ESSAYS IN INTERNATIONAL MONETARY ECONOMICS

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ABSTRACT

This dissertation focuses on the role of monetary policy in the rapidly changing global environment, characterized in particular by the adoption of the common currency by fifteen European countries and by rapid accumulation of foreign debt (or wealth) by many nations around the world. The first chapter calculates differences in welfare costs of nominal rigidities in the EMU countries. Using a two-country DSGE model with optimizing agents, monopolistic wage and price setting and government debt dynamics, I find that these costs are virtually identical for all members of the EMU, and small countries are not at a disadvantage when it comes to the setting of the common monetary policy. This conclusion is primarily due to highly correlated technological processes in Europe, which cause national and Euro-wide inflations to move together. The second chapter studies the causes of the cyclical behavior of aggregate inflation and regional inflation differentials in the EMU. The answer has strong implications for monetary policy. In the United States, inflation rates move pro-cyclically, and across the Euro Area, inflation differentials are positively correlated with growth differentials. This suggests that demand shocks are the primary determinants of the cyclical behavior of aggregate inflation and regional inflation differentials. The conclusion is that demand shocks are either missing or inadequately modeled in the in typical New Keynesian model. In the last chapter, I study the impact of net foreign wealth on the optimal monetary policy of an open economy in a two-country DSGE model with incomplete markets, sticky prices and deviations from the Law of One Price. I find that by optimally manipulating monetary policy, central banks can affect the timing of interest receipts (or payments) and therefore increase the risk-sharing role of the internationally traded asset. In particular, debtor nations find it optimal to allow their currency to float relatively more freely than do creditor nations. We also find that for most specifications of the model, central banks should target a weighted average of CPI inflation and changes in the nominal exchange rate.
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1 Welfare Implications of Country Size In a Monetary Union

1.1 Introduction

Now that several years have passed since the inception of the Euro, and various indicators of Europe’s economic activity became available, the attention of many economists has been focused on analyzing the workings of the European Monetary Union (EMU). One of the most interesting and important questions which are asked has to do with the coordination of the common monetary and national fiscal policies. Do all members of the EMU receive adequate attention from the European Central Bank (ECB), or does the monetary policy favor large countries over small? In the latter case, fiscal policies are needed to stabilize regional economies and therefore should not be constrained by the Stability and Growth Pact. In a few months, the EMU will add two more small countries: Malta and Cyprus; will the benefits to these economies from joining the Euro area outweigh the costs of losing monetary control over their national business cycles?

In this paper, we address these questions within the framework of New Neoclassical Synthesis, which is characterized by the presence of nominal inertia and therefore lends itself easily to analyzing the interaction of monetary and fiscal policies. In particular, we use a two-country general equilibrium model of a currency union to study the relationship between country size and its costs of wage and price rigidities. To the best of our knowledge, very few authors have done research in this area. Canzoneri et al (2004) use a two-country partial-equilibrium model calibrated to the EMU to find that the central bank pays less attention to small countries’ inflations. The authors calculate that small EMU countries incur up to four times higher welfare costs of price and wage rigidities (around 1.9 percent of consumption) than their large neighbors.

Recent papers that study the EMU in a general equilibrium setup do so either using a symmetric currency union model, or link country sizes directly to the size of the home bias in consumer demand
function\(^1\). Thus, consumers in a small country demand a (proportionately) small amount of domestic goods and a large amount of goods produced abroad. This assumption carries with it two important implications.

