of alternative monetary regimes between households in the tradable sector – they are the same-type as representative household in the models of Aoki et al. (2018) and Akinci and Queralto (2019), and households in the non-tradable sector. By focusing on representative household in the tradable sector, Akinci and Queralto (2019) show that using monetary policy to manage the exchange rate reduces welfare compared to a strict inflation targeting regime, and while greater currency mismatches significantly magnify the spillovers from foreign interest rate shocks, but they do not necessarily imply more desirability of exchange rate stabilization. By contrast, this paper shows, in line with their findings, that stabilizing the exchange rate always reduces the welfare of households in the tradable sector but at the same time, improves the welfare of households in the non-tradable sector. Therefore, the exchange rate stabilization can improve aggregate welfare as long as the response of monetary policy to the exchange rate is not too strong.

This paper is also related to a growing literature that studies the transmission of monetary and fiscal policies using New Keynesian models with household heterogeneity.\(^8\) While this literature mainly studies in the context of closed economies or focuses on advanced economies, this paper extends a New-Keynesian framework with

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\(^8\)See, for example, Kaplan, Moll and Violante (2018), Gornemann, Kuester and Nakajima (2014), Mckay, Nakamura and Steinsson (2016), and Guerrieri and Lorenzoni (2017) with a full distribution of households, and Gali, Lopez-Salido and Valles (2007), Bilbiie (2008),
limited household heterogeneity to the small open economy setting with financial frictions.

Finally, this paper is related to several papers that study the distributional consequences of monetary and exchange rate policies in the context of EMEs. This paper is closely related to the work of Prasad and Zhang (2015). As their work, this paper considers the relevant features of EMEs – tradable and non-tradable sectors with a limited intersectoral labor mobility, and develops a two-sector, two-agent New-Keynesian model of a small open economy. But this paper differs from their work by introducing physical capital in production and the financial channel of banks who face balance sheet constraints, which have the effect of tying domestic capital investment to bank net worth. As a result, this paper features the transmission mechanism of foreign monetary shocks, working through movements of the exchange rate and capital price, and the condition of balance sheet constraints. This allows to study the effects of alternative monetary and exchange rate regimes in the face of the spillovers from foreign monetary shocks through the financial channel.

The remainder of the paper is organized as follows. In Section 1.2, I describe my baseline model. In Section 1.3, I discuss its baseline calibration. In Section 1.4, I illustrate the spillovers of foreign interest rate shocks and analyze the effects of alternative monetary rules. In Section 1.5, I analyze the effects of alternative second instruments of monetary policy under an inflation targeting regime. Section 1.6 concludes.

See, for example, Drenik (2015), Cravino and Levchenko (2017), and Cugat (2019).
1.2 Model

I develop a small open economy model with two-sectors, two-types of households, financial frictions, and nominal rigidities. The core framework is the standard New-Keynesian small open economy framework of Gali and Monacelli (2005). In domestic economy, there are two-types of households: (i) households in tradable sector, and (ii) households in non-tradable sector; three types of producers: (i) tradable goods producers, (ii) non-tradable goods producers, and (iii) capital goods producers; banks; and the central bank and government. Households consume, save, and supply labor services to producers. Households in tradable sector save by lending funds to banks and holding domestic government bonds, and supply labor services to tradable goods producers. Households in non-tradable sector can only make deposits with banks, and supply labor services to non-tradable goods producers. Each worker in both types of households is specialized in a differentiated occupation and monopolistically competitive; workers set their nominal wages subject to the Calvo constraint on their ability to adjust wages.10 Banks receive deposits from households in domestic currency and borrow from foreign investors in foreign currency to invest in risky domestic production technologies. As in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), tradable and non-tradable goods producers obtain funds from banks to finance their acquisition of capital from capital goods producers for use in production. To acquire funds from banks, goods producers issue state contingent securities to banks (i.e., there is no frictions transferring funds between producers and banks). But there is a moral hazard problem between banks and their creditors, generating endogenously determined balance sheet constraints: banks must have enough equity capital and satisfy the balance sheet constraints to obtain funds from their creditors. As a result,

10I could add additional nominal rigidities in goods markets, i.e., sticky nominal prices, but my qualitative results would remain unchanged.
domestic capital investment depends on the condition of balance sheet constraints and hence the equity capital of banks. Combined with foreign currency borrowing in the balance sheet of banks, this creates the transmission mechanism of foreign interest rate shocks to EMEs, working through mutually-reinforcing feedback between movements in the exchange rate and capital price, and the condition of balance sheet constraints. This constitutes the arrangement whereby financial frictions act to magnify the spillovers from foreign interest rate shocks, and monetary policy with exchange rate regime has distributional consequences. The central bank conducts capital controls or sterilized foreign exchange interventions in addition to conventional monetary policy.

1.2.1 Consumption Composites

Let $c_t$ be a consumption composite of tradable and non-tradable goods. The constant elasticity of substitution (CES) index defines household preferences over tradable, $c_{Tt}$, and non-tradable, $c_{Nt}$, goods as follows:

$$c_t = \left[ \gamma \frac{1}{\rho} (c_{Tt})^{\frac{\rho-1}{\rho}} + (1 - \gamma) \frac{1}{\rho} (c_{Nt})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad (1.1)$$

where $\rho \in (0, \infty)$ is the elasticity of substitution between tradable and non-tradable goods, and $\gamma \in (0, 1)$ is the relative weight of tradable goods in the consumption composite.

Also, let $c_{Tt}$ be a consumption composite of domestically-produced and foreign-produced tradable goods. The constant elasticity of substitution (CES) index defines household preferences over domestically-produced, $c_{Ht}$, and foreign-produced, $c_{Ft}$, tradable goods as follows:

$$c_{Tt} = \left[ \gamma_T \frac{1}{\rho_T} (c_{Ht})^{\frac{\rho_T-1}{\rho_T}} + (1 - \gamma_T) \frac{1}{\rho_T} (c_{Ft})^{\frac{\rho_T-1}{\rho_T}} \right]^{\frac{\rho_T}{\rho_T-1}} \quad (1.2)$$
where $\rho_T \in (0, \infty)$ is the elasticity of substitution between domestically-produced and foreign-produced tradable goods, and $\gamma_T \in (0, 1)$ is the relative weight of domestically-produced tradable goods in the tradable consumption composite.

Let $P_{Tt}$, $P_{Nt}$, $P_{Ht}$, and $P_{Ft}$ denote, respectively, aggregate domestic price of tradable goods, aggregate domestic price of non-tradable goods, aggregate domestic price of domestically-produced tradable goods, and aggregate domestic price of foreign-produced tradable goods. Corresponding aggregate domestic consumer price index, $P_t$, aggregate domestic price of tradable goods, $P_{Tt}$, and demands for tradable, non-tradable, domestically-produced tradable, and foreign-produced tradable goods $c_{Tt}$, $c_{Nt}$, $c_{Ht}$, and $c_{Ft}$ are given by

$$P_t = [\gamma (P_{Tt})^{1-\rho} + (1 - \gamma) (P_{Nt})^{1-\rho}] \frac{1}{1-\rho}$$  \hspace{1cm} (1.3)

$$P_{Tt} = [\gamma_T (P_{Ht})^{1-\rho_T} + (1 - \gamma_T) (P_{Ft})^{1-\rho_T}] \frac{1}{1-\rho_T}$$  \hspace{1cm} (1.4)

$$c_{Tt} = \left( \frac{P_{Tt}}{P_t} \right)^{-\rho} \gamma c_t$$  \hspace{1cm} (1.5)

$$c_{Nt} = \left( \frac{P_{Nt}}{P_t} \right)^{-\rho} (1 - \gamma) c_t$$  \hspace{1cm} (1.6)

$$c_{Ht} = \left( \frac{P_{Ht}}{P_{Tt}} \right)^{-\rho_T} \gamma_T c_{Tt}$$  \hspace{1cm} (1.7)

$$c_{Ft} = \left( \frac{P_{Ft}}{P_{Tt}} \right)^{-\rho_T} (1 - \gamma_T) c_{Tt}$$  \hspace{1cm} (1.8)

From the equations above, I obtain

$$\frac{P_{Tt}}{P_t} = \left[ \gamma + (1 - \gamma) \left( \frac{P_{Nt}}{P_{Tt}} \right)^{1-\rho} \right] \frac{1}{\rho-1}$$  \hspace{1cm} (1.9)

$$\frac{P_{Nt}}{P_t} = \left[ \gamma \left( \frac{P_{Nt}}{P_{Tt}} \right)^{\rho-1} + (1 - \gamma) \right] \frac{1}{\rho-1}$$  \hspace{1cm} (1.10)

$$\frac{P_{Ht}}{P_{Tt}} = \left[ \gamma_T + (1 - \gamma_T) \left( \frac{P_{Ft}}{P_{Ht}} \right)^{1-\rho_T} \right] \frac{1}{\rho_T-1}$$  \hspace{1cm} (1.11)
\[
\frac{P_{Ft}}{P_{Ht}} = \left[ \gamma_T \left( \frac{P_{Ft}}{P_{Ht}} \right)^{\rho_T-1} + (1 - \gamma_T) \right]^{1/\rho_T-1} \tag{1.12}
\]

### 1.2.2 Terms of Trade and Real Exchange Rate

I assume that domestically-produced and foreign-produced tradable goods are sold in a world market and the law of one price holds. Let \( NOM_t \) denotes the nominal exchange rate, defined as domestic price of foreign currency; \( P_{Ht}^* \) denotes aggregate foreign price of domestically-produced tradable goods; and \( P_{Ft}^* \) denotes aggregate foreign price of foreign-produced tradable goods. Then the law of one price implies

\[
P_{Ht} = NOM_t P_{Ht}^* \tag{1.13}
\]

\[
P_{Ft} = NOM_t P_{Ft}^* \tag{1.14}
\]

The foreign country is large relative to the home country and so domestically-produced tradable goods have a negligible weight in foreign consumption. Therefore, \( P_{Ft}^* \) is approximately equal to the foreign CPI, \( P_t^* \). I also assume that there is no inflation in the foreign country so that

\[
P_t^* = P_{Ft}^* = P^* \tag{1.15}
\]

Then the terms of trade, \( TOT_t \), and real exchange rate, \( RER_t \), can be expressed as follows:

\[
TOT_t = \frac{P_{Ft}}{P_{Ht}} = \frac{NOM_t P^*}{P_{Ht}} \tag{1.16}
\]

\[
RER_t = \frac{NOM_t P^*}{P_t} = \left[ \gamma_T \left( \frac{P_{Ft}}{P_{Ht}} \right)^{\rho_T-1} + (1 - \gamma_T) \right]^{1/\rho_T-1} \left[ \gamma + (1 - \gamma) \left( \frac{P_{Nt}}{P_{Pt}} \right)^{1-\rho} \right]^{1/\rho-1} \tag{1.17}
\]

Then I obtain

\[
\frac{TOT_t}{TOT_{t-1}} = \frac{NOM_t}{NOM_{t-1}} \frac{1}{\Pi_{Ht}} \tag{1.18}
\]
\[ \frac{RER_t}{RER_{t-1}} = \frac{NOM_t}{NOM_{t-1}} \frac{1}{\Pi_t} \]  

(1.19)

Also from the relative prices, \( \frac{P_{Nt}}{P_t} \) and \( \frac{P_{Ht}}{P_{Tt}} \), I have

\[ \Pi_{Nt} = \Pi_t \left[ \frac{\gamma \left( \frac{P_{Nt}}{P_{Tt}} \right)^{1-\rho^*} + (1 - \gamma)}{\gamma \left( \frac{P_{Nt-1}}{P_{Tt-1}} \right)^{1-\rho^*} + (1 - \gamma)} \right] \]  

(1.20)

\[ \Pi_{Ht} = \Pi_{Tt} \left[ \frac{\gamma_T + (1 - \gamma_T)(TOT_t)^{1-\rho_T}}{\gamma_T + (1 - \gamma_T)(TOT_{t-1})^{1-\rho_T}} \right] \]  

(1.21)

where \( \Pi_t = \frac{P_t}{\bar{P}_t} \), \( \Pi_{Nt} = \frac{P_{Nt}}{P_{Nt-1}} \), \( \Pi_{Tt} = \frac{P_{Tt}}{P_{Tt-1}} \), and \( \Pi_{Ht} = \frac{P_{Ht}}{P_{Ht-1}} \).

The home country takes the foreign gross real interest rate \( R^*_t \) as given (which equals the gross nominal interest rate because I assume that there is no inflation in foreign country). I assume that \( R^*_t \) follows the exogenous stochastic process

\[ R^*_t - 1 = \rho_{R^*}(R^*_{t-1} - 1) + (1 - \rho_{R^*})(\bar{R}^* - 1) + \epsilon_{R^*,t} \]  

(1.22)

where \( \rho_{R^*} \) denotes the persistence of the shock, \( \bar{R}^* \) is the steady-state level of the foreign gross real interest rate, and \( \epsilon_{R^*,t} \) is an innovation which follows a normal process with a mean of zero and a standard deviation of \( \sigma_{R^*} \).

I assume that export demand for domestically-produced tradable goods by foreigners is function of the relative foreign price of domestically-produced tradable goods to foreign CPI and the level of foreign demand

\[ EX_t = \left( \frac{P_{Ht}}{NOM_t P^*} \right)^{-\rho^*} Y^* \]  

(1.23)

where \( \rho^* \) is a constant price elasticity of foreign demand, and \( Y^* \) is an exogenous parameter of foreign demand.
1.2.3 Households

The small open economy consists of two-types of infinitely lived households, referred to respectively as households in tradable sector, with population share $\mu > 0$, and households in non-tradable sector, with population share $1 - \mu$. I assume that two-types of households differ in three aspects. Households in tradable sector work for tradable goods producers, have all ownership of financial and non-financial firms, and unfettered access to domestic financial markets. Households in non-tradable sector work for non-tradable goods producers, do not have any firm ownership, and can only save by making deposits at banks.

Households in Tradable Sector

There is a continuum of households in tradable sector. Each household consumes, saves and supplies labor. Households save by lending funds to banks and holding domestic bonds. At any moment in time, each household is composed of a constant fraction $(1 - f)$ of workers and a fraction $f$ of bankers. Each worker, indexed by $j \in [0, 1]$, is specialized in a differentiated occupation and supplies labor service $L_{jt}$ at the nominal wage $W_{jt}$ to tradable goods producers and return her wage to the household of which she is a member.\textsuperscript{11} Bankers manage banks and who exit give their retained earnings to their respective household. The household provides its new bankers with some start up funds. Over time, an individual can switch between the two occupations. In particular, a banker this period stays banker next period with probability $\eta$, which is independent of history.\textsuperscript{12} Thus, every period, $(1 - \eta)f$ of bankers

\textsuperscript{11}Tradable goods producers regard each specialized labor service as an imperfect substitute for the specialized labor services of other workers.

\textsuperscript{12}The average survival time for a banker in any given period is $\frac{1}{1-\eta}$. This finite horizon for bankers insures that over time they do not reach the point where they can fund all domestic capital investment from their own equity capital.
exit and become workers. A same number of workers randomly become bankers, so keeping the relative proportion of each type fixed. Within the household, there is perfect consumption insurance.

Household $j$’s preferences over consumption, leisure, and real domestic bond holdings are given by the lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^T, L_{jt}^T, b_t^T) = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1 - \sigma} (c_t^T)^{1-\sigma} - \frac{\chi_0}{1 + \chi} (L_{jt}^T)^{1+\chi} + \theta_b \log b_t^T \right)$$

(1.24)

where $E_t$ is the mathematical expectation operator conditional on the information set available at the period $t$, $\beta \in (0, 1)$ is the subjective discount rate, $\sigma$ parameterizes the curvature of the utility function with respect to the consumption, $\chi_0$ is the utility weight of labor, $\chi > 0$ is the inverse Frisch elasticity of labor supply, and $\theta_b$ is the utility weight of real domestic bond holdings.\(^{13}\)

I assume that the contracts of deposits and domestic bond holdings are nominal, short term and non-contingent. Households who deposit $D_t^T$ at banks from period $t$ to $t+1$ will receive $(1 + i_t)D_t^T$ in period $t+1$, where $i_t$ is the nominal interest rate on deposit.\(^{14}\) Also, those who hold domestic bonds $B_t^T$ from period $t$ to $t+1$ will receive $(1 + i_{bt})B_t^T$ in period $t+1$, where $i_{bt}$ is the nominal interest rate on domestic bonds. I assume that only the households in tradable sector receive the nominal profits $\Sigma_t^T$ from non-financial and financial firms. The nominal wage income is subsidized at a fixed rate $\tau_{WT}$, and $T_t^T$ is the nominal lump-sum taxes from the government.\(^{15}\) In addition, $\Xi_{jt}^T$ is the net cash flow from household $j$’s state-contingent claims used to ensure that all households are identical in their choices of consumption, deposit, and

---

\(^{13}\)I assume that real domestic bond holdings provide non-pecuniary services as they have a collateral value and may facilitate transactions.

\(^{14}\)Since there is no interbank frictions, the nominal interest rate, $i_t$ coincides with the policy rate of the central bank.

\(^{15}\)The lump-sum taxes from the government and central bank will be described in the section for the government.
domestic bond holdings, and only differ in wage and labor supply, as in Akinci and Queralto (2019). Then the household $j$ in tradable sector’s flow budget constraint is given by\textsuperscript{16}

\[ P_t c_j^T + D_j^T + B_j^T \leq (1 + \tau_{WT})W_{j,t}^T L_{j,t}^T + (1 + i_{t-1})D_{j,t-1}^T + (1 + i_{bt-1})B_{j,t-1}^T + \Xi_{j,t} + \sum T - T_j^T \]  

(1.25)

In real terms,

\[ c_j^T + d_j^T + b_j^T \leq (1 + \tau_{WT})w_{j,t}^T L_{j,t}^T + R_{t-1}d_{j,t-1}^T + R_{bt-1}b_{j,t-1}^T + \frac{\Xi_{j,t}}{P_t} + \frac{\sum T}{P_t} - \frac{T_j^T}{P_t} \]  

(1.26)

where $d_j^T = \frac{D_j^T}{P_t}$, $b_j^T = \frac{B_j^T}{P_t}$, $w_{j,t}^T = \frac{W_{j,t}^T}{P_t}$, $R_{t-1} = (1+i_{t-1})\frac{P_{t-1}}{P_t}$, and $R_{bt-1} = (1+i_{bt-1})\frac{P_{t-1}}{P_t}$.

The real profits $\frac{\sum T}{P_t}$ distributed from production of capital goods (the first line) and banking (the second and third lines) are follows:

\[
\mu \frac{\sum T}{P_t} = \left[ q_t I_t - \frac{P_t I_t}{P_t} \left( 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right) I_t \right] \\
+ (1 - \eta)(R_{Ht}q_{t-1}s_{Ht-1} + R_{Nt}q_{t-1}s_{Nt-1} - R_{t-1}d_{t-1} - R_{t-1}d^*_{t-1}) \\
- \xi(R_{Ht}q_{t-1}s_{Ht-1} + R_{Nt}q_{t-1}s_{Nt-1})
\]  

(1.27)

The household $j$ chooses $c_j^T$, $d_j^T$, and $b_j^T$ to maximize preferences (1.24) subject to its flow budget constraint (1.26). Letting $\lambda_j^T$ be the Lagrangian multiplier associated with the budget constraint, the first order conditions for consumption, deposit, and domestic bond holdings are given by

\[ 1 = E_t \beta \frac{\lambda_j^{T+1}}{\lambda_j^T} R_t \]  

(1.28)

\[ 1 = \frac{\theta_b}{\lambda_j^T b_j^T} + E_t \beta \frac{\lambda_j^{T+1}}{\lambda_j^T} R_{bt} \]  

(1.29)

\textsuperscript{16}All variables in the budget constraint are normalized by the population share in tradable sector $\mu$. 
with $\lambda_t^T = \frac{1}{(c_t^T)}$, the marginal utility of consumption, and $\Lambda_{t,t+1}^T = \beta \frac{\lambda_{t+1}^T}{\lambda_t^T}$, the household in tradable sector’s stochastic discount factor.

I introduce nominal wage rigidities following Erceg et al. (2000). I assume that a representative labor aggregator combines specialized labor into a composite labor input $L_{Ht}$ that is used by tradable goods producers as follow:

$$L_{Ht} = \left( \int_0^1 \frac{L_{jt}^T}{W_{HT}^T} \epsilon_{WT}^{-1} dj \right)^{\frac{\epsilon_{WT}}{\epsilon_{WT}^T}}$$  \hspace{1cm} (1.30)

where $\epsilon_{WT} > 0$. The cost minimization by the labor aggregator yields a demand schedule for specialized labor $j$

$$L_{jt}^T = \left( \frac{W_{jt}^T}{W_{HT}^T} \right)^{-\epsilon_{WT}} L_{Ht}$$  \hspace{1cm} (1.31)

where $W_{jt}^T$ is the nominal wage of worker specialized in occupation $j$ and $W_{HT}^T$ is an aggregate nominal wage index paid by tradable goods producers

$$W_{HT}^T = \left( \int_0^1 W_{jt}^{T-1-\epsilon_{WT}} dj \right)^{\frac{1}{1-\epsilon_{WT}}}$$  \hspace{1cm} (1.32)

I also assume that in each period, only a fraction $(1 - \varrho_{WT})$ of households, drawn randomly from the population in tradable sector, can reset their nominal wages optimally to maximize their preferences (1.24), taking into account an isoelastic demand function for specialized labor $j$ (1.31), and subject to the flow budget constraint (1.26). The remaining fraction $\varrho_{WT}$ of households keep their nominal wages unchanged. The nominal wage newly set in the period $t$, denoted by $\overline{W}_t^T$, must satisfy the following optimality condition

$$E_t \sum_{s=0}^{\infty} (\beta \varrho_{WT})^s \left[ \frac{(1 + \tau_{WT})(\epsilon_{WT} - 1)}{\epsilon_{WT}} \Lambda_{t+s}^T \frac{W_t^T}{P_{t+s}} - \chi_0 \left( \frac{\overline{W}_t^T}{W_{HT+s}} \right)^{-\epsilon_{WT}} \right] (L_{Ht+s})^\chi \left( \frac{\overline{W}_t^T}{W_{HT+s}} \right)^{-\epsilon_{WT}} L_{Ht+s} = 0$$  \hspace{1cm} (1.33)
where $\frac{\varepsilon_{WT}}{(\varepsilon_{WT}-1)}$ is the frictionless wage markup and $\tau_{WT} = \frac{1}{\varepsilon_{WT}-1}$ is the fiscal subsidy rate that eliminates the wage markup in the steady-state.

Let $w_t^T = \frac{w_t^T}{P_t}$ be the optimal real wage and $w_{HT} = \frac{W_{HT}}{P_t}$ be the aggregate real wage index. Then the optimality condition (1.33) can be written recursively as $F_{1t} = F_{2t}$ where

$$F_{1t} = \lambda_{t}^{T}w_{t}^{T}\left(\frac{w_{HT}}{w_{t}}\right)^{\varepsilon_{WT}}L_{HT} + \beta \varrho_{WT}E_{t}\left[\Pi_{t+1}^{\varepsilon_{WT}-1}\left(\frac{w_{T}^{t+1}}{w_{t}}\right)^{\varepsilon_{WT}-1}F_{1t+1}\right]$$

(1.34)

and

$$F_{2t} = \chi_{0}\left(\frac{w_{HT}}{w_{t}}\right)^{\varepsilon_{WT}(1+\chi)}L_{HT}^{1+\chi} + \beta \varrho_{WT}E_{t}\left[\Pi_{t+1}^{\varepsilon_{WT}(1+\chi)}\left(\frac{w_{T}^{t+1}}{w_{t}}\right)^{\varepsilon_{WT}(1+\chi)}F_{2t+1}\right]$$

(1.35)

From the expression for the aggregate wage index, I obtain the following relation

$$(w_{HT})^{1-\varepsilon_{WT}} = (1 - \varrho_{WT})(\overline{w}_{t}^{T})^{1-\varepsilon_{WT}} + \varrho_{WT}(w_{HT-1})^{1-\varepsilon_{WT}}(\Pi_{t})^{\varepsilon_{WT}-1}$$

(1.36)

where $\Pi_{t} = \frac{P_{t}}{P_{t-1}}$. Lastly, wage inflation in the tradable sector is given by

$$\Pi_{t}^{WT} = \frac{W_{HT}}{W_{HT-1}} = \frac{w_{HT}}{w_{HT-1}}\Pi_{t}$$

(1.37)

**HOUSEHOLDS IN NON-TRADEABLE SECTOR**

There is a continuum of households in non-tradable sector. Each household consumes, saves, and supplies labor. Households save only by lending funds to banks. Each household is composed of only workers, and each worker, indexed by $j \in [0, 1]$, is specialized in a differentiated occupation and supplies labor service $L_{N}^{j_{t}}$ at the nominal wage $W_{N}^{j_{t}}$ to non-tradable goods producers. Household $j$ in non-tradable sector maximizes the lifetime utility

$$E_{0}\sum_{t=0}^{\infty} \beta^{t}u(c_{t}^{N},L_{N}^{j_{t}}) = E_{0}\sum_{t=0}^{\infty} \beta^{t}\left(\frac{1}{1 - \sigma}(c_{t}^{N})^{1-\sigma} - \frac{\chi_{0}}{1 + \chi}(L_{j_{t}}^{N})^{1+\chi}\right)$$

(1.38)
subject to the flow budget constraint\(^{17}\)

\[
P_t c^N_t + D^N_t \leq (1 + \tau_{WN}) W^N_{jt} L^N_{jt} + (1 + i_{t-1}) D^N_{t-1} + \Xi^N_{jt} - T^N_t \quad (1.39)
\]

where \(\Xi^N_{jt}\) is the net cash flow from household \(j\)’s state-contingent claims used to ensure that choices of consumption and bank deposit are identical for all households, the nominal wage income is subsidized at a fixed rate \(\tau_{WN}\), and \(T^N_t\) is the nominal lump-sum taxes from the government.\(^{18}\) In real terms,

\[
c^N_t + d^N_t \leq (1 + \tau_{WN}) w^N_{jt} L^N_{jt} + R_{t-1} d^N_{t-1} + \frac{\Xi^N_{jt}}{P_t} - \frac{T^N_t}{P_t} \quad (1.40)
\]

where \(d^N_t = \frac{D^N_t}{P_t}\) and \(w^N_{jt} = \frac{W^N_{jt}}{P_t}\).

The household \(j\) chooses \(c^N_t\) and \(d^N_t\) to maximize preferences (1.38) subject to its flow budget constraint (1.40). Letting \(\lambda^N_t\) be the Lagrangian multiplier associated with the budget constraint, the first order conditions for consumption/saving are given by

\[
1 = E_t \beta \frac{\lambda^N_{t+1}}{\lambda^N_t} R_t \quad (1.41)
\]

with \(\lambda^N_t = \frac{1}{(c^N_t)^{\sigma}}\), the household in non-tradable sector’s marginal utility of consumption, and \(\Lambda^N_{t,t+1} = \beta \frac{\lambda^N_{t+1}}{\lambda^N_t}\), the household in non-tradable sector’s stochastic discount factor.

Analogous to the households in tradable sector, there is a representative labor aggregator that combines specialized labor into a composite labor input \(L_{Nt}\) as follow:

\[
L_{Nt} = \left( \int_0^1 L^N_{jt} \frac{c^N_{WN,t-1}}{c^N_{WN}} dj \right)^{\frac{c^N_{WN}}{c^N_{WN,t-1}}} \quad (1.42)
\]

\(^{17}\)All variables in the budget constraint are normalized by \(1 - \mu\).

\(^{18}\)The lump-sum taxes levied to households in non-tradable sector equal to the fiscal subsidy on their nominal wages (i.e., \(T^N_t = \tau_{WN} W^N_t L^N_t\)).
where $\varepsilon_{WN} > 0$. The cost minimization by the labor aggregator yields a demand schedule for specialized labor $j$

$$L^N_{jt} = \left( \frac{W^N_{jt}}{W^N_{Nt}} \right)^{-\varepsilon_{WN}} L_{Nt}$$  \hspace{1cm} (1.43)

where $W^N_{jt}$ is the nominal wage for specialized labor $j$ and $W^N_{Nt}$ is an aggregate nominal wage index paid by non-tradable goods producers

$$W^N_{Nt} = \left( \int_0^1 W^N_{jt}^{1 - \varepsilon_{WN}} dj \right)^{\frac{1}{1 - \varepsilon_{WN}}}$$  \hspace{1cm} (1.44)

Also, in each period, only a fraction $(1 - \varrho_{WN})$ of households, drawn randomly from the population in non-tradable sector, can reset their nominal wages optimally to maximize their preferences (1.38), taking into account an isoelastic demand function for specialized labor $j$ (1.43), and subject to their flow budget constraint (1.40). The remaining fraction $\varrho_{WN}$ of households keep their nominal wages unchanged. The nominal wage newly set in period $t$, denoted by $W^N_t$, must satisfy the following optimality condition

$$E_t \sum_{s=0}^{\infty} (\varrho_{WN})^s \left[ \left( \frac{1 + \tau_{WN}}{\varepsilon_{WN}} \right)^{\lambda^N_{t+s}} \frac{W^N_t}{P_{t+s}} - \chi_0 \left( \frac{W^N_t}{W_{Nt+s}} \right)^{-\varepsilon_{WN}} \right] \left( \frac{L_{Nt+s}}{W_{Nt+s}} \right)^{\varepsilon_{WN}} L_{Nt+s} = 0$$  \hspace{1cm} (1.45)

where $\frac{\varepsilon_{WN}}{\varepsilon_{WN}-1}$ is the frictionless wage markup and $\tau_{WN} = \frac{1}{\varepsilon_{WN}-1}$ is the fiscal subsidy rate that eliminates the wage markup in the steady-state.

Let $\bar{w}^N_t = \frac{W^N_t}{P_t}$ be the optimal real wage and $w_{Nt} = \frac{W^N_{Nt}}{P_t}$ be the aggregate real wage index. Then the optimality condition (1.45) can be written recursively as $G_{1t} = G_{2t}$, where

$$G_{1t} = \lambda^N_t \bar{w}^N_t \left( \frac{w_{Nt}}{\bar{w}^N_t} \right)^{\varepsilon_{WN}} L_{Nt} + \beta_{WN} E_t \left[ \Pi_{t+1}^{\varepsilon_{WN}-1} \left( \frac{\bar{w}^N_{t+1}}{\bar{w}^N_t} \right)^{\varepsilon_{WN}-1} G_{1t+1} \right]$$  \hspace{1cm} (1.46)
and 

$$G_{2t} = \chi_0 \left( \frac{w_{Nt}}{\bar{w}_t^{WN}} \right)^{\varepsilon_{WN}(1+\chi)} L_{Nt}^{1+\chi} + \beta \varrho_W E_t \left[ \Pi_t^{\varepsilon_{WN}(1+\chi)} \left( \frac{\bar{w}_t^{WN}}{\bar{w}_t} \right)^{\varepsilon_{WN}(1+\chi)} G_{2t+1} \right]$$

(1.47)

From the expression for the aggregate wage index, I obtain

$$(w_{Nt})^{1-\varepsilon_{WN}} = (1 - \varrho_W)\left(\bar{w}_t^{WN}\right)^{1-\varepsilon_{WN}} + \varrho_W (w_{Nt-1})^{1-\varepsilon_{WN}} (\Pi_t)^{\varepsilon_{WN}-1}$$

(1.48)

Lastly, wage inflation in the non-tradable sector is given by

$$\Pi_t^{WN} = \frac{W_{Nt}}{W_{Nt-1}} = \frac{w_{Nt}}{w_{Nt-1}} \Pi_t$$

(1.49)

1.2.4 PRODUCERS

There are three types of non-financial firms: tradable goods producers, non-tradable goods producers, and capital goods producers.

TRADABLE GOODS PRODUCERS

There is a continuum of competitive tradable goods producers, each of which produces output using an identical Cobb-Douglas production function with capital and labor as inputs. Aggregate output $Y_{Ht}$ can be expressed as a function of aggregate capital $K_{Ht-1}$ and aggregate labor $L_{Ht}$ as follow:

$$Y_{Ht} = A_H (K_{Ht-1})^\alpha (L_{Ht})^{1-\alpha}$$

(1.50)

where $A_H$ is the aggregate productivity in tradable sector.

In the end of period $t$, tradable goods producers acquire capital $\mu K_{Ht}$ from capital goods producers for use in production in the subsequent period $t+1$. After production in period $t+1$, the tradable goods producers sell the depreciated capital to capital goods producers. All variables in the production function are normalized by the population share in tradable sector $\mu$. 

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goods producers. The tradable goods producers finance its capital acquisition each period by obtaining funds from banks. To acquire the funds to buy capital, the producers issue $s_{Ht}$ claims equal to the number of units of capital acquired $\mu K_{Ht}$ and price each claim at the price of a unit of capital $q_t$. That is, $q_t \mu K_{Ht}$ is the value of capital acquired and $q_t s_{Ht}$ is the value of claim against this capital as

$$q_t \mu K_{Ht} = q_t s_{Ht} \quad (1.51)$$

Let $z_{Ht}$ and $w_{Ht}$ be the real rental price of capital and the real wage rate. Then the minimized unit cost of production is

$$\left( \frac{P_{Ht}}{P_t} \right) = \frac{z_{Ht}^{\alpha} w_{Ht}^{1-\alpha}}{A_H \alpha^{\alpha}(1 - \alpha)^{1-\alpha}} \quad (1.52)$$

The cost minimization by the tradable goods producers implies

$$w_{Ht} = \left( \frac{P_{Ht}}{P_t} \right) (1 - \alpha) \frac{Y_{Ht}}{L_{Ht}} \quad (1.53)$$

$$z_{Ht} = \left( \frac{P_{Ht}}{P_t} \right) \alpha \frac{Y_{Ht}}{K_{Ht-1}} \quad (1.54)$$

$$\frac{w_{Ht}L_{Ht}}{z_{Ht}K_{Ht-1}} = \frac{1 - \alpha}{\alpha} \quad (1.55)$$

**Non-Tradable Goods Producers**

There is a continuum of competitive non-tradable goods producers, each producing output using an identical Cobb-Douglas production function with capital and labor as inputs. Aggregate output $Y_{Nt}$ can be expressed as a function of aggregate capital $K_{Nt-1}$ and aggregate labor $L_{Nt}$ as follow:\textsuperscript{20}

$$Y_{Nt} = A_N (K_{Nt-1})^{\alpha} (L_{Nt})^{1-\alpha} \quad (1.56)$$

where $A_N$ is the aggregate productivity in non-tradable sector.

\textsuperscript{20}All variables in the production function are normalized by $1 - \mu$.  

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In the end of period $t$, non-tradable goods producers acquire capital $(1 - \mu)K_{Nt}$ from capital goods producers for use in production in the subsequent period. After production in period $t + 1$, the non-tradable producers sell the depreciated capital to capital goods producers. The non-tradable goods producers finance its capital acquisition each period by obtaining funds from banks. To acquire the funds to buy capital, the non-tradable producers issue $s_{Nt}$ claims equal to the number of units of capital acquired, $(1 - \mu)K_{Nt}$ and price each claim at the price of a unit of capital $q_t$. Accordingly,

$$q_t(1 - \mu)K_{Nt} = q_t s_{Nt}$$ (1.57)

Let $z_{Nt}$ and $w_{Nt}$ be the rental price of capital and the wage rate in terms of consumption composite goods. Then the minimized unit cost of production is

$$\left(\frac{P_{Nt}}{P_t}\right) = \frac{z_{Nt}^{\alpha} w_{Nt}^{1-\alpha}}{A_N \alpha^\alpha (1 - \alpha)^{1-\alpha}}$$ (1.58)

The cost minimization by the non-tradable goods producers implies

$$w_{Nt} = \left(\frac{P_{Nt}}{P_t}\right) (1 - \alpha) \frac{Y_{Nt}}{L_{Nt}}$$ (1.59)

$$z_{Nt} = \left(\frac{P_{Nt}}{P_t}\right) \frac{\alpha Y_{Nt}}{K_{Nt-1}}$$ (1.60)

$$\frac{w_{Nt} L_{Nt}}{z_{Nt} K_{Nt-1}} = \frac{1 - \alpha}{\alpha}$$ (1.61)

**Capital Goods Producers**

In the end of period $t$, using input of tradable composite goods, competitive capital goods producers build new capital subject to adjustment costs of investment. Combining the depreciated capital stock $(1 - \delta)K_{t-1}$ and new investment $I_t$, they sell the new capital $K_t$ to the tradable and non-tradable goods producers at the price $q_t$. The
discounted profits for a capital goods producer are then given by

$$\max_{I_t} E_0 \left\{ \sum_{t=0}^{\infty} A_{0,t}^T \left[ q_t I_t - \frac{P_{T_t}}{P_t} \left( 1 + f \left( \frac{I_t}{I} \right) \right) I_t \right] \right\} \quad (1.62)$$

where $f(1) = f'(1) = 0$, and $f''(1) > 0$.\(^{21}\)

The first order condition for investment is

$$q_t = \frac{P_{T_t}}{P_t} + \frac{P_{T_t} \kappa_I I_t}{2} \left( \frac{I_t}{I} - 1 \right)^2 + \frac{P_{T_t} \kappa_I}{P_t} \left( \frac{I_t}{I} - 1 \right) \left( \frac{I_t}{I} \right) \quad (1.63)$$

1.2.5 Banks

Competitive banks make loans to tradable and non-tradable goods producers who need to finance their capital acquisitions, by issuing deposits to households, borrowings from foreign investors, and using their own equity capital.\(^{22}\) The deposits are denominated in domestic currency while the foreign borrowings are denominated in foreign currency.

Let $n_t$ denotes the amount of equity capital or net worth that an individual bank has in the end of period $t$, $D_t$ the deposits the bank obtains from households, $D^*_t$ the borrowings the bank obtains from foreign investors, $s_{Ht}$ and $q_t$ the quantity and price of financial claims on tradable goods producers that the bank holds, $R_{Ht}$ the realized return on these financial claims from period $t - 1$ to $t$, $s_{Nt}$ and $q_t$ the quantity and price of financial claims on non-tradable goods producers that the bank holds, $R_{Nt}$ the

\(^{21}\)Following Aoki et al. (2018),

$$f \left( \frac{I_t}{I} \right) = \frac{\kappa_I}{2} \left( \frac{I_t}{I} - 1 \right)^2$$

\(^{22}\)Following Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), there is no friction in transferring funds between banks and producers. Banks finance tradable and non-tradable goods producers by purchasing their state-contingent securities.
realized return on these financial claims from $t-1$ to $t$. The variables with superscript $\ast$ are denominated in foreign currency, while the ones without the superscript are denominated in domestic currency. The balance sheet for the bank is then given by

$$
\left(1 + \frac{\kappa b}{2} x_t^2\right) (P_t q_t s_{Ht} + P_t q_t s_{Nt}) = P_t n_t + D_t + NOM_t D_t^* \tag{1.64}
$$

where \(\frac{\kappa b}{2} x_t^2 (P_t q_t s_{Ht} + P_t q_t s_{Nt})\) is the additional resource cost depending on the fraction of bank’s assets financed by the borrowings from foreign investors. This assumption is described later in this section.