First, as Table 1 shows, while smaller countries do tend to be more open, their share of trade in GDP is by no means connected to their relative size in the union, which makes it difficult to justify small home biases in smaller countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>Exports, % GDP</th>
<th>Imports, % GDP</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>48.5 51.0 53.4 56.1</td>
<td>45.2 46.7 48.5 50.4</td>
<td>2.6</td>
</tr>
<tr>
<td>Belgium</td>
<td>81.0 83.4 86.3 87.7</td>
<td>76.6 79.4 83.3 85.1</td>
<td>3.3</td>
</tr>
<tr>
<td>Finland</td>
<td>38.6 39.9 41.8 44.5</td>
<td>30.3 31.9 36.2 39.3</td>
<td>1.7</td>
</tr>
<tr>
<td>France</td>
<td>25.6 25.7 26.0 26.9</td>
<td>24.6 25.6 26.9 28.3</td>
<td>19.9</td>
</tr>
<tr>
<td>Germany</td>
<td>35.7 38.2 40.7 45.1</td>
<td>31.7 33.2 35.5 39.6</td>
<td>26.1</td>
</tr>
<tr>
<td>Greece</td>
<td>19.6 20.8 20.8 21.3</td>
<td>29.2 29.7 28.0 30.1</td>
<td>3.5</td>
</tr>
<tr>
<td>Ireland</td>
<td>83.8 84.1 81.3 79.7</td>
<td>67.7 69.2 68.6 68.4</td>
<td>1.3</td>
</tr>
<tr>
<td>Italy</td>
<td>24.6 25.4 26.1 27.8</td>
<td>24.0 24.7 26.1 28.7</td>
<td>18.6</td>
</tr>
<tr>
<td>Luxem.</td>
<td>134.0 148.3 159.3 177.2</td>
<td>113.4 127.4 138.0 149.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Nether.</td>
<td>63.0 66.5 69.9 74.2</td>
<td>56.7 59.3 62.2 66.5</td>
<td>5.2</td>
</tr>
<tr>
<td>Portugal</td>
<td>28.0 28.4 28.5 31.1</td>
<td>34.5 36.2 37.1 38.9</td>
<td>3.3</td>
</tr>
<tr>
<td>Slovenia</td>
<td>55.8 60.0 64.6 69.2</td>
<td>55.9 61.2 65.1 69.9</td>
<td>0.6</td>
</tr>
<tr>
<td>Spain</td>
<td>26.3 26.0 25.5 26.1</td>
<td>28.7 30.0 30.9 32.3</td>
<td>13.7</td>
</tr>
</tbody>
</table>

Table 1: Trade in the European Monetary Union. Country size measures national population as a percent of the EMU total. Source: Eurostat.

\(^1\)See, for example, Benigno (2004), Duarte and Wolman (2003), and Ferrero (2005), among many others.
Second, equating home bias and country size means that both members of the currency union have the same composition of their consumption baskets (e.g., 10 percent of the small country good) and therefore pay the same price for them. The two CPIs equalize and move in lockstep unless a preference shock perturbs the home biases themselves. This of course minimizes inflation differentials (and therefore welfare costs of having the common monetary authority) in the two countries.

The main contribution of this paper lies in separating country size from home bias to better understand the impact of various shocks on national inﬂations and on monetary policy responses in a currency union. In addition, we augment the standard New-Keynesian two-country DSGE model with a more realistic economic structure by allowing for incomplete international asset markets, distortionary taxes, labor market rigidities, and non-zero steady state government debt. Within this framework we pose the following two questions: (1) are the welfare costs of nominal rigidities in the EMU related to country size, and (2) which feature(s) of the model have the biggest influence on the differences in these costs across countries. Most recent studies only address one of these issues; EMU papers generally have a positive focus, while welfare literature concentrates on the United States.

The ﬁnding of this study is that the welfare costs of nominal inertia are very similar for the EMU countries regardless of their relative size. Productivity correlations and intra-union trade act as price-correcting mechanisms following country-speciﬁc shocks and so help to stabilize the monetary union by reducing national inﬂation differentials and lowering the costs of idiosyncratic disturbances. This result is robust to different speciﬁcations of international asset markets, the monetary policy rule, trade openness, and several key parameter values.

It should be noted at the outset that New Keynesian models over-emphasize the importance of technological shocks in explaining variability of inﬂation (we will further discuss this issue below). Admittedly, some of our results may be driven by this problem; therefore, this study should be viewed as the ﬁrst step in analyzing the relationship between country size and its welfare.

The rest of the paper is structured as follows: the theoretical model is presented in section 1.2;
section 1.3 discusses calibration; section 1.4 presents the results of our simulations and robustness exercises; finally, section 1.5 concludes.