I consider capital controls as levying taxes on foreign borrowing of banks and the revenues from taxing foreign borrowings are rebated to households in the tradable sector through lump-sum transfers. Letting $\tau_t^{D^*}$ be a time-varying tax rate on foreign borrowings, the balance sheet for the bank is then given by

$$
\left(1 + \frac{\kappa b}{2} x_t^2\right) (P_t q_t s_{Ht} + P_t q_t s_{Nt}) = P_t n_t + D_t + (1 - \tau_t^{D^*}) NOM_t D_t^* \tag{1.65}
$$

In real terms,

$$
\left(1 + \frac{\kappa b}{2} x_t^2\right) (q_t s_{Ht} + q_t s_{Nt}) = n_t + d_t + (1 - \tau_t^{D^*}) RER_t d_t^* \tag{1.66}
$$

where $d_t = \frac{D_t}{P_t}$, $d_t^* = \frac{D_t^*}{P_t^*}$, and $RER_t = \frac{NOM_t P_t^*}{P_t}$. I assume that $\tau_t^{D^*}$ follows the exogenous stochastic process

$$
\tau_t^{D^*} = \rho_{D^*} \tau_{t-1}^{D^*} + (1 - \rho_{D^*}) \overline{\tau}^{D^*} + \epsilon_{D^* t} \tag{1.67}
$$

where $\overline{\tau}^{D^*}$ is the steady state level of tax rate on foreign borrowings of banks, and $\epsilon_{D^* t}$ is an innovation that follows a normal process with a mean of zero and a standard deviation of $\sigma_{D^*}$.\(^{23}\)

\(^{23}\)I will conduct an impulse response exercise with the AR(1) process of $\tau_t^{D^*}$ to show how this policy affects the real economy. But $\tau_t^{D^*}$ will also be an instrument for the Ramsey planner when I turn to the Ramsey policy.
I assume that foreign borrowings $D_t^*$ are non-contingent riskless short-term debt that pays the gross nominal return of $(1 + i_t^*), \text{ in terms of foreign currency, from } t \text{ to } t + 1. \text{ Then, the equity capital or net worth of the bank evolves as the difference between earnings on assets and interest payments on liabilities as follow:}

$$P_t n_t = R_{Ht} P_t q_{t-1} s_{Ht-1} + R_{Nt} P_t q_{t-1} s_{Nt-1} - (1 + i_{t-1}) D_{t-1} - NOM_t (1 + i_{t-1}) D_{t-1}^*$$

(1.68)

where $R_{Ht} = \frac{z_{Ht} + (1 - \delta) q_t}{q_{t-1}}$ and $R_{Nt} = \frac{z_{Nt} + (1 - \delta) q_t}{q_{t-1}}$.

In real terms,

$$n_t = R_{Ht} q_{t-1} s_{Ht-1} + R_{Nt} q_{t-1} s_{Nt-1} - R_{t-1} d_{t-1} - R_{t-1}^* RER_t d_{t-1}$$

(1.69)

where $d_t = \frac{D_t}{P_t}$, $d_t^* = \frac{D_t^*}{P_t^*}$, $RER_t = \frac{NOM_t P_t^*}{P_t}$, $R_{t-1} = (1 + i_{t-1}) \frac{P_{t-1}}{P_t}$, and $R_{t-1}^* = (1 + i_{t-1}^*) \frac{P_{t-1}^*}{P_t^*}$.

Given that a banker pays dividends only when she exits, taking prices as given, a banker in period $t$ maximizes expected discounted value of terminal dividends paid to her members of household

$$V_t = E_t \sum_{\tau=t+1}^{\infty} (1 - \eta) \eta^{t-1} \Lambda_{t,\tau}^T n_{\tau}$$

(1.70)

where a banker values payoffs in each period and state using $\Lambda_{t,\tau}^T$ the stochastic discount factor of households in tradable sector, which is equal to the marginal rate of substitution between consumption of period $t + i$ and period $t$ of households in tradable sector.

Following Aoki et al (2018), to motivate a constraint on the banker’s ability to obtain funds from households and foreign investors, I assume the following moral hazard problem: after purchasing the securities of tradable and non-tradable goods producers, the banker can decide whether to operate honestly or divert from her assets and transfer some fraction of assets to the household of which she is a member. If
the banker diverts assets, then her bank defaults on its debt and is shut down, and its creditors reclaim the remaining fraction of assets. Since creditors recognize the banker’s incentive to divert funds, they will restrict the amount of funds they lend to the banks. In this way a borrowing constraint arises.

Also, I assume that banker’s ability to divert assets depends upon the source of funds. Specifically,

$$\Theta(x_t) = \theta_0 \exp(-\theta x_t)$$

(1.71)

where $\theta_0 > 0$, $\theta > 0$, and $x_t = \frac{RER_t d_t^*}{q_t s_{Ht} + q_t s_{Nt}}$ is the fraction of assets financed by foreign borrowings. $\theta > 0$ implies that the banker can divert a smaller fraction of assets when she raises the fraction of assets financed by foreign borrowings.

In order to ensure a banker not to divert assets so that creditors are willing to supply funds to the banker, the following incentive constraint must be satisfied:

$$V_t \geq \Theta(x_t)(q_t s_{Ht} + q_t s_{Nt})$$

(1.72)

This inequality indicates that the loss from diverting a fraction of assets, the franchise value of bank $V_t$, should be greater than or equal to the gain from doing so.

Moreover, I assume that foreign borrowings are costly in terms of resources

$$\chi^b(RER_t d_t^*, q_t s_{Ht} + q_t s_{Nt}) = \frac{k^b}{2} x_t^2 (q_t s_{Ht} + q_t s_{Nt})$$

(1.73)

Therefore, although an increase in borrowing from foreign investors may relax the bank’s borrowing constraint, it also incurs an increase in resource costs.

Switching to the recursive formulation for the franchise value of bank in the end of period $t$, I can write the banker’s problem as follow:

$$V_t = \max_{s_{Ht}, s_{Nt}, d_t, d_t^*} E_t \{ \Lambda^T_t \{ (1 - \eta) n_{t+1} + \eta V_{t+1} \} \}$$

(1.74)
subject to the balance sheet constraint (1.66), (1.69) and the incentive constraint (1.72):

\[
\left(1 + \frac{k^b}{2} x_t^2\right) (q_t s_{Ht} + q_t s_{Nt}) = n_t + d_t + (1 - \tau_t^D) RER_t d_t^* \\
n_t = R_{Ht} q_{t-1} s_{Ht-1} + R_{Nt} q_{t-1} s_{Nt-1} - R_{t-1} d_{t-1} - R_{t-1}^* RER_t d_{t-1}^* \\
V_t \geq \Theta(x_t)(q_t s_{Ht} + q_t s_{Nt})
\]

The associated optimality conditions with the banker’s problem where the incentive constraint is binding, \(V_t = \Theta(x_t)(q_t s_{Ht} + q_t s_{Nt})\), are given by

\[
\frac{\mu^*_t}{\nu_t} = \left(\frac{1}{\theta + x_t}\right) \left[\left(\theta + \frac{x_t}{2}\right) \chi^b x_t - \frac{\mu_{Ht}}{\nu_t}\right] \tag{1.75}
\]

\[
\mu_{Ht} = E_t[\Omega_{t+1}(R_{Ht+1} - R_t)] = \mu_{Nt} = E_t[\Omega_{t+1}(R_{Nt+1} - R_t)] > 0 \tag{1.76}
\]

\[
\mu^*_t = E_t \left\{ \Omega_{t+1} \left[ (1 - \tau_t^D) R_t - \frac{RER_{t+1}^*}{RER_t} R_{t+1}^* \right] \right\} > 0 \tag{1.77}
\]

\[
\nu_t = E_t[\Omega_{t+1} R_t] \tag{1.78}
\]

\[
\Omega_{t+1} = \Lambda_{t+1}^T (1 - \eta + \eta \psi_{t+1}) \tag{1.79}
\]

\[
\psi_t = \Theta(x_t) \phi_t \tag{1.80}
\]

where \(\mu_{Ht}\) is the expected discounted excess return on capital in the tradable sector over deposits, \(\mu_{Nt}\) is the expected discounted excess return on capital in the non-tradable sector over deposits, \(\mu^*_t\) is the expected discounted cost advantage of foreign borrowings over deposits, \(\nu_t\) is the expected discounted marginal cost of deposits, and \(\Omega_{t+1}\) is the stochastic discount factor of banker. Equation (1.75) shows that given \(\theta\) and \(\chi^b\), the relationship between \(\frac{\mu_{Ht}}{\nu_t}\) and \(\frac{\mu^*_t}{\nu_t}\) depends on \(x_t\), as bankers adjust the latter to equalize the marginal benefit of foreign borrowings with their marginal cost.

Also, I obtain a unique \(x_t = \frac{RER_t d_t^*}{q_t s_{Ht} + q_t s_{Nt}} > 0\) as follow:

\[
x_t = \frac{\mu^*_t}{\kappa^b \nu_t} - \frac{1}{\theta} + \sqrt{\left(\frac{\mu^*_t}{\kappa^b \nu_t}\right)^2 + \left(\frac{1}{\theta}\right)^2 + 2 \frac{\mu_{Ht}}{\kappa^b \nu_t}} \tag{1.81}
\]
Moreover, the leverage ratio $\phi_t = \frac{q_t s_{Ht} + q_t s_{Nt}}{n_t}$ satisfies:

$$\phi_t = \frac{\nu_t}{\Theta(x_t) + \frac{\mu^*}{\nu^*} x_t^2} - (\mu_{Ht} + \mu_{*t} x_t)$$  \hspace{1cm} (1.82)

Equations (1.81) and (1.82) show that the fraction of assets financed by foreign borrowings $x_t$ is an increasing function of $\frac{\mu_{Ht}}{\nu_t}$ and $\frac{\mu^*}{\nu^*}$, while the leverage ratio $\phi_t$ is decreasing in $\Theta(x_t)$ the fraction of assets bankers can divert and increasing in $\frac{\mu_{Ht}}{\nu_t}$ and $\frac{\mu^*}{\nu^*}$.

Given that $x_t$ and $\phi_t$ are independent of bank-specific factors, I can sum across individual banks to obtain the relation for the demand for total bank assets $q_t s_{Ht} + q_t s_{Nt}$ as a function of total net worth $n_t$ as follow:

$$q_t s_{Ht} + q_t s_{Nt} = \phi_t n_t$$  \hspace{1cm} (1.83)

An equation of motion for total net worth in banking sector $n_t$ is the sum of the net worth of existing bankers, $n_{et}$, and the net worth of entering bankers, $n_{nt}$

$$n_t = n_{et} + n_{nt}$$  \hspace{1cm} (1.84)

The net worth of existing bankers equals the earnings on assets held in the previous period net the costs of foreign borrowings and deposits, multiplied by the fraction of bankers that survive until the current period $\eta$

$$n_{et} = \eta \{ R_{Ht} q_{lt-1} s_{Ht-1} + R_{Nt} q_{lt-1} s_{Nt-1} - R_{lt-1} d_{lt-1} - R_{lt-1}^e R E R_{lt-1}^e \}$$  \hspace{1cm} (1.85)

Assuming that the household transfers to each new banker a fraction $\frac{\xi}{1-\eta}$ of the total assets of exiting bankers in period $t$, $(1-\eta)(R_{Ht} q_{lt-1} s_{Ht-1} + R_{Nt} q_{lt-1} s_{Nt-1})$, the aggregate net worth of new bankers is then as follow:

$$n_{nt} = \xi (R_{Ht} q_{lt-1} s_{Ht-1} + R_{Nt} q_{lt-1} s_{Nt-1})$$  \hspace{1cm} (1.86)
Total net worth of bankers is now

\[ n_t = \eta(R_{Ht}q_{t-1}s_{Ht-1} + R_{Nt}q_{t-1}s_{Nt-1} - R_{t-1}d_{t-1} - R^*_t RER_t d^*_t) \]

\[ + \xi(R_{Ht}q_{t-1}s_{Ht-1} + R_{Nt}q_{t-1}s_{Nt-1}) \]  \quad (1.87)

1.2.6 Monetary Policy

As the baseline policy, monetary policy is characterized by a simple Taylor rule that targets inflation under flexible exchange rate arrangement, with the smoothing parameter \( \rho_i \). Let \( i_t \) be the net nominal interest rate, \( \bar{i} \) the steady-state nominal rate. Then,

\[ i_t = \rho_i i_{t-1} + (1 - \rho_i) \bar{i} + \kappa \pi (\Pi_t - 1) \]  \quad (1.88)

I consider two alternative monetary policy scenarios. In first scenario, I assume a managed exchange rate regime, in which monetary policy is conducted according to an augmented Taylor rule that responds to fluctuations of the nominal exchange rate in addition to inflation:

\[ i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[ \bar{i} + \frac{\kappa_{NOM}}{\kappa_{NOM}} \pi (\Pi_t - 1) + \frac{\kappa_{NOM}}{1 - \kappa_{NOM}} \log \left( \frac{NOM_t}{NOM_{t-1}} \right) \right] \]  \quad (1.89)

where \( \kappa_{NOM} \in [0, 1] \): this specification represents a strict inflation targeting regime when \( \kappa_{NOM} = 0 \), and higher values of \( \kappa_{NOM} \) represent cases in which the exchange rate stabilization is more important than inflation targeting.\(^{24}\)

In second scenario, I assume a fixed exchange rate regime where the nominal interest rate is set to maintain the nominal exchange rate at a constant value, i.e.,

\[ NOM_t = \overline{NOM}, \quad \forall t \]  \quad (1.90)

\(^{24}\)I follow the formulation of managed exchange rate regime in Gali and Monacelli (2016).
1.2.7 Sterilized Foreign Exchange Intervention and the Government

In addition to monetary policy rules, I assume that the central bank holds foreign bonds for use in sterilized foreign exchange interventions. In particular, the central bank intervenes in the foreign financial markets by purchasing or selling foreign bonds, along with an open market operation in the opposite direction. In this framework, domestic bonds and bank deposits are imperfect substitutes (as shown in Equation (1.29)) and a deviation from the uncovered interest rate parity condition endogenously arises due to imperfect arbitrage resulting from financial frictions (as shown in Equation (1.75)). As a result, a portfolio balance sheet channel emerges and thus sterilized interventions to change the quantities of domestic and foreign bonds in the portfolio of the central bank are effective in influencing the exchange rate. Also, I assume that all profits or losses generated from the portfolio of bonds held by the central bank are rebated to households in the tradable sector through lump-sum transfers.

Let $B_{ct}$ be the nominal domestic bonds and $B^*_{ct}$ the nominal foreign currency bonds held by the central bank in period $t$. The central bank’s balance sheet is then given by

$$B_{ct} - B_{ct-1} + NOM_t(B^*_{ct} - B^*_{ct-1}) = 0$$  \hspace{1cm} (1.91)

In real terms,

$$b_{ct} - \frac{b_{ct-1}}{\Pi_t} + RER_t (b^*_{ct} - b^*_{ct-1}) = 0$$  \hspace{1cm} (1.92)

where $b_{ct} = \frac{B_{ct}}{P_t}$ and $b^*_{ct} = \frac{B^*_{ct}}{P^*_t}$. I assume that $b^*_{ct}$ follows the exogenous stochastic process

$$\log b^*_{ct} = \rho_{b^*} \log b^*_{ct-1} + (1 - \rho_{b^*}) \log \bar{b}^*_{ct} + \epsilon_{b^*t}$$  \hspace{1cm} (1.93)

where $\bar{b}^*_{ct}$ is the steady state level of foreign bonds held by the central bank, and $\epsilon_{b^*t}$ is an innovation to the shock and follows a normal process with a mean of zero and
a standard deviation of $\sigma_{b_t^*}$. The real profits generated from the portfolio of bonds held by the central bank are given by

$$\frac{TR_{st}}{P_t} = R_{bt-1}b_{ct-1} - b_{ct} + R^*_t \left( RER_t b^*_t - RER_t b^*_t \right)$$  \hfill (1.94)

The government finances its expenditures and interest payments for matured domestic bonds by issuing new domestic bonds and levying lump-sum taxes to households in the tradable sector. Let $G_t$ denote the nominal government expenditures, $B_{gt}$ the supply of nominal domestic bonds, and $T_{gt}$ the nominal lump sum taxes. The government’s flow budget constraint is then given by

$$G_t + (1 + i_{bt-1})B_{gt-1} = B_{gt} + T_{gt}$$  \hfill (1.95)

where $i_{bt-1}$ is the net nominal interest rate on domestic bonds from $t-1$ to $t$.

In real terms,

$$g_t + R_{bt-1}b_{gt-1} = b_{gt} + \frac{T_{gt}}{P_t}$$  \hfill (1.96)

where $g_t = \frac{G_t}{P_t}$, $R_{bt-1} = \frac{1+i_{bt-1}}{P_t}$, and $b_{gt} = \frac{B_{gt}}{P_t}$.

I assume that real government expenditures and the supply of real domestic bonds are exogenously fixed at constant levels, i.e., $g_t = \bar{g} = 0$ and $b_{gt} = \bar{b}_g$. The real lump-sum taxes $\frac{T_{gt}}{P_t}$, are then endogenously determined to satisfy the government’s budget constraint:

$$\frac{T_{gt}}{P_t} = (R_{bt-1} - 1)\bar{b}_g$$  \hfill (1.97)

Finally, total real lump-sum taxes levied from the government and central bank to households in the tradable sector are given by

$$\mu_t \frac{T^T_{t}}{P_t} = \frac{T_{gt}}{P_t} - \frac{TR_{st}}{P_t} + \mu \tau_{W^T w_t} L_t^T - \tau_t^{*} RER_t d_t^*$$  \hfill (1.98)

\footnote{I will conduct an impulse response exercise with the AR(1) process of $b_{ct}^*$. But I will also use $b_{ct}^*$ as an instrument for the Ramsey planner in the optimal policy analysis.}
where the third term is the real lump sum taxes from the fiscal subsidy on nominal wages of households in the tradable sector, and the last term is the real lump sum transfers from taxing foreign borrowing of banks.\footnote{I assume that the government and central bank cannot use the tax policy to redistribute income across agents. To avoid the redistribution through the tax policy, the lump-sum taxes or transfers are used only to undo all the other taxes levied or subsidies given to households in the tradable sector.}

1.2.8 Market Equilibrium

The market clearing conditions for non-tradable and domestically-produced tradable goods are

\[
(1 - \mu)Y_{Nt} = c_{Nt} = \left(\frac{P_{Nt}}{P_t}\right)^{-\rho} (1 - \gamma)c_t \tag{1.99}
\]

\[
\mu Y_{Ht} = \left(\frac{P_{Ht}}{P_{Tt}}\right)^{-\rho_T} \gamma_T \left\{ c_{Tt} + \left[ 1 + \frac{\kappa_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] I_t + \left( \frac{P_{Tt}}{P_t} \right)^{-1} \frac{\kappa b}{2} x_t^2 q_t K_t \right\} + E X_t \tag{1.100}
\]

where

\[
c_t = \mu c^T_t + (1 - \mu) c^N_t \tag{1.101}
\]

\[
K_t = \mu K_{Ht} + (1 - \mu) K_{Nt} \tag{1.102}
\]

The real aggregate output is given by

\[
Y_t = \left(\frac{P_{Ht}}{P_t}\right) \mu Y_{Ht} + \left(\frac{P_{Nt}}{P_t}\right) (1 - \mu) Y_{Nt} \tag{1.103}
\]

The market equilibrium for bank deposits implies

\[
d_t = \mu d^T_t + (1 - \mu) d^N_t \tag{1.104}
\]

The market equilibrium for domestic bonds implies

\[
b_{gt} = \bar{b}_g = b_{ct} + \mu b_t^T \tag{1.105}
\]
The aggregate capital stock evolves according to

\[ K_t = (1 - \delta) K_{t-1} + I_t \]  

(1.106)

The real trade balance is defined as

\[ TB_t = \frac{P_{Ht}}{P_t} E X_t - \frac{P_{Ft}}{P_t} \left( \frac{P_{Ft}}{P_{Tt}} \right)^{-\rho_T} (1 - \gamma_T) \]

\[ \left\{ c_{Tt} + \left[ 1 + \frac{\kappa_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] I_t + \left( \frac{P_{Tt}}{P_t} \right)^{-1} \frac{\kappa^b}{2} x_t q_t K_t \right\} \]  

(1.107)

The real current account is given by

\[ CA_t = TB_t + \text{RER}_t (R^*_t - 1)b_{ct-1}^* - \text{RER}_t (R^*_t - 1)d_{t-1}^* \]  

(1.108)

The balance of payments implies that the real current account equals the net foreign capital outflows:

\[ CA_t = \text{RER}_t (b_{ct}^* - b_{ct-1}^*) - \text{RER}_t (d_t^* - d_{t-1}^*) \]  

(1.109)

1.3 Model Calibration

I calibrate the home economy to represent a small open economy in EMEs. Table 1.1 summarizes parameter values in baseline calibration. The model is calibrated at a quarterly frequency. I set the home discount factor \( \beta \) to 0.99, implying a steady-state annual real interest rate of 4 percent, which is a value commonly used in the real business-cycle literature. The foreign real interest rate is set to 2 percent annually, following an estimate used in recent studies (see, for example, Akinci and Queralto 2019). The inverse of the intertemporal elasticity of substitution \( \sigma \) is set to 2, a standard value used in the literature on emerging market economies (see, for example, Aguiar and Gopinath, 2007; Garcia-Cicco et al., 2010). I set the utility weight of real domestic bond holdings for households in tradable sector \( \theta_b \) to 0.0007 so that
a steady state interest rate spread between bank deposits and domestic bonds is 27 basis points annually, which is consistent with the average spread in Mexico over the period 2009-2018. In terms of parameters related to labor markets, I set the inverse of Frisch elasticity of labor supply $\chi$ to 3, as in Gali and Monacelli (2005). Given the value of $\chi$, the disutility of labor scale parameter $\chi_0$ is calibrated to 100.33 so that a steady-state weighted average of households’ labor hours are 1/3 of the time endowment. I assume that tradable and non-tradable sectors share the same Cobb-Douglas production technology. I set the capital share $\alpha$ to 0.33, and the depreciation rate of capital $\delta$ to 0.025, which are conventional values in the literature. The cost parameter of adjusting investment goods production $\kappa_I$ is calibrated to 0.67 so that the price elasticity of investment is consistent with a value in Aoki et al. (2018). To calibrate baseline values of the share of non-tradable goods in consumption composite $1 - \gamma$, and the population share of households in non-tradable sector $1 - \mu$, I use estimates for emerging market economies from Prasad and Zhang (2015). Accordingly, I set them to 0.4.

Turning to parameters related to the international trade, I set the elasticity of substitution between domestically and foreign produced tradable goods $\rho_T$ to 1.5, as in Backus et al. (1994) and Heathcote and Perri (2016). The weight of domestically produced tradable goods in tradable consumption composite $\gamma_T$, and the exogenous parameter of foreign demand for domestically produced tradable goods $Y^*$ are, respectively, set to 0.8 and 0.37, so that a steady-state ratio of home country’s export to output of 20 percent, which is consistent with an estimate for emerging market economies used in Akinci and Queralto (2019). The price elasticity of export demand, $\rho^*$, is set to a conventional value of 1. The elasticity of substitution between tradable and non-tradable goods $\rho$ is set to 0.44, which is consistent with an estimate for Argentina in Gonzalez Rozada et al. (2004).
Regarding parameters related to the sticky nominal wages, I calibrate the degree of monopolistic competition for labor markets by assuming a steady state markup of 20 percent, which implies $\varepsilon_{WT} = \varepsilon_{WN} = 6$, which are commonly assumed values in the literature. The Calvo parameters of wage rigidities $\varrho_{WT}$ and $\varrho_{WN}$ are set to 0.7, as a conventional value, which corresponds to the average duration of wage contract of 3.33 quarters.

Regarding parameters governing banking sector, I rely on estimates for emerging market economies from Aoki et al. (2018) and Akinci and Queralto (2019). I set the survival probability of financial intermediaries $\eta$ to 0.94, implying an expected horizon of 4.16 years. I set $\theta$ to 0.1, implying that a rise of the fraction of bank assets financed by foreign borrowings by 10 percent lowers the fraction of bank assets banker can divert by 1 percent. I calibrate three parameters, the fraction of bank assets transferred to new entering banks $\xi$, the fraction of assets that can be diverted $\theta_0$, and the resource costs for foreign borrowings $\kappa_b$, to jointly match the following three targets: a steady state bank leverage ratio of 5, a steady state spread between lending and deposit rates of 200 basis points annually, and a steady state ratio of foreign borrowings to deposits of 30 percent. These imply that $\xi = 0.0049$, $\theta_0 = 0.362$, and $\kappa_b = 0.0302$.

In the baseline experiments, I focus on a steady state equilibrium with zero inflation $\Pi = 1$, zero tax on foreign borrowings of banks $\tau^{D^*} = 0$. I calibrate the steady state levels of the government’s supply of domestic bonds $\delta_g$ and the central bank’s foreign bond holdings $\delta_c^*$ to hit the following two targets: a steady state annual ratio of domestic government debt to output of 26 percent, and a steady state annual ratio of the central bank’s holdings of foreign reserve to output of 12 percent, which are consistent with the average ratios in Mexico over the period 2004-2018. I set the coefficients of monetary policy rule as $\kappa_{\pi} = 1.5$ and $\rho_i = 0.8$ quarterly, as conven-
tional values. I set the coefficient of managed exchange rate regime $\kappa_{NOM} = 0.6$ as the welfare-maximizing value from $\kappa_{NOM} \in [0, 1]$. Following Aoki et al. (2018), the foreign real interest rate ($R_t^* - 1$) follows AR(1) process with a persistent coefficient of 0.9 quarterly and a standard deviation for innovations of 1 percent annually.

1.4 Optimal Monetary Policy Rule

In analyzing the optimal monetary policy rule, I present the quantitative results of alternative monetary policy rules in two steps. First, I analyze the impulse responses of key aggregate and sectoral variables to an unexpected rise in foreign real interest rate for the model under alternative monetary policy rules. Second, I analyze the consequences for welfare and volatility associated with alternative monetary policy rules.

1.4.1 Aggregate and Distributional Effects of Monetary Policy Rules

I consider three alternative monetary policy scenarios. In first scenario, I assume that the nominal interest rate is set according to an inflation targeting rule under flexible exchange rate regime. In second scenario, the nominal interest rate is set according to a managed exchange rate regime with the optimal degree of the exchange rate stabilization. In third scenario, a fixed exchange rate regime, in which the central bank sets its nominal interest rate to maintain the nominal exchange rate at a constant value.

$^{27}$In the model with baseline calibration, there is a value for the response to the exchange rate in managed exchange rate regime ($\kappa_{NOM} = 0.6$) that maximizes aggregate welfare. Figure 1.7 presents welfare gains associated with $\kappa_{NOM} \in (0, 1]$ relative to a strict inflation targeting regime (when $\kappa_{NOM} = 0$).
The impulse response functions are simulated with first order approximation of the decision rules around the non-stochastic steady state. I consider a twenty-five basis point positive shock to the foreign net real interest rate \((R^*_t - 1)\) that follows a first-order autoregressive process that persists at a rate of 0.9 per quarter (in Equation (1.22)). The impulse responses of key aggregate variables to the shock under alternative monetary policy rules are presented in Figure 1.1. The blue-solid line represents the responses under an inflation targeting regime. For comparison, the red-dashed line represents the responses under a manage exchange rate regime with the optimal degree of exchange rate stabilization, while the green-dashed line represents the responses under a fixed exchange rate regime.

The positive shock to foreign real interest rate produces an immediate depreciation of the domestic currency and a sudden reversal of capital inflows to the small open economy. The green-dashed line shows that under a fixed exchange rate regime, on impact of the shock, the nominal interest rate is forced to rise in order to maintain the nominal exchange rate at a constant value. The tightening of monetary policy induces a significant increase in real interest rate, which in turn causes a contraction in investment, consumption and output. The financial accelerator amplifies the initial domestic responses to the shock. The combination of the real exchange rate depreciation and capital outflows, and the increase in real interest rate induced by monetary tightening leads to a substantial decrease in bank net worth and a significant rise in the cost of borrowing for banks from both domestic and foreign financial markets and thus a sharp tightening of balance sheet constraints of banks. This leads to a further contraction in investment and a decline in capital price and thus a further worsening of domestic balance sheets. This feedback loop eventually sends the economy into a recession. In addition, the sharp contraction of real activity associated with the strongly stabilizing the exchange rate leads to a sharp decrease in inflation.
Figure 1.1 also illustrates that the responses are significantly more stable under an inflation targeting regime than those under a fixed exchange rate regime. Under an inflation targeting regime, the nominal interest rate is no longer strongly tied to the foreign interest rate. The nominal interest rate is instead governed by a Taylor rule that pursues inflation stability. In this case, the rise in foreign real interest rate that produces a depreciation of the exchange rate leads to an increase in exports and a rise in inflation. According to an inflation targeting rule, the nominal interest rate is forced to rise in order to fight the higher inflation. This monetary tightening, however, induces a much less degree of increase in the real interest rate compared to the case under a fixed exchange rate regime and thus a moderate initial contraction in investment and consumption. Moreover, this leads to a moderate increase in the cost of borrowing for banks and thus a less tightening of balance sheet constraints, which in turn produces a much lower degree of financial accelerator that amplifies the initial impact of the shock. As a consequence, output and investment fall to a lesser extent than under a fixed exchange rate regime. Furthermore, under an inflation targeting regime, inflation rises as the exchange rate depreciates but inflation volatility is smaller than under a fixed exchange rate regime.

Turning to the responses under a managed exchange rate regime with the optimal degree of exchange rate stabilization, the figure shows that the optimal degree of exchange rate stabilization induces a rise in the real interest rate to a greater extent than under an inflation targeting regime, but to a lesser extent compared to a fixed exchange rate regime, thereby leading to a contraction of real activity to the degree in between those under the two alternative regimes. But the optimal degree of stabilization also has the effect of stabilizing inflation compared to the alternative regimes. As so far discussed, given inefficiencies arising from the banking sector that features financial frictions, the tradeoffs that monetary policy faces become richer.
to an inflation targeting regime, a monetary tightening, which pursues the exchange rate stabilization, raises the cost of borrowing for banks and thus tightens the balance sheet constraints, which triggers the financial accelerator that eventually depresses the economy to a greater extent; however, the optimal degree of exchange rate stabilization has more stable inflation, while strongly stabilizing (or fixing) the exchange rate leads to a sharp reduction in inflation and in turn a higher inflation volatility. Therefore, monetary policy now needs to navigate a tradeoff between inflation stability and exchange rate stability.

Figure 1.2 presents the impulse responses of key sectoral variables to the shock under alternative monetary policy rules. The blue-solid line represents the responses under an inflation targeting regime, while the red-dashed line represents the responses for the optimal degree of exchange rate stabilization, and the green-dashed line represents the responses under a fixed exchange rate regime. Under an inflation targeting regime, the depreciation of the exchange rate caused by the shock improves the trade competitiveness, leading to a rise in the demand for domestically-produced tradable goods and thus a slight rise in output in the tradable sector, despite of the contraction of real activity. Also, in the presence of nominal wage rigidities, the ensuing increased inflation associated with the depreciation reduces real wages. However, the exchange rate stabilization leads to a more tightening of balance sheet constraints and a further contraction of real activity compared to an inflation targeting regime as shown above, but the stabilization also degrades the competitiveness and therefore reduces the demand for domestically-produced tradable goods. Consequently, the exchange rate stabilization results in a greater degree of contraction of output in the tradable sector than in the non-tradable sector.

Moreover, the contraction in consumption and output associated with the stabilization leads to an increase in households’ marginal utility of consumption and a
decrease in marginal utility of leisure, thereby reducing nominal wages in order to restore the balance between marginal utilities of consumption and leisure. But the size of that reduction in nominal wages is smaller than the size of the reduction in inflation associated with the exchange rate stabilization, leading to an increase in real wages. Since nominal wages slightly rise and real wages fall under an inflation targeting regime, the optimal degree of exchange rate stabilization has the effect of stabilizing real wages; however, strongly stabilizing (or fixing) the exchange rate causes a sharp rise in real wages, as the size of reduction in inflation is much larger than the size of reduction in nominal wages. Hence, monetary policy faces another dimension of tradeoffs. Relative to an inflation targeting regime, the exchange rate stabilization can stabilize real wages, but this also entails a worsening of the trade competitiveness and a further tightening of financial constraints. This in turn causes a further contraction of output but to a greater degree in the tradable sector than in the non-tradable sector.

1.4.2 Welfare Analysis

In this section, I examine performances of alternative monetary policy rules by conducting welfare evaluations around the non-stochastic steady state, as in Schmitt-Grohe and Uribe (2007). I define $\lambda^g_j$ as the fraction that has to be added to the consumption process for household $j$ under the baseline policy to yield a level of welfare of household $j$ equivalent to that under alternative monetary policy rule. Then let $\lambda^T_j$ (and $\lambda^N_j$) denotes the welfare gain of household $j$ in tradable sector (and that of household $j$ in non-tradable sector) of adopting alternative monetary policy rule instead of the baseline policy, conditional on a particular state in period zero.\textsuperscript{28}

\textsuperscript{28}Since the non-stochastic steady state is the same across alternative monetary policy rules, computing expected welfare conditional on the non-stochastic steady state ensures
Formally, $\lambda^T_g$ and $\lambda^N_g$ are defined as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{1 - \sigma} \left( (1 + \lambda^T_g) \sum_{t=0}^{T_g} \right) \right]^{1 - \sigma} - \frac{\chi_0}{1 + \chi} (L_{jt}^{T_b})^{1 + \chi} + \theta_b \log b_t^{T_b}$$

(1.110)

and

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{1 - \sigma} \left( (1 + \lambda^N_g) \sum_{t=0}^{N_g} \right) \right]^{1 - \sigma} - \frac{\chi_0}{1 + \chi} (L_{jt}^{N_b})^{1 + \chi}$$

(1.111)

where $c_t^b$, $L_{jt}^b$, and $b_t^b$ ($c_t^a$, $L_{jt}^a$, and $b_t^a$), respectively, denote the contingent plans for consumption, labor supply, and domestic bond holdings of household $j$ under the baseline policy (under the alternative monetary policy rule).

Thus, a positive value of $\lambda^g_j$ indicates that the alternative policy rule achieves a higher welfare relative to the baseline policy. In order to obtain correct welfare rankings of alternative policy rules and their accurate welfare gains relative to the baseline policy, I approximate $\lambda^g_j$ up to second order of accuracy by conducting a second order approximation of the policy functions and welfares.

I calculate the welfare gain of households in tradable sector $\lambda^T_g$ and that of households in non-tradable sector $\lambda^N_g$ as:

$$\lambda^T_g = \int_0^1 \lambda^T_g dj$$

(1.112)

and

$$\lambda^N_g = \int_0^1 \lambda^N_g dj$$

(1.113)

I then calculate the welfare gain for aggregate household $\lambda^g$ as the population-weighted average of the welfare gains of the two types of households:

$$\lambda^g = \mu \lambda^T_g + (1 - \mu) \lambda^N_g$$

(1.114)

that the economy begins from the same initial point under all possible monetary policy rules.
Table 1.2 shows the welfare gains for two-type of households and for aggregate household under the two alternative regimes (the optimal degree of exchange rate stabilization and a fixed exchange rate regime) relative to the baseline policy (an inflation targeting regime). As clearly shown, welfare gains under alternative regimes considerably vary with the type of households. In particular, with regard to households in the tradable sector, the exchange rate stabilization always raises welfare loss. Turning to households in the non-tradable sector, the optimal degree of exchange rate stabilization leads to welfare gain, but the gain shrinks when monetary policy strongly targets the exchange rate. As a result, for aggregate household, the optimal degree of stabilization leads to welfare gain, while strongly stabilizing the exchange rate causes welfare loss compared to an inflation targeting regime.

The table also complements the welfare analysis by showing the volatilities of key aggregate and sectoral variables under alternative regimes. As shown above, the volatilities of output, consumption, and investment all rise along with the increases in volatilities of bank net worth and real interest rate, as monetary policy targets the exchange rate. Also, the volatilities of inflation and real wages reduce under the optimal degree of stabilization, while they rise under a fixed exchange rate regime, except real wages in the non-tradable sector. Consistent with aggregate consumption volatility, the volatility of consumption for each type of households increases under the exchange rate stabilization, but the size of that rise for each type of households differs and is larger for households in the tradable sector compared to the size in the non-tradable sector. Moreover, the exchange rate stabilization is less successful in bringing down the volatility of real wages in the tradable sector: under the optimal degree of exchange rate stabilization, the size of reduction is smaller than the size in the non-tradable sector, and under a fixed exchange rate regime, the volatility of real wages in the tradable sector eventually rises, while the volatility in the non-tradable sector still
shrinks. Therefore, as argued in the analysis of impulse responses, for households in the tradable sector, the adverse effects from greater financial instability are dominant and raise welfare losses as monetary policy targets the exchange rate. However, under the optimal degree of exchange rate stabilization, the benefits from increased stability of real wages is significant and more than offset by the losses associated with greater financial instability for households in the non-tradable sector, thus leading to the welfare gain under the optimal degree of stabilization.

1.5 Second Instrument in an Inflation Targeting Regime

In this section, I consider the use of taxes on foreign borrowings of banks or sterilized foreign exchange interventions as a second instrument of monetary policy, while the nominal interest rate is set according to an inflation targeting regime. To analyze the effects of the second instruments, I present the quantitative results of alternative second instruments in two steps. First, I analyze the impulse responses of key aggregate and sectoral variables in the face of shocks to alternative second instruments for the model under an inflation targeting regime. Second, I analyze the impulse responses to an unexpected rise in foreign interest rate for the model under the optimal use of alternative second instruments with an inflation targeting regime. To characterize the optimal use of second instrument, I focus on the Ramsey approach under which the planner sets alternative second instruments to maximize aggregate welfare, subject to all the equations characterizing the decentralized equilibrium.\textsuperscript{29} In solving for the

\begin{align*}
\mathcal{W} &= \mu \mathcal{W}^T + (1 - \mu) \mathcal{W}^N \\
\mathcal{W}^T &= \int_0^1 \mathcal{W}_j^T \, dj = \int_0^1 E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^T, L_{jt}^T, b_t^T) \, dj \\ 
\mathcal{W}^N &= \int_0^1 \mathcal{W}_j^N \, dj = \int_0^1 E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^N, L_{jt}^N) \, dj.
\end{align*}
optimal policy, I follow the timeless perspective as in Woodford (2003) and use the toolkit developed in Bodenstein et al. (2019).

1.5.1 Aggregate and Distributional Effects of Second Instrument

I assume that the tax rate on foreign borrowings of banks and the log level of foreign bond holdings of the central bank ($\tau^D_t$, $\log b^*_ct$) in Equations (1.67) and (1.93), respectively, follow independent AR(1) process with a persistent coefficient of 0.8. I consider the standard deviation of shocks to ($\tau^D_t$, $\log b^*_ct$) to be a negative 1% in quarterly rate.