1.2 The Model

The currency union is composed of two countries, home \( (H) \) and foreign \( (F) \). Both countries are populated by infinitely lived households of measure \( M \) in the home country and \( M^* \) in the foreign; there is no migration. Households consume all varieties of home and foreign goods and have access to international markets where they can trade a state-contingent nominal bond (this assumption will be relaxed later). Each country has a measure one of firms, which use labor and capital to produce a continuum of domestic goods, which are then traded internationally. Firms are monopolistically competitive, and the prices they set for their products are sticky. International goods markets are not segmented, so the Law of One Price holds. Labor and capital are assumed to be country-specific, and capital investment is subject to adjustment costs.

We model each country as having only the traded goods sector. Empirical studies provide mixed evidence on the relative importance of traded and non-traded goods sectors in generating the observed cross-country inflation differentials\(^2\). Therefore, for modeling simplicity, we assume that all goods are traded.

1.2.1 Firms

Each country has a continuum of firms indexed by \( f \) on the unit interval. At time \( t \), each home firm rents capital \( K_{t-1}(f) \) from the domestic households at the rate \( R_t \), hires a labor bundle \( N_t(f) \) at the rate \( W_t \) and produces one of the varieties of the domestic good. Each firm is free to set its own price level \( P_{H,t}(f) \) (\( P_{F,t}(f) \) in the foreign country). The Law of One Price holds, so every product

---

\(^2\)Altissimo et al (2005) present evidence that inflation dispersion in the EMU stems primarily from the non-traded goods sectors; on the other hand, Andrés et al (2006) find that there is a lower but still persistent inflation dispersion in the industrial goods (excluding energy).
sells for the same price at home and abroad.

As a matter of notation, subscripts $H$ and $F$ will refer to a good’s country of origin; asterisks will indicate that it is consumed in country $F$. For example, $C_{H}^{*}$ denotes consumption of country $H$’s good in country $F$. The two economies have a similar structure; therefore, most of the equations will be presented only for the home country.

Home firms use the following Cobb-Douglas technology to produce output:

$$Y_{H,t}(f) = Z_{t}K_{t-1}(f)^{\nu}N_{t}(f)^{1-\nu},$$

where $0 < \nu < 1$, and $Z_{t}$ denotes the level of productivity enjoyed by all the home firms at time $t$.

Productivity in the two countries evolves according to the following autoregressive process:

$$[Z_{t}] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} [Z_{t-1}] + [\varepsilon_{z,t}] + [\varepsilon_{z,t}^{*}],$$

and $\varepsilon$’s aren’t serially or spatially correlated.

From the firms’ cost minimization problem it follows that

$$\frac{R_{t}}{W_{t}} = \frac{\nu}{1-\nu} \frac{N_{t}(f)}{K_{t-1}(f)},$$

$$MC_{t}(f) = \frac{R_{t}^{\alpha}W_{t}^{1-\nu}}{Z_{t}^{\nu}(1-\nu)^{1-\nu}}$$

All goods varieties are then bundled into a composite home and foreign goods using the Dixit-Stiglitz aggregator:

$$Y_{H,t} = \left[ \int_{0}^{1} Y_{H,t}(f) \frac{\sigma-1}{\sigma} df \right]^{\frac{1}{\sigma-1}}, \quad Y_{F,t} = \left[ \int_{0}^{1} Y_{F,t}(f) \frac{\sigma-1}{\sigma} df \right]^{\frac{1}{\sigma-1}}, \quad (1.1)$$

where $\sigma > 1$. These composite goods can then be used for public and private consumption or private investment.