Figure 1.3 presents the impulse responses of key aggregate and sectoral variables to an unexpected decrease in the tax rate on foreign borrowings of banks for the model under an inflation targeting regime. The decrease in tax rate on foreign borrowings initially lowers the cost of foreign borrowing for banks and thus relaxes the bank balance sheet constraints. This induces a rise in financial intermediation and an appreciation of the domestic currency as foreign borrowings of banks rise. As a result, capital price rises and the net worth of banks increases substantially due to both the rise of capital price and the exchange rate appreciation with associated reduction in foreign debt burden. Also, the ensuing lower inflation with the exchange rate appreciation requires the central bank to lower the nominal interest rate and thus further relaxes the balance sheet constraints. This in turn leads to a sharp rise in investment and consumption, and makes the expansionary effects of decreased tax rate on foreign borrowings significant in the domestic economy. The last two rows of the figure illustrate the responses of sectoral variables. Although the exchange rate appreciation degrades the trade competitiveness and depresses output in tradable sector, the rise in output in the non-tradable sector associated with the rise in aggregate demand is more than offset by the output fall in the tradable sector, resulting in a rise in
aggregate output. In addition, in the presence of nominal wage rigidities, the lower inflation induces a rise in real wages in both sectors.

Figure 1.4 illustrates the impulse responses to an unexpected decrease in the foreign bond holdings of the central bank along with sterilized interventions under the model with an inflation targeting regime. The sale of foreign bonds combined with purchasing domestic bonds initially causes an appreciation of the domestic currency and a decline in domestic bond supply. Then domestic demand for bank deposits rises as the supply of domestic bonds reduces, and the ensuing lower inflation associated with the exchange rate appreciation induces a decrease in the nominal interest rate. These together lead to a reduction in the cost of domestic borrowing for banks, which in turn causes a rise in the share of domestic deposits in bank liability and a rise in capital outflows. Since the rise in capital outflows mitigates the initial appreciation of the exchange rate, the degree of relaxation of the bank balance sheet constraints due to the exchange rate appreciation with associated drop in foreign debt burden is insignificant. As a consequence, in this case of sterilized intervention shock, the expansionary effect from the relaxed balance sheet constraints is smaller than the contractionary effect from the worsening of trade competitiveness, leading to a contraction in aggregate output.

1.5.2 Optimal Second Instrument with an Inflation Targeting Regime

In this section, to evaluate the effectiveness of alternative second instruments with an inflation targeting regime, I allow the Ramsey planner to use alternative second instruments while the nominal interest rate follows an inflation targeting rule under flexible exchange rate regime in response to foreign interest rate shocks.

Figure 1.5 presents the impulse responses to a positive foreign real interest rate shock for the model under the optimal time-varying tax rate on foreign borrowings
of banks chosen by the Ramsey planner (the red-dashed line). For comparison, the blue-solid line represents the responses for the model without the tax instrument. In both cases, monetary policy follows an inflation targeting regime. In the face of an adverse foreign interest rate shock, the planner initially reduces the tax rate on foreign borrowings of banks followed by smaller decreases in the tax rates. The reduced tax rate lowers the cost of foreign borrowing for banks, leading to a rise in the share of foreign borrowing in bank liability and hence an appreciation of the exchange rate. As a result, this mitigates the exchange rate depreciation associated with capital outflows triggered by the shock, leading to smaller fluctuations in the exchange rate, capital price, inflation, and the nominal interest rate than those in the economy without such Ramsey optimal policy. Therefore, the Ramsey planner can effectively insulate domestic financial markets from capital outflows caused by the shock and the economy can avoid a persistent drop in aggregate output despite of the more worsening of trade competitiveness. In addition, the resulting lower inflation stabilizes real wages in both sectors.

Figure 1.6 presents the impulse responses to a positive foreign interest rate shock for the model under the optimal sterilized interventions chosen by the Ramsey planner (the red-dashed line). As before, the blue-solid line illustrates the responses for the model without such sterilized interventions for comparison. In both cases, monetary policy follows an inflation targeting regime. With an unexpected foreign interest rate hike, the planner chooses to reduce the central bank’s holdings of foreign bonds followed by lower reductions, but the magnitudes of interventions are very small. The economy with the optimal sterilized interventions experiences somewhat lower domestic deposits and higher foreign borrowings than the economy does without such optimal policy, but other variables are virtually unaffected. Consistent with the discussion in the section above, in this framework, the use of sterilized foreign exchange
intervention would not be effective in dealing with the capital outflows with associated
disruption of domestic financial markets in the face of foreign monetary tightening
shock.

1.6 Conclusion

In this paper, I develop a two-sector, two-agent New-Keynesian model of a small
open economy with financial frictions and foreign currency debt in balance sheets.
Focusing on foreign interest rate shocks, I analyze the distributional and aggregate
welfare implications of alternative monetary policy rules to account for the desir-
ability of exchange rate stabilization in emerging market economies. I also evaluate
the effectiveness of the use of capital controls or sterilized foreign exchange interven-
tions with an inflation targeting regime for macroeconomic stabilization. There are
three main findings.

First, relative to an inflation targeting regime, as long as the response of mone-
tary policy to the exchange rate is not too strong, managed exchange rate regimes
can improve aggregate welfare. Such exchange rate regimes also induce welfare redis-
tribution across the two-types of households: they stabilize inflation and real wages,
while leading to a greater contraction of output in the tradable sector, relative to the
non-tradable sector. This, in turn, improves welfare of households in the non-tradable
sector at the expense of households in the tradable sector.

Second, in comparison with more flexible exchange rate regimes, managed
exchange rate regimes always reduce welfare of households in the tradable sector.
However, strongly stabilizing (or fixing) the exchange rate raises the volatility of
inflation and real wages, and also leads to a reduction in welfare of households in the
non-tradable sector.
Lastly, under an inflation targeting regime, the use of capital controls is effective in alleviating financial instability and thus enhancing macroeconomic stability, while the use of sterilized foreign exchange interventions is nearly ineffective in this environment.
Table 1.1: Baseline Parameters for the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Home discount rate</td>
<td>0.99</td>
</tr>
<tr>
<td>$R^*$</td>
<td>Foreign risk-free real interest rate</td>
<td>1.005</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Inverse of intertemporal elasticity of substitution</td>
<td>2</td>
</tr>
<tr>
<td>$\theta_b$</td>
<td>Utility weight of domestic bond holdings</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Inverse Frisch elasticity of labor supply</td>
<td>3</td>
</tr>
<tr>
<td>$\chi_0$</td>
<td>Disutility of labor scale parameter</td>
<td>100.33</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>0.33</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\kappa_I$</td>
<td>Investment adjustment cost parameter</td>
<td>0.67</td>
</tr>
<tr>
<td>$\rho_T$</td>
<td>Home and foreign-produced elasticity of substitution</td>
<td>1.5</td>
</tr>
<tr>
<td>$1-\gamma$</td>
<td>Share of non-traded goods in consumption</td>
<td>0.4</td>
</tr>
<tr>
<td>$1-\mu$</td>
<td>Share of households working in non-traded sector</td>
<td>0.4</td>
</tr>
<tr>
<td>$\gamma_T$</td>
<td>Share of home-produced in tradable goods consumption</td>
<td>0.8</td>
</tr>
<tr>
<td>$Y^*$</td>
<td>Foreign demand parameter of home-produced traded goods</td>
<td>0.37</td>
</tr>
<tr>
<td>$\rho^*$</td>
<td>Price elasticity of export demand</td>
<td>1</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Traded and non-traded goods elasticity of substitution</td>
<td>0.44</td>
</tr>
<tr>
<td>$\varepsilon_{WT}$</td>
<td>Traded sector labor elasticity of substitution</td>
<td>6</td>
</tr>
<tr>
<td>$\varepsilon_{WN}$</td>
<td>Non-traded sector labor elasticity of substitution</td>
<td>6</td>
</tr>
<tr>
<td>$\varrho_{WT}$</td>
<td>Calvo probability of keeping traded sector wages fixed</td>
<td>0.7</td>
</tr>
<tr>
<td>$\varrho_{WN}$</td>
<td>Calvo probability of keeping non-traded sector wages fixed</td>
<td>0.7</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Bank survival rate</td>
<td>0.94</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of leverage with respect to foreign borrowings</td>
<td>0.1</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>Bank asset divertible fraction</td>
<td>0.362</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Bank asset transfer rate</td>
<td>0.0049</td>
</tr>
<tr>
<td>$\kappa^b$</td>
<td>Resource costs for foreign borrowings</td>
<td>0.0302</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Steady state inflation rate</td>
<td>1</td>
</tr>
<tr>
<td>$\pi^{D^*}$</td>
<td>Steady state tax rate on bank’s foreign borrowings</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{b}_g$</td>
<td>Steady state domestic bond supply</td>
<td>1.115</td>
</tr>
<tr>
<td>$\bar{b}_c$</td>
<td>Steady state foreign bond holdings of the central bank</td>
<td>0.785</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Interest rate smoothing parameter</td>
<td>0.8</td>
</tr>
<tr>
<td>$\kappa_\pi$</td>
<td>Inflation coefficient of Taylor rule</td>
<td>1.5</td>
</tr>
<tr>
<td>$\kappa_{NOM}$</td>
<td>Exchange rate stabilization coefficient of managed regime</td>
<td>0.6</td>
</tr>
<tr>
<td>$\rho_{R^*}$</td>
<td>Persistence of foreign interest rate shocks</td>
<td>0.9</td>
</tr>
<tr>
<td>$\sigma_{R^*}$</td>
<td>Standard deviation of foreign interest rate shocks</td>
<td>0.025</td>
</tr>
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</table>
## Table 1.2: Welfare Gain and Volatility under Alternative Monetary Policy Rules

<table>
<thead>
<tr>
<th></th>
<th>Inflation Targeting (Baseline Policy)</th>
<th>Optimal Stabilization</th>
<th>Fixed Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Welfare Gain</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tradable sector ((\lambda^{Tg} \times 100))</td>
<td>-</td>
<td>-0.0177</td>
<td>-0.0268</td>
</tr>
<tr>
<td>Non-Tradable sector ((\lambda^{Ng} \times 100))</td>
<td>-</td>
<td>0.0518</td>
<td>0.0043</td>
</tr>
<tr>
<td>Aggregate Household ((\lambda^{g} \times 100))</td>
<td>-</td>
<td>0.0101</td>
<td>-0.0143</td>
</tr>
<tr>
<td><strong>Volatility</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_Y)</td>
<td>0.0018</td>
<td>0.0033</td>
<td>0.0099</td>
</tr>
<tr>
<td>(\sigma_c)</td>
<td>0.0033</td>
<td>0.0041</td>
<td>0.0079</td>
</tr>
<tr>
<td>(\sigma_I)</td>
<td>0.0229</td>
<td>0.0267</td>
<td>0.0409</td>
</tr>
<tr>
<td>(\sigma_n)</td>
<td>0.0734</td>
<td>0.0874</td>
<td>0.1346</td>
</tr>
<tr>
<td>(\sigma_{\Pi})</td>
<td>0.0021</td>
<td>0.0013</td>
<td>0.0028</td>
</tr>
<tr>
<td>(\sigma_{RER})</td>
<td>0.0134</td>
<td>0.0118</td>
<td>0.0052</td>
</tr>
<tr>
<td>(\sigma_R)</td>
<td>0.0010</td>
<td>0.0016</td>
<td>0.0029</td>
</tr>
<tr>
<td>(\sigma_{R^*})</td>
<td>0.0042</td>
<td>0.0042</td>
<td>0.0042</td>
</tr>
<tr>
<td>(\sigma_{cT})</td>
<td>0.0043</td>
<td>0.0051</td>
<td>0.0097</td>
</tr>
<tr>
<td>(\sigma_{cN})</td>
<td>0.0049</td>
<td>0.0052</td>
<td>0.0080</td>
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<tr>
<td>(\sigma_{wH})</td>
<td>0.0047</td>
<td>0.0045</td>
<td>0.0055</td>
</tr>
<tr>
<td>(\sigma_{wN})</td>
<td>0.0055</td>
<td>0.0048</td>
<td>0.0046</td>
</tr>
</tbody>
</table>

**Notes:** The welfare gain (\(\lambda^{g} \times 100\)) under each regime is expressed as percentage of quarterly consumption equivalent relative to an inflation targeting regime. The volatility \(\sigma_x\) denotes the standard deviation of the variable \(x\), where \(x\) denotes aggregate output (\(Y\)), aggregate consumption (\(c\)), aggregate investment (\(I\)), bank net worth (\(n\)), inflation (\(\Pi\)), real exchange rate (\(RER\)), real interest rate (\(R\)), foreign real interest rate (\(R^*\)), consumption of households in tradable sector (\(c^T\)), consumption of households in non-tradable sector (\(c^N\)), employment in tradable sector (\(L_H\)), employment in non-tradable sector (\(L_N\)), real wages of households in tradable sector (\(w_H\)), and real wages of households in non-tradable sector (\(w_N\)).
Figure 1.1: Foreign Interest Rate Shock: Aggregate Variables

Notes: This figure presents the impulse responses of key aggregate variables to an unexpected 1% annual increase in the foreign real interest rate for the model under alternative monetary policy rules. The blue-solid line represents the responses under an inflation targeting regime, while the red-dashed line (the green-dashed line) represents the responses under a managed exchange rate regime (under a fixed exchange rate regime). All responses are in log-deviations from the steady-state except the nominal policy rate and the foreign real interest rate, whose responses are in level deviations from the steady-state.
Figure 1.2: Foreign Interest Rate Shock: Sectoral Variables

Notes: This figure presents the impulse responses of key sectoral variables to an unexpected 1% annual increase in the foreign real interest rate for the model under alternative monetary policy rules. The blue-solid line represents the responses under an inflation targeting regime, while the red-dashed line (the green-dashed line) represents the responses under a managed exchange rate regime (under a fixed exchange rate regime). All responses are in log-deviations from steady-state except the foreign real interest rate, whose response is in a level deviation from steady-state.
Figure 1.3: Capital Control Policy Shock

Notes: This figure presents the impulse responses of key aggregate and sectoral variables to an unexpected 1% quarterly decrease in tax rate on foreign borrowings of banks for the model under an inflation targeting regime. All responses are in log-deviations from the steady-state except the nominal policy rate and the tax rate on foreign borrowing, whose responses are in level deviations from the steady-state.
Figure 1.4: Sterilized Intervention Policy Shock

Notes: This figure presents the impulse responses of key aggregate and sectoral variables to an unexpected 1% quarterly decrease in foreign bond holdings of the central bank along with the sterilized interventions for the model under an inflation targeting regime. All responses are in log-deviations from the steady-state except the nominal policy rate, whose response is in a level deviation from the steady-state. T, NT, F, and CB, respectively, represent tradable sector, non-tradable sector, foreign, and the central bank.
**Figure 1.5: Optimal Capital Controls with an Inflation Targeting Regime**

*Notes:* This figure presents the impulse responses of key variables to an unexpected 1% annual *increase* in foreign real interest rate for the model under the optimal tax on foreign borrowings of banks with an inflation targeting regime. The red-dashed line represents the responses under an inflation targeting regime with the Ramsey optimal time-varying tax rate on foreign borrowing of banks, while the blue-solid line represents the responses under an inflation targeting regime without the tax instrument. All responses are in log-deviations from the steady-state except the nominal policy rate, the tax rate on foreign borrowings of banks and the foreign interest rate shock, whose responses are in level deviations from the steady-state. T, NT, F, and FBs, respectively, represent tradable sector, non-tradable sector, foreign, and foreign borrowings.
Figure 1.6: Optimal Sterilized Interventions with an Inflation Targeting Regime

Notes: This figure presents the impulse responses of key variables to an unexpected 1% annual increase in foreign real interest rate for the model under the optimal sterilized foreign exchange intervention with an inflation targeting regime. The red-dashed line represents the responses under an inflation targeting regime with the Ramsey optimal sterilized interventions, while the blue-solid line represents the responses under an inflation targeting regime without the sterilized interventions. All responses are in log-deviations from the steady-state except the nominal policy rate and the foreign interest rate shock, whose responses are in level deviations from the steady-state. T, NT, F, and CB, respectively, represent tradable sector, non-tradable sector, foreign, and the central bank.
Figure 1.7: Welfare Gain under Alternative Managed Exchange Rate Regimes

Notes: This figure presents the welfare gains associated with alternative managed exchange rate regimes ($\kappa_{NOM} \in (0, 1]$) relative to a strict inflation targeting regime (when $\kappa_{NOM} = 0$) for the model with baseline calibration. Welfare gains are expressed as percentage of quarterly consumption. * in the black (solid) line indicates the optimal value of $\kappa_{NOM}$ where the welfare gain for aggregate household is maximized.
Chapter 2

Did the Unconventional Monetary Policy of the US Hurt Emerging Markets?

2.1 Introduction

Emerging economies have been buffeted by financial shocks emanating from the United States, and by the Federal Reserve’s response to the ensuing recession. The Fed lowered the fed funds rate to (near) zero in Dec 2008, and kept it there for seven years. Quantitative easing – and the rapid expansion of bank reserves held by the Fed – started about the same time. A byproduct of these policies was a weak dollar and a capital flow to emerging economies. There were immediate complaints of currency manipulation.\(^1\) Later, as the Fed was preparing to raise the fed funds rate, Raghuram Rajan (then head of the Reserve Bank of India) had a testy exchange with Ben Bernanke at the Brookings Institution. Rajan (2014) claimed that Fed’s policy was bad for emerging economies, and that the Fed should coordinate policies with central banks in emerging market countries. Bernanke replied that the Fed was reacting to domestic problems, and that emerging economies would have been worse off if the Fed had not acted to contain the crisis. In this paper, we offer an evaluation of these claims.\(^2\)

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\(^1\)For example, in September of 2010, Guido Mantega, Brazil’s finance minister, accused the Fed of starting a currency war. See, The Economist, June 22nd, 2019.

\(^2\)Our model does not capture all of the interactions that Rajan and Bernanke discussed. Other models, focusing on different features, may come to different conclusions.
The standard New Keynesian model is not well suited for such an evaluation. Holding the interest rate fixed for a long period of time results in price indeterminacy unless some ad hoc terminal conditions are imposed, and even then Del Negro et al. (2015) and others have shown that the model is flawed because of a "forward guidance puzzle." So here we draw upon the work of Diba and Loisel (2019). We explicitly model the market for bank reserves: the demand for reserves derives from the fact that they reduce the costs of banking, and the supply of reserves is set by the Fed’s open market operations (or quantitative easing/tightening). Diba and Loisel show that, as long as the demand for reserves is not fully satiated, the interest rate on reserves can be held constant for an indefinite period of time with no price indeterminacy or forward guidance puzzle. In this framework then, the Fed has two policy instruments: the interest rate on reserves and the supply of reserves.

In what follows, we evaluate the claims of Rajan and Bernanke using a two country model consisting of the United States (the US) and an emerging market country (the EM). The US is the core country from which large financial shocks flow; as stated above, its monetary policy instruments are the interest rate on, and the supply of, bank reserves. The EM may be thought of as a collection of emerging market countries (such as Mexico, India, South Africa, Turkey, South Korea and Russia) that experienced similar capital flows during the Great Recession; its monetary policy instrument is the interest rate on bank deposits. The EM collective is large enough to induce strategic interactions with the US.

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3Canzoneri et al. (2018) show that there is also a forward guidance puzzle for fiscal policy.  
4Alternatively, we could have resorted to the Fiscal Theory of the Price Level. We decided against this approach for reasons outlined in new paper and more generally in AER paper.  
5See Burger et al. (2017) and the IRF’s therein. See also Banerjee et al. (2016) and the references therein.
Capital flows may be of little policy interest unless there are important market inefficiencies (in addition to nominal price rigidities). In our model, banks in the US and the EM face the financial frictions described by Gertler and Karadi (2011); in addition, (1) US banks incur costs in making loans that can only be ameliorated by holding reserves at the Fed, and (2) EM banks and households inherit the "original sin" of having their foreign borrowing and lending denominated in dollars. These financial frictions raise the possibility that policy interventions may be desirable, and that there may be gains from some sort of policy coordination.

So, how do we plan to evaluate the claims of Rajan and Bernanke? We set the stage by assuming that: (1) the US has been hit by Gertler et al. (2012)'s capital quality shock, and (2) the Fed has already fixed the interest rate on reserves (near) zero. As we shall see, the capital quality shock can be interpreted as a deterioration of banks' net worth; Gertler et al. (2012) and others have used this shock to model the Great Recession. First, we show how this shock passes through the US and on to the EM when there is no policy response from the Fed; that is, when the supply of reserves is held constant. Then, we see how a self-centered response by the Fed would change the outcome; here a Ramsey planner sets the US policy for reserves to maximize US welfare while the EM policy is governed by a Taylor Rule. Would the EM be made better off, despite the ensuing capital flows and exchange rate fluctuations, as claimed by Bernanke? And finally, we see how a cooperative response would affect the outcome; here the Ramsey planner sets US and EM policies to maximize a weighted...
average of US and EM welfare, as suggested by Rajan. We use the toolkit developed in Bodenstein et al. (2019) for all of these calculations.

The two papers closest to ours are Aoki et al. (2018) and Banerjee et al. (2016). Both papers incorporate the Gertler and Karadi (2011) banking frictions, and both papers feature original sin. Aoki et al model a small open emerging economy, and discuss an increase in the real US interest rate. Banerjee et al have a two country model similar to ours, but they focus on US monetary policy prior to the protracted zero lower bound. Within that environment, they calculate Nash and Cooperative responses to a (different) financial shock.

Our paper draws upon Curdia and Woodford (2011) and Canzoneri et al. (2017); both papers assume that reserves reduce the costs of banking. And of course there are many papers that study, empirically and theoretically, the spillovers of US and ECB policy shocks. Aoki et al. (2018) and Banerjee et al. (2016) provide a number of references. None of these papers is closely related to ours.

Our paper proceeds as follows: In Section 2.2, we outline our two country model. In Section 2.3, we discuss its calibration. In Section 2.4, we illustrate the effects of US monetary policy shocks. In particular, we discuss an increase in the interest rate on reserves while holding the supply of reserves constant, and we discuss an increase in the supply of reserves while holding the interest rate constant. In Section 2.5, we discuss the effects of a capital quality shock, and we assess the claims of Rajan and Bernanke. Section 2.6 concludes with a discussion of future work.

2.2 A Two Country Model of the US and Emerging Markets

Households and banks are at the heart of our model, and we begin with them. Some of the mathematical detail is relegated to an appendix.

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9Rajan did not actually call for full cooperation in this sense.
2.2.1 Banks and Their Managers

Following Gertler and Karadi (2011), each household in the US and the EM has a continuum of members on the unit interval. A constant fraction $f$ of the members are bankers; the remaining fraction $1-f$ work for intermediate goods producers (described below). But any individual banker may not remain a banker for long. At the end of every period, the banker retires with probability $1-\sigma$ and transfers the bank’s net worth to the banker’s own household; then, the household provides a new banker, armed with a fixed endowment of funds, to keep the fraction of bankers constant. The new banker cannot return to the household for more funds; in addition, a moral hazard problem (described below) limits the banker’s ability to fund new loans by issuing deposits to other households or banks. The banker can save and fund loans through accumulated net worth, but the probability of retirement limits the banker’s ability to save enough to avoid the moral hazard problem.

These basic Gertler and Karadi (2011) frictions result in too few loans being made in the US and the EM, and this fact will play a major role in what follows. In particular, capital inflows that bring new bank funding can be advantageous; capital outflows can similarly be disadvantageous. The remaining features of US and EM banks differ in significant ways; they require separate explanations.

US Banks

The value of a US bank to its own household is

$$V_{t}^{us} = E_t \{ A_{t,t+1}^{us} \left[ (1-\sigma) n_{t+1}^{us} + \sigma V_{t+1}^{us} \right] \}$$

(2.1)

where $n_t$ is the bank’s net worth and $A_{t,t+1}^{us}$ is its household’s stochastic discount factor. The bank’s balance sheet can be written as
\[ q_t^{us} K_t^{us} + t_t^{em} + m_t^{us} + \Gamma(q_t^{us} K_t^{us} + l_t^{em}, m_t^{us}) = d_t^{us} + d_t^{em} + n_t^{us} \] (2.2)

The first term on the asset side represents loans to intermediate goods firms; these firms have to borrow from banks in period \( t \) to purchase the capital they will use in period \( t+1 \). Firms cannot borrow directly from households, and this is another financial friction since bank intermediation is costly. \( q_t^{us} \) is the real price of capital \( K_t^{us} \). The second term is loans to EM banks, and the third term is reserves held at the FED. Following Diba and Loisel (2019), the last term on the asset side is the bank’s cost of making loans; these costs increase with the total loans made and decrease with the amount of reserves held at the FED. The first term on the liabilities side is deposits issued to US households (other than the banker’s own household). The second term is deposits issued to EM households. The final term is the bank’s net worth.

A US bank’s net worth evolves according to

\[ n_t^{us} = R_{k,t}^{us} q_{t-1}^{us} K_{t-1}^{us} + R_{l,t-1}^{us} t_{t-1}^{em} + R_{m,t-1}^{us} m_{t-1}^{us} - R_{d,t-1}^{us} d_{t-1}^{us} - R_{d,t-1}^{us} d_{t-1}^{em} \] (2.3)

The first three terms on the RHS are the real returns on loans that the bank made last period. The first term is the return on loans to intermediate goods firms. \( R_{k,t}^{us} \) will turn out to be the return on owning capital valued at \( q_{t-1}^{us} K_{t-1}^{us} \) last period. \( R_{k,t}^{us} \) was not known when the bank made the loan, and this "loan" would be better described as an equity position in the firm. Indeed, the "banks" in our model represent both commercial banks and investment banks. The second and third terms are the returns on the loans made to EM banks and the FED. \( R_{m,t}^{us} \) is the real return on bank reserves; the FED sets its net nominal rate, \( i_t^{us} \) (\( \equiv E_t \left[ R_{m,t}^{us} H_{t+1}^{us} \right] - 1 \)). The last two terms are

\(^{10}\)Bank reserves are the only "base money" asset we have in the model, hence the symbol \( m_t^{us} \). It will be seen however that banks deposits have transactions value.

\(^{11}\)The functional form used in our numerical calculations is given in the Appendix.
interest on the deposits of US and EM households; the return on these dollar deposits is the same.

It should be noted that there are no exchange rates in (2.2) or (2.3). US loans to EM banks and deposits issued to EM households are contracted in terms of dollars.

There is one further restriction on a US bank’s operations. A moral hazard problem inhibits the bank’s acquisition of deposits from US and EM households. In particular, at the beginning of any period, the banker can divert a fraction $\theta_{us}$ of its assets back to its own household. If the bank does so, then it is in default and shut down; the bank’s depositors can reclaim the remaining fraction $1 - \theta_{us}$. US and EM households understand the banker’s temptation to cheat, and they will only make deposits if an incentive compatibility constraint

$$V_t^{us} \geq \theta_{us} (d_t^{us} K_t^{us} + l_t^{em} + m_t^{us})$$

(2.4)

eliminates that temptation: (2.4) says that the continuation value of the bank is greater than or equal to the gain from diverting assets. This financial friction, in conjunction with the bank’s lack of access to equity markets, keeps the bank from lending as much as it would otherwise like.

So finally, the US banker’s problem is the choose $\{K_t^{us}, l_t^{em}, m_t^{us}, d_t^{us}, d_t^{em}\}$ to maximize (2.1) subject to (2.2), (2.3) and (2.4). The details of this optimization are not very revealing; they may be found in the Appendix.

**Emerging Market Banks**

Emerging market countries did not engage in massive quantitative easing, and we will not model a market for bank reserves. Here, we assume that the commercial bank deposit rate is the instrument of monetary policy, and we adopt Aoki et al. (2018)’s modeling of EM banks.
The value of an EM bank to its household is

\[ V_{t}^{em} = E_t \{ A_{t,t+1}^{em} \left[ (1 - \sigma) n_{t+1}^{em} + \sigma V_{t+1}^{em} \right] \} \tag{2.5} \]

which is analogous to (2.1). But, the EM bank’s balance sheet is

\[ q_t^{em} K_t^{em} \left[ 1 + \left( \kappa/2 \right) x_t^2 \right] = d_t^{em} + \epsilon_t^{em} l_t^{em} + n_t^{em} \tag{2.6} \]

where \( \epsilon_t^{em} \) is the EM’s real exchange rate, and \( x_t = \frac{\epsilon_t^{em} q_t^{em}}{q_t^{em} K_t^{em}} \) is the fraction of the bank’s lending that is financed by US banks. As the LHS shows, it is more costly to borrow from US banks than to issue deposits to EM households. On the RHS are the bank’s liabilities: deposits issued to EM households, loans from US banks, and of course net worth.

The bank’s net worth accumulates as follows

\[ n_t^{em} = R_{k,t}^{em} q_{t-1}^{em} K_{t-1}^{em} - R_{d,t-1}^{em} d_{t-1}^{em} - \epsilon_t^{em} R_{l,t-1}^{em} l_{t-1}^{em} \tag{2.7} \]

The first term on the RHS is the real return on loans to intermediate goods firms. The second and third terms are the real costs of borrowing from EM households and US banks.

Note that the real exchange rate appears in both (2.6) and (2.7) because of original sin. And here, a real depreciation makes the repayment of loans from US banks more expensive.

Following Aoki et al. (2018), EM bankers can divert a fraction \( \theta^{em}(x_t) = \theta_0^{em} \exp(-\theta_1^{em} x_t) \) of their assets back to their households. Increasing loans from US banks (or \( x_t \)) lowers this fraction; it is harder to cheat international bankers; similarly, a real depreciation lowers the fraction. This temptation is understood by households and US banks, so EM banks are limited in their borrowing by the incentive compatibility constraint

\[ V_{t}^{em} \geq \theta^{em}(x_t) q_t^{em} K_{b,t}^{em} \tag{2.8} \]
Cheating does not pay.

So finally, the EM banker’s problem is the choose \( \{K, l^e_t, d^e_t\} \) to maximize (2.5) subject to (2.6), (2.7) and (2.8). Once again, the details of this optimization are not very illuminating, and they are relegated to the Appendix.

2.2.2 Households

The division of households between workers and bankers was described at the beginning of the section on banks; this basic structure holds for both US and EM households. However, the dollar is a vehicle currency, and US financial assets have transactions value for both US and EM households.\(^{12}\) We begin with US households.

US Households

A US household’s utility is given by

\[
U^u_t = E_t \sum_{j=0}^{\infty} \beta^{j-t} \left[ \log \left( c^u_t + h c^u_{t-1} \right) - \frac{\nu_0}{1 + \zeta} \left( L^u_t \right)^{1+\zeta} + \nu_1 \log \left( d^u_t \right) \right] \tag{2.9}
\]

where \( c^u_t \) is a CES aggregate of US and EM goods (to be defined below), \( L^u_t \) is the supply of labor to intermediate goods firms, and \( h \) measures the degree of habit formation. \( d^u_t \) are household deposits at US banks; they appear in the utility function because they have transactions value. The household’s budget constraint is

\[
c^u_t + d^u_t = w^u_t L^u_t + R^u_{d,t-1} d^u_{t-1} + \Upsilon^u_t + T^u_t \tag{2.10}
\]

where \( w^u_t \) is the real wage rate; \( \Upsilon^u_t \) are transfers from the household’s bank and profits from intermediate goods firms. The Fed earns seigniorage \( T^u_t \); we assume it is transferred directly to the households.

\(^{12}\)A large fraction of international trade is invoiced in dollars, and importers utilize dollar transactions balances.
The household chooses \( \{c^u_t, L^u_t, d^u_t\} \) to maximize (2.9) subject to (2.10). Letting \( \lambda^u_t \) be the Lagrange multiplier for the budget constraint, the first order conditions are

\[
\lambda^u_t = \frac{1}{c^u_t - hc^u_{t-1}} \quad (2.11)
\]
\[
\lambda^u_t w^u_t = v_0 (L^u_t)^{\zeta} \quad (2.12)
\]
\[
1 = \frac{v_1}{\lambda^u_t d^u_t} + \beta E_t \left( \frac{\lambda^u_{t+1}}{\lambda^u_t} \right) R^u_{d,t} \quad (2.13)
\]

The conditions for \( c^u_t \) and \( L^u_t \) are standard; the condition for \( d^u_t \) is not. If \( v_1 \) were equal to 0, then \( R^u_t \) would be the standard CAPM rate. But here \( v_1 > 0 \); deposits provide transactions services, and they will be held at a lower rate of interest.

**Emerging Market Households**

EM households are similar to US households. But, they hold deposits in US banks – \( d^em_t \) – for their transactions services, and since those deposits are denominated in dollars, the real exchange rate becomes a factor. The EM household’s utility is given by

\[
U^em_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[ \log \left(c^em_{t+j} - hc^em_{t+j-1}\right) - \frac{v_0}{1+\zeta} (L^em_{t+j})^{1+\zeta} + v_1 \log (d^em_{t+j}) \right] \quad (2.14)
\]

and its budget constraint is

\[
c^em_t + b^em_t + \epsilon_t d^em_t = w^em_t L^em_t + R^em_{b,t-1} b^em_{t-1} + \epsilon^em_{t-1} R^us_{d,t-1} d^em_{t-1} + \Upsilon^em_t \quad (2.15)
\]

where \( b^em_t \) are the household’s deposits in EM banks. Note that these deposits are essentially standard risk free bonds since they do not appear in the EM household’s utility function; \( R^em_{b,t-1} \) is the real CAPM rate. Note also that a real depreciation increases the return on deposits in the US banks since the household is holding the appreciating asset.
The household chooses \( \{c_t^{em}, L_t^{em}, b_t^{em}, d_t^{em}\} \) to maximize (2.14) subject to (2.15). Letting \( \lambda_t^{em} \) be the Lagrange multiplier for the budget constraint, the first order conditions are

\[
\lambda_t^{em} = \frac{1}{c_t^{em} - h_c^{em}} \quad (2.16)
\]

\[
\lambda_t^{em} w_t^{em} = v_0 (L_t^{em})^\zeta \quad (2.17)
\]

\[
1 = \beta E_t \left( \frac{\lambda_{t+1}^{em}}{\lambda_t^{em}} \right) R_{b,t}^{em} \quad (2.18)
\]

\[
1 = \frac{v_1}{\lambda_t^{em} \epsilon_t d_t^{em}} + \beta E_t \left( \frac{\epsilon_{t+1} \lambda_{t+1}^{em}}{\epsilon_t \lambda_t^{em}} \right) R_{d,t}^{us} \quad (2.19)
\]

Once again, the first two conditions are standard. The condition for \( b_t^{em} \) is the standard Euler equation. The condition for \( d_t^{em} \) is like the condition for \( d_t^{us} \) above, since deposits in US banks have transactions value. However, the real exchange rate gets into this condition because the deposits are in dollars. And in particular, a depreciating EM exchange rate will increase the household’s demand for dollar deposits since that is the appreciating currency.

2.2.3 FIRMS

There are four kinds of firms in our model: intermediate goods producers, final goods producers, consumption goods producers, and capital goods producers. With one exception, EM firms are the same as US firms, so in most of this section we can dispense with the "us" and "em" superscripts.

INTERMEDIATE GOODS PRODUCERS AND THE CAPITAL QUALITY SHOCK

A continuum of intermediate goods firms – indexed by \( i \) on the unit interval – produce differentiated goods. Banks take equity positions in these firms, allowing them to purchase capital in period \( t - 1 \) for use in period \( t \). Their production functions are
identical

\[ y_{i,t} = A \left( \xi_t^K K_{t-1} \right)^\alpha (L_{i,t})^{1-\alpha} \tag{2.20} \]

where \( \xi_t^K \) is what Gertler and Karadi (2011) call a capital quality shock. Here, if less than one, the shock is best thought of as random obsolescence of usable capital.

Labor markets are competitive, and \( w_t \) is the market clearing real wage rate. Letting \( z_t \) be the rental rate on capital, cost minimization implies that marginal cost is given by

\[ mc_t = \frac{z_t^\alpha w_t^{1-\alpha}}{A_t \alpha^\alpha (1-\alpha)^{1-\alpha}} \tag{2.21} \]

And the real ex post return on the bank’s effective ownership of a unit of capital is

\[ R_{k,t} = \frac{[z_t + (1-\delta)q_t] \xi_t^k}{q_{t-1}} \tag{2.22} \]

where \( \delta \) is the non-stochastic rate of capital depreciation. Here, we see that \( \xi_t^K \) can also be interpreted as a shock to net worth in equations (2.3) and (2.7).

**Final Goods Producers**

The mechanics of monopolistic competition are well known in the DSGE literature, and our description of it can be brief. Competitive final goods producers buy the differentiated intermediate goods \( y_{i,t} \) at price \( p_{i,t} \) and bundle them into a final good

\[ Y_t = \left( \int_0^1 \frac{y_{i,t}}{y_{i,t}} \, di \right)^{\frac{\varepsilon}{1-\varepsilon}} \tag{2.23} \]

They then sell the final good to consumers and capital goods producers at a price \( P_t \). Cost minimization (and the zero profit condition) implies

\[ P_t = \left( \int_0^1 p_{i,t}^{\frac{1}{1-\varepsilon}} \, di \right)^{\frac{1}{1-\varepsilon}} \tag{2.24} \]
It is costly for the final goods producers to change their prices. They choose $P_t$ to maximize expected profits

$$E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[ \left( \frac{p_{i,t+j}}{P_{t+j}} - m c_{t+j} \right) y_{i,t+j} - \frac{\kappa_P}{2} \left( \frac{p_{i,t+j}}{p_{t+j}} - 1 \right)^2 Y_{t+j} \right]$$  \hspace{1cm} (2.25)

where $\Lambda_{t,t+j}$ is the household’s stochastic discount factor, and $\kappa_P$ measures the severity of the cost of changing prices.\textsuperscript{13}

**Capital Goods Producers**

Our modeling of capital goods producers follows Gertler and Karadi (2011). At the end of period $t$, capital goods producers buy – at price $q_t$ – the remaining capital of intermediate goods producers. They costlessly refurbish any capital that was hit by a quality shock, and they produce new capital. They then sell – again, at price $q_t$ – the capital back to intermediate goods producers for their use in period $t + 1$. While it is costless to refurbish old capital, there are adjustment costs associated with producing new capital.

The perfectly competitive capital producers choose investment $I_t$ to maximize discounted profits

$$E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[ (q_t - 1)I_t - \frac{\kappa_I}{2} \left( \frac{I_t}{I} - 1 \right)^2 \right]$$  \hspace{1cm} (2.26)

where $I$ is steady state investment, and $\kappa_I$ measures the severity of the capital adjustment costs. The capital producer’s first order condition is

$$q_t = 1 + \frac{\kappa_I}{2} \left( \frac{I_t}{I} - 1 \right)^2 + \left( \frac{I_t}{I} \right) \kappa_I \left( \frac{I_t}{I} - 1 \right)$$  \hspace{1cm} (2.27)

And finally, capital accumulation is then given by

$$K_t = I_t + (1 - \delta) \xi^t K_{t-1}$$  \hspace{1cm} (2.28)

\textsuperscript{13}The Rotemberg parameter $\kappa_P$ produces the same inflation dynamics as the familiar Calvo parameter $\theta$ if $\frac{(1-\theta)(1-\beta \theta)}{\theta} = \frac{\varepsilon - 1}{\kappa_P}$. 

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Consumption Good Producers

Finally, competitive bundlers in each country combine home and foreign final goods to produce the household’s consumption good. Here unfortunately, we have to bring back the "us" and "em" superscripts. In the US, the consumption good is

$$c^us_t = \left( \gamma^us \right)^{\frac{1}{\rho^{us}}} (Y^us_t)^{\frac{\rho^{us}-1}{\rho^{us}}} + (1 - \gamma^us) \left( Y^em_t \right)^{\frac{\rho^{us}}{\rho^{us}-1}} \right)^{\frac{1}{\rho^{us}-1}}$$ (2.29)

where $\frac{1}{1-\rho^{us}}$ is the elasticity of substitution, and $\gamma^{us}$ is a measure of home bias. The price of the consumption good is

$$P^{us}_t = \left[ \gamma^{us} (P^{us}_t)^{1-\rho^{us}} + (1 - \gamma^{us}) (P^{em}_t)^{1-\rho^{us}} \right]^{\frac{1}{1-\rho^{us}}}$$ (2.30)

The modeling for the EM consumption good is analogous.