Imposing the zero-profit condition, the prices (in the common currency) of the bundles are given by

$$P_{H,t} = \left[ \int_{0}^{1} P_{H,t}(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}, \quad P_{F,t} = \left[ \int_{0}^{1} P_{F,t}(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}, \quad (1.2)$$
and therefore demand for firm $f$’s product is

$$Y_{H,t}^d(f) = \left[ \frac{P_{H,t}(f)}{P_{H,t}} \right]^{-\sigma} Y_{H,t}$$

$$Y_{F,t}^d(f) = \left[ \frac{P_{F,t}(f)}{P_{F,t}} \right]^{-\sigma} Y_{F,t}$$

Here $Y_{H,t}$ and $Y_{F,t}$ are the aggregate demands (to be defined below) for home and foreign goods bundles from all agents in both economies (i.e., households, investors and governments).

As in Calvo (1983), home and foreign firms reset their prices each period with a constant probability $(1-\alpha)$ and $(1-\alpha^*)$, respectively; otherwise, the old prices remain in effect. If a (home) firm $f$ gets to announce a new price in period $t$, it chooses $\tilde{P}_{H,t}(f)$ to maximize its expected future profits

$$E_t \sum_{j=t}^{\infty} \Omega_{t,j} \alpha^{j-t} \left[ \tilde{P}_{H,t}(f)Y_{H,j}^d(f) - TC_{H,j}(Y_{H,j}^d(f)) \right]$$

$\Omega_{t,j}$ is the home households’ stochastic discount factor: $\Omega_{t,j} = \beta^{j-t} \left( \frac{\lambda_t}{\lambda_{t-j}} \right)$, where $\lambda_t$ is the marginal utility of nominal wealth (defined in the next subsection).

The optimal price is given by

$$\tilde{P}_{H,t} = \varphi_p \frac{P_{B_t}}{P_{A_t}}$$

$$\varphi_p = \frac{\sigma}{\sigma - 1}$$

$$P_{B_t} = \alpha \beta E_t[P_{B_{t+1}} + \lambda_t P_{H,t}^2 MC_{H,t}(f)]$$

$$P_{A_t} = \alpha \beta E_t[P_{A_{t+1}} + \lambda_t P_{H,t}^2 Y_{H,t}]$$

We consider a symmetric equilibrium in which every firm that gets a chance to reset its price in period $t$ will set it to the same value; therefore, optimal price isn’t denoted by a firm subscript $(f)$. As $\alpha \to 0$ and prices become perfectly flexible, each period firms set a new $\tilde{P}_{H,t} \to \varphi_p MC_{H,t}(f)$.

Since the monopolistic markup $\varphi_p > 1$, prices are set above the marginal cost and so output is inefficiently low.

Given the price-setting behavior of individual firms, the aggregate price level in the country can be written as

$$P_{H,t}^{1-\sigma} = (1-\alpha)\tilde{P}_{H,t}^{1-\sigma} + \alpha P_{H,t-1}^{1-\sigma}$$

(1.4)
1.2.2 Households

There is a continuum of households in the home country, indexed by $i$ on the interval $[0, M]$. A representative household maximizes expected lifetime utility

$$U_t(h) = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \frac{C_j(h)^{1-\Theta}}{1-\Theta} - (1 + \chi)^{-1} L_j(h)^{1+\chi} \right\}$$

Here $C_t(h)$ denotes the household’s consumption of the composite good, which is aggregated from home and foreign goods using the CES aggregator:

$$C_t(h) = \left[ \mu_t C_{H,t}(h)^{\frac{1}{\eta}} + (1 - \mu_t) \frac{1}{\eta} C_{F,t}(h)^{\frac{1}{\eta}} \right]^{\frac{\eta}{1-\eta}}$$

$\eta > 1$ is the elasticity of substitution between home and foreign goods, and $0 < \mu_t < 1$ determines the degree of home bias. The time subscript on $\mu$ will allow us to model demand (preference) shocks. $C_t^*(h)$ denotes consumption of the aggregate good (with a different home bias $\mu_t^*$) in the foreign country.