Once again, the Phillips curves are the same in the two countries

$$(\chi_t - 1)\Pi_t = \frac{1}{\kappa_{P}} \left[ \varepsilon mc_t + S(1 - \varepsilon) \left( \frac{P^c_t}{P_t} \right) \right] + E_t \left[ \Lambda_{t, t+1} \frac{Y^c_{t+1}}{Y_t} (\Pi_{t+1} - 1)\Pi_{t+1} \right]$$ (2.31)

where $S \equiv \frac{\varepsilon}{\varepsilon - 1}$ is a fiscal subsidy that eliminates the country’s monopoly power. We do not want the Ramsey Planner trying exploit this distortion; we want the Planner to focus on macroeconomic distortions.

2.2.4 The US Current Account

In equilibrium, the US current account can be expressed as

$$CA^{us}_t = (l^us_t - l^us_{t-1}) - (d^{em}_t - d^{em}_{t-1})$$ (2.32)

The US current account is equal to net capital outflow, or the growth in US loans to EM banks minus the growth in EM household deposits in US banks.
2.3 Model Calibration

Table 2.1 summaries the parameter values in baseline calibration. The model is calibrated at a quarterly frequency. We assume the model consisting of two equal-sized economies: the United States (the US) and a collection of emerging market countries (the EM). Parameters are set symmetrically across the two economies except those shaping financial frictions and monetary policy.

Most of the parameters governing households and producers are standard in the related literature. We use a standard separable utility function with logarithmic consumption, so the intertemporal elasticity of substitution for consumption is equal to one. The discount rate $\beta$ is equal to 0.99, a standard value for a quarterly model. The inverse of Frisch elasticity of labor supply $\zeta$ is set to 0.276 as in Gertler and Karadi (2011), and the degree of habit formation $h$ set to 0.7. Given the value of $\zeta$, the disutility of labor scale parameter $\zeta_0$ is calibrated to 3.606 so that a steady-state labor supply is $1/3$. To calibrate the utility weight for the logarithm of US dollar deposit $v_1$, we rely on estimates of net nominal interest rate in the US during the great recession from Diba and Loisel (2019) of 35 basis points per annum, and accordingly calibrate $v_1$ to 0.0624. We set the capital share $\alpha$ to 0.33, and the depreciation rate of capital $\delta$ to 0.025 as standard values. The cost parameter of adjusting investment goods production $\kappa_I = f''(1)$ is calibrated to 0.67 so that the price elasticity of investment is consistent with a value estimated in Aoki et al. (2018). For the price setting nexus, we set the elasticity of substituion between differentiated goods $\varepsilon$ to 6 as in Banerjee et al. (2016), and the Rotemburg parameter of price rigidities $\kappa_P$ based on the Calvo parameter choice of $\varrho_P = 0.66$ as in Aoki et al. (2018). For the parameters related to international trade, we set the elasticity of substitution between home and foreign-produced goods $\rho$ to 0.9, as in Heathcote and Perri (2002) and Kollmann (2006). The
share of home-produced goods in consumption composite is set to 0.8 as a standard value, implying a steady-state ratio of export to output of about 20 percent.

Regarding the parameters shaping financial frictions in the EM, we rely on estimates for emerging market economies from Aoki et al. (2018). We choose the survival probability of banker \( \sigma \) to 0.94, implying an expected horizon of 4.16 years. We set the elasticity of bank leverage with respect to foreign borrowing for the EM \( \vartheta_{1em} \) to 0.1, implying that a rise of the fraction of bank assets financed by foreign borrowing by 10 percent lowers the fraction of bank assets banker can divert by 1 percent. We calibrate remaining three parameters, the fraction of bank assets transferred to new entering banks \( \xi \), the fraction of assets that can be diverted \( \vartheta^e_0 \), and the resource costs for foreign borrowing \( \kappa^{em} \), to jointly match the following three targets: a steady-state bank leverage ratio of 5, a steady-state ratio of foreign borrowing to bank assets of 25 percent, and a steady-state spread between lending and deposit rates of 200 basis points per annum. These imply that \( \xi = 0.0046, \vartheta^e_0 = 0.375, \) and \( \kappa^{em} = 0.032 \).

Turning to the parameters governing the banking cost function and monetary policy in the US, we follow Diba and Loisel (2019) and calibrate them to a steady-state equilibrium that matches some features of US economy during the great recession, while maintaining the standard properties of the banking cost function. We set the exponent parameters \( \varsigma^u_s \) and \( \varsigma^m_s \), and the cost parameter of US bank loans and reserves \( \kappa^{us}_{lm} \) to hit the following three targets: a steady-state spread between interest rates on reserves and bank deposit of 10 basis points per annum, a steady-state ratio of reserves to bank loans of 11 percent, and a steady-state rate of return on bank loans of 3.25 percent per annum, which are the values prevailing in November 2010 in the US. These result in that \( \varsigma^u_s = 2, \varsigma^m_s = 2, \) and \( \kappa^{us}_{lm} = 0.0000054 \).

In our baseline experiments, we focus on a steady-state equilibrium with zero inflation. We set the coefficients of monetary policy rule in the EM as \( \kappa_{\pi} = 1.5 \) and
\( \rho_i = 0.8 \), as standard values. Turning to monetary policy in the US, we focus on a steady-state net nominal interest rate on reserves of 25 basis points per annum and a steady state growth rate of nominal reserves of zero.

2.4 AN EVALUATION OF RAJAN AND BERNANKE’S CLAIMS

We begin by assuming that the US has been hit by Gertler and Karadi (2011)’s capital quality shock. First, we look at how the quality shock affects the US and passes on to the EM when the Fed remains passive; that is, neither the interest on reserves nor the supply of reserves responds to the shock. Note that standard New Keynesian models cannot support this passive equilibrium; for determinacy, those models need the interest rate to follow something like a Taylor Rule.\(^{14}\) As far as we are aware, this paper is the first to look at policy spillovers in this passive environment.

Next, we assume that the Fed has already set the interest on reserves at 25 basis points, and we shock the growth rate of reserves. This shows how unconventional monetary policy can be superimposed upon the passive policy equilibrium. And finally, we calculate the self-centered and cooperative equilibria described in the introduction.

**CAPITAL QUALITY SHOCK**

Figure 2.1 shows the effects of a negative capital quality shock; this is a 1 standard deviation shock with an autoregressive parameter of 0.8. As explained above, this shock has two components: it is a negative productivity shock, and it worsens a US bank’s balance sheet. Both components can be seen in Figure 2.1.

As with any negative productivity shock, the capital quality shock is stag-
imflationary; it causes a recession and it is inflationary. This by itself creates a

\(^{14}\)Banerjee et al. (2016), for example, consider a shock to the Taylor Rule in a model that is otherwise similar to ours.
challenge for monetary policy. But the Gertler and Karadi (2011) financial frictions produce added complications. Theses complications can be seen in the bank’s balance sheet, equation (2.2), its net worth, equation (2.3), and in its return on loans to firms, equation (2.22). The shock decreases the ex-post return on loans, $R^{us}_{k,t}$, which in turn lowers net worth. US banks curb their lending to US firms and EM banks, and they economize on reserves which increases the cost of making loans. Moreover, the decrease in net worth exacerbates the moral hazard problem, as seen in the widening spread $R^{us}_{k,t} - R^{us}_{t}$. This in turn leads to a decrease in US and EM household deposits in US banks, further worsening the balance sheet. Investment spending rises to replace the depleted capital stock, raising the price of capital, $q_t$, but it is not sufficient to replace the capital firms are no longer purchasing; the US capital stock falls.

The crucial transmission to the EM goes through capital flows and the real exchange rate. EM households repatriate their deposits from US banks, and this repatriation is larger than the US banks’ curtailing of loans to EM banks. Thus, there is a net capital inflow to the EM; its current account deteriorates and its exchange rate starts to appreciate. This lowers EM consumption via expenditure switching, and lowers inflation. But, the EM policy rate falls in line with its Taylor Rule. This causes a real depreciation instead. Once again, the Gertler and Karadi (2011) frictions produce added complications. The withdrawal of loans from US banks worsens the EM banks balance sheets. In addition, original sin comes back to haunt the EU banks; the depreciation increases the remaining loan payment to US banks. EM banks decrease loans to EM firms and the capital stock falls. All of this make the situation in the EM worse.
Unconventional Monetary Policy Shock

Figures 2.2 and 2.3 show the effects of an increase in the growth rate of the stock of reserves, while keeping the interest on reserves fixed at 25 basis points. As before, this is a 1 standard deviation shock with an autoregressive parameter of 0.8.

Figure 2 illustrates the fed funds market. The demand for reserves derives from the fact that reserves lower the cost of making loans; recall equation (2.2). The opportunity cost for reserves is the spread between fed funds rate and the rate on reserves, or $I_{ff} - I_r$ in the figure. The supply of reserves is set by the Fed’s open market operations. An increase in the supply of reserves decreases the opportunity cost, and since we are holding the interest on reserves fixed, the fed funds rate falls.

Figure 2.3 illustrates the response of the rest of the economy. In our model, the fed funds rate can be interpreted as the CCAPM rate, and the fall in the CCAPM rate has the standard effect of raising consumption and investment. More interesting however is what happens to US banks, which can be seen in the bank’s balance sheet (2.2) and the bank’s net worth (2.3). As the figure shows, the shock ultimately increases inflation, decreasing the real demand for reserves. This allows loans to EM banks and intermediate goods producers. The increased demand for capital raises its price, and this in turn increases the real return on capital ownership, $R^{us}_{k,t}$; net worth increases. Higher net worth decreases the financial frictions and the moral hazard problem, as illustrated by the decrease in the spread $R^{us}_{k,t} - R^{us}_{t}$. This makes makes US and EM household increase their deposits, further improving the bank’s balance sheet.

Once again, the crucial transmission to the EM goes through capital flows and the exchange rate. Lower US interest rates lead to a real appreciation in the EM, which has a contractionary impact via expenditure switching. But this is overwhelmed by developments in the EM banking sector, as seen in the balance sheet (2.6) and net
worth accumulation (2.7). The increase in US bank lending allows the EM bank to lend more to intermediate goods firms, and the rise in the demand for capital increases its price. This in turn increases the real return to capital ownership, and improves the bank’s net worth. Higher net worth ameliorates the financial frictions and eases the moral hazard problem, as illustrated by the fall in the spread $R_{k,t} - R_{t}$. As a result, EM households increase their deposits. Furthermore, the real appreciation decreases the cost of debt repayment. All of this improves the balance sheet. And finally, the appreciation reduces inflation, and EM central bank, following a Taylor Rule, decreases its policy rate. Consumption and investment fall.

Next we superimpose on this passive equilibrium two different policy reactions.

**The Self-Centered Equilibrium**

Figure 2.4 shows the modification made by Fed’s self-centered policy reactions; this is our interpretations of the equilibrium that Rajan complained about. Here, the Ramsey planner sets the growth in reserves to maximize the US household’s utility.

The blue (solid) lines represent the passive equilibrium, but there are two changes from Figure 2.3: (1) the capital quality shock has been increased to two standard deviations, and (2) the IRF’s represent deviations from the Ramsey optimal steady state. The red (dashed) lines represent the Ramsey equilibrium.

The Ramsey planner faces a dilemma. The shock is inflationary which would call for a tighter monetary policy, but the shock is also recessionary which would call for a looser monetary policy. The planner chooses to have an immediate increase in reserves, followed by a smaller decrease in reserves. The result is a somewhat lower level of consumption and output, but less inflation. The the real exchange rate

\footnote{Gertler and Karadi (2011) use a 5 standard deviation to describe the Great Recession. We are calculating linear approximations around the steady state, so we settled for a two standard deviation shock.}
depreciates a little more, but the current account is largely unaffected. So, emerging
market complaints about the real exchange rate and current account are mostly due
to the shock itself, and not US policy. EM consumption is lower, but investment is
higher at least initially. The welfare gains or losses will be reported below.

THE COOPERATIVE EQUILIBRIUM

Figure 2.5 describes the cooperative equilibrium, which we interpret as Rajan’s plea
for coordination. In the cooperative equilibrium, the Ramsey Planner chooses the
growth rate of US reserves and the EM deposit rate to maximize a weighted average
of US and EM household utilities; in the benchmark case, the two countries are
of equal size, and the weights are equal. In Figure 2.5, the blue (solid) lines once
again represent the passive equilibrium, while the green (dashed) lines describe the
cooperative equilibrium.

Comparing Figures 2.4 and 2.5, the Planner chooses a similar, but more aggres-
sive, path for reserves. The initial increase in the rate of growth is bigger, and the
subsequent fall is also bigger. US consumption, investment and output are somewhat
lower, but inflation is much lower; this is how the Planner resolves the stag-inflation
dilemma. The EM interest rate follows also follows a whiplash path, initially con-
tractionary and then expansionary. As a result of both US and EM policies, EM
investment and output are much higher. The real exchange rate depreciates some-
what more and the US current account is much the same.

WELFARE NUMBERS

So, what does this all add up to in terms of welfare? Table 2.2 reports the percent
gains and losses relative to the passive equilibrium, and in terms of consumption
equivalents. As Bernanke suggested, the US Self-Centered policy brings for both the
US and the EM. Simply reacting to the shock is good for both countries. As Rajan suggested, the Cooperative policy brings additional gains for both the US and the EM.

In both cases, the gains for the US are an order of magnitude higher than for the EM. This is because the capital quality shock originates in the US does the most damage there.

2.5 Conclusion

In this paper, we present an assessment of the competing claims of Raghuram Rajan and Ben Bernanke. Rajan (2015) complained that the unconventional (or quantitative easing) policy in the US was driven solely by conditions in the US, and to the detriment of emerging market countries. Bernanke (2015) responded that an unconventional monetary policy was necessary to keep the US economy growing, and that this was in everyone's interest.

To make this assessment, we developed a two country model for the United States (US) and an aggregate emerging market country (EM). In this model, we assume that the Great Recession is driven by a Gertler et al. (2012) capital quality shock, and that each country is encumbered by Gertler and Karadi (2011) financial frictions. These frictions imply that banks are under capitalized. So, the initial capital inflow is beneficial to the EM; the subsequent outflow is not. We assume that the Fed has already lowered the fed funds rate to 25 basis points, and that the Fed keeps it there. The Fed can however, engage in the unconventional policy of quantitative easing. We consider three different equilibria. The first is a passive equilibrium in which there is no active response to the capital quality shock: the Fed keeps the level of bank reserves constant, and the EM monetary policy follows a Taylor rule. The second is
a *self-centered equilibrium* in which the EM continues to follow a Taylor rule, but a Ramsey Planner sets the level of US reserves to maximize the US household’s utility. And finally, the third is a *cooperative equilibrium* in which the Ramsey Planner sets both the level of US reserves and the EM policy rate to maximize the sum of household utility in the US and the EM.

Our results essentially support Bernanke’s position. Welfare in the self-centered equilibrium is higher than in the passive equilibrium for both the US and the EM. And welfare rises even further in the cooperative equilibrium. However, the welfare increases in the US are an order of magnitude higher than in the EM.
## Table 2.1: Baseline Parameters for the Two-Country Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Elasticity of substitution btw home and foreign-produced goods</td>
<td>0.9</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Share of home goods in consumption composite for EM</td>
<td>0.8</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.99</td>
</tr>
<tr>
<td>$v_1$</td>
<td>Utility weight for the logarithm of US dollar deposit</td>
<td>0.0624</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Inverse of Frisch elasticity of labor supply</td>
<td>0.276</td>
</tr>
<tr>
<td>$v_0$</td>
<td>Relative utility weight of labor</td>
<td>3.609</td>
</tr>
<tr>
<td>$h$</td>
<td>Habit formation parameter</td>
<td>0.7</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Bank survival probability</td>
<td>0.94</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Fraction of total assets brought by new banks</td>
<td>0.0046</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Management cost for foreign borrowing for EM</td>
<td>0.032</td>
</tr>
<tr>
<td>$\varphi_{em}$</td>
<td>Elasticity of leverage w.r.t. foreign borrowing for EM</td>
<td>0.1</td>
</tr>
<tr>
<td>$\varphi_{0m}$</td>
<td>Divertible proportion of assets for EM</td>
<td>0.375</td>
</tr>
<tr>
<td>$\theta_{us}$</td>
<td>Divertible proportion of assets for US</td>
<td>0.375</td>
</tr>
<tr>
<td>$\xi_{us}$</td>
<td>Exponent parameter of bank loans for US</td>
<td>2</td>
</tr>
<tr>
<td>$\xi_{us}$</td>
<td>Exponent parameter of bank reserves for US</td>
<td>2</td>
</tr>
<tr>
<td>$\kappa_{lm}$</td>
<td>Cost parameter of bank loans and reserves for US</td>
<td>0.0000054</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of capital in output</td>
<td>0.33</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate of capital</td>
<td>0.025</td>
</tr>
<tr>
<td>$\kappa_I$</td>
<td>Investment adjustment cost parameter</td>
<td>0.67</td>
</tr>
<tr>
<td>$\kappa_P$</td>
<td>Probability of keeping prices fixed</td>
<td>0.66</td>
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<tr>
<td>$\epsilon$</td>
<td>Final goods producer’s elasticity of substitution btw varieties</td>
<td>6</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Interest rate smoothing parameter for EM</td>
<td>0.8</td>
</tr>
<tr>
<td>$\kappa_\pi$</td>
<td>Inflation coefficient of Taylor rule for EM</td>
<td>1.5</td>
</tr>
<tr>
<td>$\bar{i}_m$</td>
<td>Steady-state nominal interest rate on US reserves (in annual rate)</td>
<td>0.0025</td>
</tr>
<tr>
<td>$M$</td>
<td>Steady-state stock of nominal US reserves</td>
<td>1.2632</td>
</tr>
<tr>
<td>$G_m$</td>
<td>Steady-state growth rate of nominal US reserves</td>
<td>1</td>
</tr>
<tr>
<td>$G^*_m$</td>
<td>Ramsey optimal steady-state growth rate of nominal US reserves</td>
<td>1.010143</td>
</tr>
</tbody>
</table>
Table 2.2: Welfare Gain under Alternative Policy Responses

<table>
<thead>
<tr>
<th>Policy</th>
<th>US Welfare Gain</th>
<th>EM Welfare Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Self-Centered Ramsey Policy</td>
<td>2.86</td>
<td>0.21</td>
</tr>
<tr>
<td>Cooperative Ramsey Policy</td>
<td>3.04</td>
<td>0.47</td>
</tr>
</tbody>
</table>

*Notes:* The welfare gain under each policy is expressed as percentage of quarterly consumption equivalent relative to a passive equilibrium.

Figure 2.1: Capital Quality Shock

*Notes:* This figure presents the impulse responses of key variables to an unexpected 1% quarterly decrease in US capital quality. All responses are in log-deviations from the steady-state except the nominal policy rate (EM) whose response is in a level deviation from the steady-state.
Figure 2.2: Federal Funds Market

Notes: This figure illustrates the fed funds market. $I_{ff} - I_r$ represents the spread between fed funds rate and the rate on reserves, while $S$ represents the supply of reserves set by the Fed.
Figure 2.3: Unconventional Policy Shock

Notes: This figure presents the impulse responses of key variables to an unexpected 1% quarterly increase in growth rate of nominal US reserves. All responses are in log-deviations from the steady-state except the nominal policy rate (EM) whose response is in a level deviation from the steady-state.
**Notes:** This figure presents the impulse responses of key variables to an unexpected 2% quarterly *decrease* in US capital quality. The blue-solid line represents a passive equilibrium in which there is no active response to the shock: the Fed keeps the level of reserves constant, and the EM monetary policy follows a Taylor rule. For the comparison, the red-dashed line represents a self-centered equilibrium in which the EM monetary policy continues to follow a Taylor rule, but a Ramsey planner sets the level of US reserves to maximize the US household’s utility. All responses are in log-deviations from the steady-state except the current account and the nominal policy rate (EM) whose responses are in level deviations from the steady-state.
Notes: This figure presents the impulse responses of key variables to an unexpected 2% quarterly decrease in US capital quality. The blue-solid line represents a passive equilibrium in which there is no active response to the shock: the Fed keeps the level of reserves constant, and the EM monetary policy follows a Taylor rule. For the comparison, the green-dashed line represents a cooperative equilibrium in which a Ramsey planner sets both the level of US reserves and the EM policy rate to maximize the sum of household utility in the US and the EM. All responses are in log-deviations from the steady-state except the current account and the nominal policy rate (EM) whose responses are in level deviations from the steady-state.
APPENDIX A

THE DETAILS OF THE OPTIMIZATION FOR THE US BANKER’S PROBLEM

The US banker’s problem is given by

\[
V_t^{us} = \max_{\{K_t^{us}, l_t^{em}, m_t^{us}, d_t^{us}, d_t^{em}\}} E_t \left\{ \Lambda_{t,t+1} \left[ (1 - \sigma) n_{t+1}^{us} + \sigma V_{t+1}^{us} \right] \right\}
\]  \hspace{1cm} (A.1)

subject to

\[
q_t^{us} K_t^{us} + l_t^{em} + m_t^{us} + \Gamma(q_t^{us} K_t^{us} + l_t^{em}, m_t^{us}) = d_t^{us} + d_t^{em} + n_t^{us}
\]  \hspace{1cm} (A.2)

\[
n_t^{us} = R_{k,t} q_{t-1}^{us} K_{t-1}^{us} + R_{l,t-1} l_{t-1}^{em} + R_{m,t-1} m_{t-1}^{us} - R_{d,t-1} d_{t-1}^{us} - R_{d,t-1} d_{t-1}^{em}
\]  \hspace{1cm} (A.3)

\[
V_t^{us} \geq \theta^{us} (q_t^{us} K_t^{us} + l_t^{em} + m_t^{us})
\]  \hspace{1cm} (A.4)

with the functional form of banking cost

\[
\Gamma(q_t^{us} K_t^{us} + l_t^{em}, m_t^{us}) = \kappa_{lm}^{us} \left[ \frac{(q_t^{us} K_t^{us} + l_t^{em})^\eta}{(m_t^{us})^\zeta} \right] (q_t^{us} K_t^{us} + l_t^{em})
\]  \hspace{1cm} (A.5)

To solve this problem, we first guess that the value function \(V_t^{us}\) is a linear object of the form as

\[
V_t^{us} = v_{kt} q_t^{us} K_t^{us} + v_{lt} l_t^{em} + v_{mt} m_t^{us} - v_{dt} d_t^{us} - v_{de} d_t^{em}
\]  \hspace{1cm} (A.6)
Let \( \lambda_t^{us} \) be the Lagrangian multiplier for the incentive constraint (A.4). Using the Lagrangian

\[
\mathcal{L} = V_t^{us} + \lambda_t^{us} \left[ V_t^{us} - \theta^{us} (q_t^{us} K_t^{us} + l_t^{em} + m_t^{us}) \right]
\]

\[
= (1 + \lambda_t^{us}) \left[ v_{kt} q_t^{us} K_t^{us} + v_{lt} l_t^{em} + v_{mt} m_t^{us} - v_{dt} d_t^{us} - v_{dt} d_t^{em} \right]
\]

\[-\lambda_t^{us} \theta^{us} (q_t^{us} K_t^{us} + l_t^{em} + m_t^{us})
\]

\[
= (1 + \lambda_t^{us}) \left[ v_{kt} q_t^{us} K_t^{us} + v_{lt} l_t^{em} + v_{mt} m_t^{us} - v_{dt} d_t^{us}
\]

\[-v_{dt} \left( q_t^{us} K_t^{us} + l_t^{em} + m_t^{us} + \Gamma (q_t^{us} K_t^{us} + l_t^{em}, m_t^{us} - d_t^{us} - n_t^{us}) \right) \]

\[-\lambda_t^{us} \theta^{us} (q_t^{us} K_t^{us} + l_t^{em} + m_t^{us}) \]

The first order conditions for \( \{K_t^{us}, l_t^{em}, m_t^{us}, d_t^{us}, \lambda_t^{us}\} \) associated with this problem are given by

\[
(1 + \lambda_t^{us}) \left\{ v_{kt} - v_{dt}^{em} \left[ 1 + \kappa_t^{us} (q_t^{us} K_t^{us} + l_t^{em})\varsigma (m_t^{us})^{-\varsigma m} \right. \right.
\]

\[
+ \kappa_t^{us} \varsigma_l (q_t^{us} K_t^{us} + l_t^{em})\varsigma (m_t^{us})^{-\varsigma m} \left. \right] \right\} = \lambda_t^{us} \theta^{us} \quad (A.8)
\]

\[
(1 + \lambda_t^{us}) \left\{ v_{kt} - v_{dt}^{em} \left[ 1 + \kappa_t^{us} (q_t^{us} K_t^{us} + l_t^{em})\varsigma (m_t^{us})^{-\varsigma m} \right. \right.
\]

\[
+ \kappa_t^{us} \varsigma_l (q_t^{us} K_t^{us} + l_t^{em})\varsigma (m_t^{us})^{-\varsigma m} \left. \right] \right\} = \lambda_t^{us} \theta^{us} \quad (A.9)
\]

\[
(1 + \lambda_t^{us}) \left\{ v_{lt} - v_{dt}^{em} \left[ 1 + \kappa_t^{us} (q_t^{us} K_t^{us} + l_t^{em})\varsigma (m_t^{us})^{-\varsigma m} \right. \right.
\]

\[
+ \kappa_t^{us} \varsigma_l (q_t^{us} K_t^{us} + l_t^{em})\varsigma (m_t^{us})^{-\varsigma m} \left. \right] \right\} = \lambda_t^{us} \theta^{us} \quad (A.10)
\]

\[
(1 + \lambda_t^{us}) \left\{ v_{mt} - v_{dt}^{em} \left[ 1 - \varsigma m \kappa_t^{us} (q_t^{us} K_t^{us} + l_t^{em})\varsigma^{+1} (m_t^{us})^{-\varsigma m-1} \right. \right.
\]

\[
\left. \right] \right\} = \lambda_t^{us} \theta^{us} \quad (A.11)
\]

\[
(1 + \lambda_t^{us}) (v_{dt} - v_{dt}^{em}) = 0 \quad (A.12)
\]

\[
(v_{kt} - v_{dt}^{em} - \theta^{us}) q_t^{us} K_t^{us} + (v_{lt} - v_{dt}^{em} - \theta^{us}) l_t^{em} + (v_{mt} - v_{dt}^{em} - \theta^{us}) m_t^{us}
\]

\[
- v_{dt} \left[ \kappa_t^{us} (q_t^{us} K_t^{us} + l_t^{em})\varsigma^{+1} (m_t^{us})^{-\varsigma m} - n_t^{us} \right] \geq 0 \quad (A.13)
\]
Combining (A.8) and (A.9), and combining (A.8) and (A.10), respectively, yield

\[ v_{kt} = v_{lt} \quad (A.14) \]

\[
\left[ (1 + \varsigma_l) + \varsigma_m \left( q_t^{us} K_t^{us} + l_t^{em} \right) \left( m_t^{us} \right)^{-1} \right] \kappa_{lm}^{us} \left( q_t^{us} K_t^{us} + l_t^{em} \right) \varsigma_l \left( m_t^{us} \right)^{-\varsigma_m} = \frac{v_{kt} - v_{mt}}{v_{dt}^{em}} \quad (A.15)
\]

Then rearrange the incentive constraint (A.4) and the first order conditions imply

\[ q_t^{us} K_t^{us} + l_t^{em} + m_t^{us} \leq \phi_t^{us} n_t^{us} \quad (A.16) \]

where

\[ \phi_t^{us} = \frac{v_t^{em}}{\theta^{us} - \left( v_{kt} - v_{dt}^{em} \right) + v_{dt}^{em} \kappa_{lm}^{us} \left( q_t^{us} K_t^{us} + l_t^{em} \right) \varsigma_l \left( m_t^{us} \right)^{-\varsigma_m} \left( 1 + \varsigma_l m_t^{us} + \varsigma_m \left( q_t^{us} K_t^{us} + l_t^{em} \right) \right) \phi_t^{us} + v_{dt}^{em}}{q_t^{us} K_t^{us} + l_t^{em} + m_t^{us}} \quad (A.17) \]

If the incentive constraint is binding, then the assets the banker can acquire will positively depend on her own equity capital as

\[ q_t^{us} K_t^{us} + l_t^{em} + m_t^{us} = \phi_t^{us} n_t^{us} \quad (A.18) \]

Given the binding incentive constraint, substituting the first order conditions and the incentive constraint into the value function yields

\[ V_t^{us} = v_{kt} q_t^{us} K_t^{us} + v_{lt} l_t^{em} + v_{mt} m_t^{us} - v_{dt} d_t^{us} - v_{dt} v_t^{em} \]

\[ = \left\{ \left( v_{kt} - v_{dt}^{em} \right) - v_{dt}^{em} \kappa_{lm}^{us} \left( q_t^{us} K_t^{us} + l_t^{em} \right) \varsigma_l \left( m_t^{us} \right)^{-\varsigma_m} \left( 1 + \varsigma_l m_t^{us} + \varsigma_m \left( q_t^{us} K_t^{us} + l_t^{em} \right) \right) \phi_t^{us} + v_{dt}^{em} \right\} n_t^{us} \quad (A.19) \]

Substituting this expression for period \( t+1 \) into (A.1) yields

\[ V_t^{us} = E_t \left\{\Lambda_{t,t+1}^{us} \left( 1 - \sigma \right) n_{t+1}^{us} + \sigma \left\{ \left( v_{kt+1} - v_{dt+1}^{em} \right) - v_{dt+1}^{em} \kappa_{lm}^{us} \left( q_{t+1}^{us} K_{t+1}^{us} + l_{t+1}^{em} \right) \varsigma_l \left( m_{t+1}^{us} \right)^{-\varsigma_m} \left( 1 + \varsigma_l m_{t+1}^{us} + \varsigma_m \left( q_{t+1}^{us} K_{t+1}^{us} + l_{t+1}^{em} \right) \right) \phi_{t+1}^{us} + v_{dt+1}^{em} \right\} n_{t+1}^{us} \right\} = E_t \Omega_{t+1}^{us} n_{t+1}^{us} \quad (A.20) \]
where
\[
\Omega_{t+1}^{us} = \Lambda_{t,t+1}^{us} \left\{ 1 - \sigma + \sigma \left[ \left( v_{kt+1} - v_{dt+1}^{em} \right) - v_{dt+1}^{em} q_{t+1}^{us} K_{t+1}^{us} + l_{t}^{em} \right] \right\} \left( m_{t+1}^{us} \right)^{-\varsigma m} \left( 1 + \frac{s m_{t+1}^{us} + \zeta m_{t}^{us} \left( q_{t+1}^{us} K_{t+1}^{us} + l_{t}^{em} \right) \right) \left[ \phi_{t+1}^{us} + v_{dt+1}^{em} \right] \right}\]
(A.21)

is the marginal value of net worth for the banker, who exits with probability \( 1 - \sigma \) and stays active with probability \( \sigma \). Applying the method of undetermined coefficient
\[
V_{t}^{us} = v_{kt}^{t} K_{t}^{us} + v_{lt}^{t} + v_{mt}^{t} m_{t}^{us} - v_{dt}^{t} q_{t}^{us} - v_{dt}^{t} l_{t}^{em}
\]
(A.22)

We let \( \mu_{t}^{us}, \mu_{l,t}^{us}, \mu_{m,t}^{us}, \) and \( \nu_{t}^{us} \) denote
\[
\mu_{t}^{us} = E_{t} \Omega_{t+1}^{us} \left( R_{k,t+1}^{us} - R_{d,t}^{us} \right)
\]
(A.23)
\[
\mu_{l,t}^{us} = E_{t} \Omega_{t+1}^{us} \left( R_{l,t}^{us} - R_{d,t}^{us} \right)
\]
(A.24)
\[
\mu_{m,t}^{us} = E_{t} \Omega_{t+1}^{us} \left( R_{m,t}^{us} - R_{d,t}^{us} \right)
\]
(A.25)
\[
\nu_{t}^{us} = E_{t} \Omega_{t+1}^{us} R_{d,t}^{us}
\]
(A.26)

Then from (A.21), (A.17), (A.14), and (A.15), we have
\[
\Omega_{t+1}^{us} = \Lambda_{t,t+1}^{us} \left\{ 1 - \sigma + \sigma \left[ \left( \mu_{t+1}^{us} - \nu_{t+1}^{us} \right) q_{t+1}^{us} K_{t+1}^{us} + l_{t}^{em} \right] \right\} \left( m_{t+1}^{us} \right)^{-\varsigma m} \left( 1 + \frac{s m_{t+1}^{us} + \zeta m_{t}^{us} \left( q_{t+1}^{us} K_{t+1}^{us} + l_{t}^{em} \right) \right) \left[ \phi_{t+1}^{us} + \nu_{t+1}^{us} \right] \right}\]
(A.27)
\[
\phi_{t}^{us} = \frac{\nu_{t}^{us}}{\theta_{t}^{us} - \mu_{t}^{us} + \mu_{l,t}^{us} K_{t}^{us} + l_{t}^{em}} \left( m_{t}^{us} \right)^{-\varsigma m} \left( 1 + \frac{s m_{t+1}^{us} + \zeta m_{t}^{us} \left( q_{t+1}^{us} K_{t+1}^{us} + l_{t}^{em} \right) \right) \left[ \nu_{im}^{us} \left( q_{t}^{us} K_{t}^{us} + l_{t}^{em} \right) \right] \left( m_{t}^{us} \right)^{-\varsigma m}
\]
(A.28)
\[
\mu_{t}^{us} = \mu_{l,t}^{us}
\]
(A.29)
\[
\mu_{t}^{us} - \mu_{m,t}^{us} = \nu_{t}^{us} \left[ \left( 1 + s \right) + \zeta \left( q_{t}^{us} K_{t}^{us} + l_{t}^{em} \right) \right] \left( m_{t}^{us} \right)^{-1} \right\} \left( q_{t}^{us} K_{t}^{us} + l_{t}^{em} \right) \left( m_{t}^{us} \right)^{-\varsigma m}
\]
(A.30)
APPENDIX B

THE DETAILS OF THE OPTIMIZATION FOR THE EMERGING MARKET BANKER'S PROBLEM

The EM banker’s problem is given by

\[ V_{t}^{em} = \max_{\{K_t^{em}, l_t^{em}, d_t^{em}\}} E_t \left\{ A_{t+1}^{em} \left[ (1 - \sigma) n_{t+1}^{em} + \sigma V_{t+1}^{em} \right] \right\} \]  (B.1)

subject to

\[ q_t^{em} K_t^{em} \left[ 1 + (\kappa/2)x_t^2 \right] = d_t^{em} + \epsilon_t^{em} l_t^{em} + n_t^{em} \]  (B.2)

\[ n_t^{em} = R_{k,t}^{em} q_{t-1}^{em} K_{t-1}^{em} - R_{d,t-1}^{em} d_{t-1}^{em} - \epsilon_t^{em} R_{us,t-1}^{em} l_{t-1}^{em} \]  (B.3)

\[ V_t^{em} \geq \theta^{em}(x_t) q_t^{em} K_t^{em} \]  (B.4)

with

\[ x_t = \frac{\epsilon_t^{em} l_t^{em}}{q_t^{em} K_t^{em}} \]  (B.5)

\[ \theta^{em}(x_t) = \theta_0^{em} \exp(-\varphi^{em} x_t) \]  (B.6)

To solve the problem, we first guess that the value function \( V_t^{em} \) is linear in bank net worth as

\[ V_t^{em} = \psi_t^{em} n_t^{em} \]  (B.7)

Using the definition of leverage ratio \( \phi_t^{em} = \frac{q_t^{em} K_t^{em}}{n_t^{em}} \), we can rewrite the EM banker’s problem as

\[ \psi_t^{em} = \max_{\{\phi_t^{em}, x_t\}} \left[ (\mu_t^{em} + \mu_{lt}^{em} x_t) \phi_t^{em} + \left( 1 - \frac{\kappa}{2} x_t^2 \phi_t^{em} \right) V_t^{em} \right] \]  (B.8)
subject to

\[
\left( \mu_{em}^t + \mu_{lt}^t x_t \right) \phi_t^em + \left( 1 - \frac{\kappa}{2} x_t^2 \phi_t^em \right) \nu_t^em \right) \geq \theta^em(x_t) \phi_t^em = \psi_0^em \exp(-\vartheta^em_1 x_t) \phi_t^em
\]

(B.9)

where

\[
\mu_t^em = E_t \Omega_{t+1}^em \left( R_{k,t+1}^{em} - R_{d,t}^{em} \right)
\]

(B.10)

\[
\mu_{lt}^em = E_t \Omega_{t+1}^em \left( R_{d,t}^{em} - \epsilon_{t+1}^{em} R_{us}^{em} \right)
\]

(B.11)

\[
\nu_t^em = E_t \Omega_{t+1}^em R_{d,t}^{em}
\]

(B.12)

\[
\Omega_{t+1}^em = \Lambda_{t,t+1}^em \left( 1 - \sigma + \varphi_t^em \right)
\]

(B.13)

Let \( \lambda_t^em \) be the Lagrangian multiplier for the incentive constraint (B.9). Then using the Lagrangian

\[
L = \psi_t^em + \lambda_t^em \left( \psi_t^em - \vartheta_0^em \exp(-\vartheta_1^em x_t) \phi_t^em \right)
\]

(B.14)

The first order conditions with respect to \( \{ \phi_t^em, x_t \} \) imply

\[
(1 + \lambda_t^em) \left( (\mu_t^em + \mu_{lt}^em x_t) \phi_t^em + \left( 1 - \frac{\kappa}{2} x_t^2 \phi_t^em \right) \nu_t^em \right) - \lambda_t^em \vartheta_0^em \exp(-\vartheta_1^em x_t) \phi_t^em = 0
\]

(B.15)

\[
(1 + \lambda_t^em) \left( \kappa x_t \nu_t^em - \mu_{lt}^em \right) = \lambda_t^em \vartheta_1^em \vartheta_0^em \exp(-\vartheta_1^em x_t)
\]

(B.16)

Combining (B.15) and (B.16) yields

\[
\frac{K}{2} x_t^2 \nu_t^em \vartheta_1^em + (\kappa \nu_t^em - \vartheta_1^em \mu_{lt}^em) x_t = \vartheta_1^em \mu_t^em + \mu_{lt}^em
\]

(B.17)

Then given the binding incentive constraint, we have

\[
x_t = \frac{\mu_{lt}^em}{\kappa \nu_t^em} - \frac{1}{\vartheta_1^em} + \sqrt{\left( \frac{\mu_{lt}^em}{\kappa \nu_t^em} \right)^2 + \left( \frac{1}{\vartheta_1^em} \right)^2 + 2 \frac{\mu_t^em}{\kappa \nu_t^em}}
\]

(B.18)

\[
\psi_t^em = \theta^em(x_t) \phi_t^em
\]

(B.19)

\[
\phi_t^em = \frac{\nu_t^em}{\theta^em(x_t) + \kappa x_t^2 \nu_t^em - (\mu_t^em + \mu_{lt}^em x_t)}
\]

(B.20)
Bibliography


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