The prices of the two final goods, which also represent the countries' CPIs, are given by

$$P_t = \left[ \mu_t P_{H,t}^{1-\eta} + (1 - \mu_t) P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

$$P_t^* = \left[ \mu_t^* P_{F,t}^{1-\eta} + (1 - \mu_t^*) P_{H,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

Given these prices, the household $(h)$ demands the following quantities of the composite goods:

$$C_{H,t}(h) = \mu_t \left[ \frac{P_{H,t}}{P_t} \right]^{-\eta} C_t(h), \quad C_{F,t}(h) = (1 - \mu_t) \left[ \frac{P_{F,t}}{P_t} \right]^{-\eta} C_t(h)$$

Households supply differentiated labor services to all the firms in their country. The composite labor bundle used in production by any given home firm is given by

$$N_t(f) = M^{\frac{1}{1-\phi}} \left[ \int_0^M L_t(h, f)^{\frac{1}{\phi-1}} dh \right]^{\frac{\phi-1}{\phi}}$$

$\phi^3$The scaling factor $M^{\frac{1}{1-\phi}}$ is necessary to maintain the aggregate relationship $N_t = \int_0^M N_t(f) df = ML_t(h)$. Together with the expression $K_t = MK_t(h)$ this will ensure that the production function exhibits constant returns to scale. Additionally, in the steady state the aggregate wage $W$ will equal the individual wage $W(h)$. 

7
Correspondingly, the aggregate labor demand is

$$N_t = \int_0^1 N_t(f)df$$

(1.6)

Each household enjoys a degree of monopolistic power in setting its wage $W_t(h)$. Similar to the price of the bundle of goods, the expression for aggregate wage can be written as

$$W_t = M^{\frac{1}{1-\phi}} \left[ \int_0^M W_t(h)^{1-\phi} dh \right]^{\frac{1}{1-\phi}}$$

The demand for household $(h)$’s labor services given its wage is

$$L_t^d(h) = \left( \frac{W_t(h)}{W_t} \right)^{-N_t/M}$$

(1.7)

Each household faces the following budget constraint:

$$E_t[\Delta_{t,t+1}D_{t+1}(h)] + B_{t+1}^d(h) + P_t(1 + \tau_{c,t})C_t(h) + P_{I,t}I_t(h) + P_tT_t(h) =$$

$$= (1 - \tau_{c,t})W_t(h)L_t^d(h) + D_t(h) + (1 + \delta_{t-1})B_{t}^d(h) +$$

$$+ (1 - \tau_{k,t})R_tK_{t-1}(h) + \tau_{k,t}\delta P_{I,t}K_{t-1}(h) + \Pi_t(h)$$

(1.8)

The first term on the left-hand side is the price of a portfolio of state-contingent bonds traded internationally, and $D_t$ is the payoff of such portfolio in period $t$. $B_t^d(h)$ represents household’s demand for the riskless one-period nominal domestic government bond. Households receive transfers $P_tT_t(h)$ from their government. $\tau_{c,t}$, $\tau_{w,t}$ and $\tau_{k,t}$ are taxes on consumption, labor income and capital income, respectively; effectively, households pay tax on nominal capital income and get an inflation-adjusted rebate on depreciated capital. $\Pi_t(h)$ represent household’s dividend income. Finally, since below we let investment good $I_t(h)$ have a different aggregator than the one for the consumption good, it has a distinct price $P_{t,i}$. 

4In the presence of the complete set of Arrow securities $D_t$, government bonds are redundant for the purposes of risk-sharing; we introduce them to model the dynamics of national debt.
The household’s capital accumulation is given by

\[ K_t(h) = (1 - \delta)K_{t-1}(h) + I_t(h) - \frac{1}{2} \psi \left[ \frac{I_t(h)}{K_{t-1}(h)} - \delta \right]^2 K_{t-1}(h) \]  \hspace{1cm} (1.9)

Here, the investment good \( I_t(h) \) is aggregated from home and foreign bundles as follows:

\[ I_t(h) = \left[ \vartheta_t^\frac{1}{2} I_{H,t}(h) \frac{\varrho_t - \eta}{\eta} + (1 - \vartheta_t)^\frac{1}{2} I_{F,t}(h) \frac{\varrho_t - \eta}{\eta} \right]^\frac{\eta}{\varrho_t} \]

where \( \vartheta \neq \mu \) to allow for a different home bias in the investment good\(^5\). Similarly to the price of the consumption good, the price and the corresponding demands for the investment good are

\[ P_{I,t} = \left[ \vartheta_t P_{H,t}^{1-\eta} + (1 - \vartheta_t) P_{F,t}^{1-\eta} \right]^\frac{1}{1-\eta} \]

\[ I^d_{H,t}(h) = \vartheta_t \left[ \frac{P_{H,t}}{P_t} \right]^{-\eta} I_t(h), \quad I^d_{F,t}(h) = (1 - \vartheta_t) \left[ \frac{P_{F,t}}{P_t} \right]^{-\eta} I_t(h) \]

Households maximize utility (1.5) subject to the budget constraint (1.8), labor demand (1.7) and capital accumulation constraint (1.9) by choosing wage rate \( W_t(h) \), consumption \( C_t(h) \), portfolio holdings \( D_{t+1}(h) \) and \( B_{t+1}(h) \), and investment \( I_t(h) \).

Wages are sticky, and in any given period a household gets to reset its wage with probability \((1 - \omega) ((1 - \omega^*) \text{ abroad})\). The optimal new wage satisfies

\[ \bar{W}_t^{\phi+1} = \varphi \frac{WB_t}{WA_t} \]

\[ \varphi_w = \frac{\phi}{\phi - 1} \]

\[ WB_t = W_t^{\phi(\chi+1)} \left( \frac{N_t}{M} \right)^{1+\chi} + (\omega \beta)E_t[WB_{t+1}] \]

\[ WA_t = \lambda_t(1 - \tau_{w,t})W_{t}^{\phi} \frac{N_t}{M} + (\omega \beta)E_t[WA_{t+1}] \]

Assuming that every household that chooses its wage in period \( t \) sets it to the same new value, we drop the subscript \((h)\) on the optimal wage rule. If \( \omega = 0 \) (flexible wages), households optimally

\[ \frac{5}{5} I_{H,t}(h) \] (and later \( G_{H,t} \)) has the same composition with respect to the differentiated intermediate home goods as \( Y_{H,t} \) in (1.1). Similarly, \( I_{F,t}(h) \) and \( G_{F,t} \) have the same composition with respect to intermediate foreign goods as \( Y_{F,t} \) in (1.1). Notice, however, that the degree of home bias in investment \( (\vartheta_t) \) may be different from consumption home bias \( (\mu_t) \) and government home bias \( (\zeta_t) \); the latter is defined below. Consequently, the price of investment \( P_{I,t} \) may be different from the price of consumption \( P_t \).
set \( \tilde{W}_{t}^{\phi+1} = \varphi_{w} \frac{W_{t}^{\phi} L_{t}(h)}{\lambda_{t}(1-\tau_{w,t})} \), from which it follows that \( \lambda_{t}(1-\tau_{w,t})\tilde{W}_{t} = \varphi_{w} L_{t}(h) \). Households don’t set their marginal disutility of work equal to the marginal product of labor, which creates an inefficiency in their (too low) labor allocation.

Similar to the derivations of the aggregate price level given firms’ first-order conditions, the aggregate wage level is given by

\[
W_{t}^{1-\phi} = (1-\omega)\tilde{W}_{t}^{1-\phi} + \omega W_{t-1}^{1-\phi}
\]

The rest of the first order conditions (with respect to consumption, bond holdings, investment, and capital stock, in that order) for the household problem are as follows:

\[
\frac{1}{C_{t}^{\phi} P_{t}} = \lambda_{t}(1 + \tau_{c,t})
\]

\[
\beta E_{t}[\frac{\lambda_{t+1}}{\lambda_{t}}] = \frac{1}{1 + \delta t} \equiv I_{t}^{-1}
\]

\[
\lambda_{t} P_{t,t} = \xi_{t} [1 - \psi(\frac{I_{t}(h)}{K_{t-1}(h)} - \delta)]
\]

\[
\xi_{t} = \beta E_{t}[\lambda_{t+1} ((1 - \tau_{k,t+1}) R_{t+1} + \tau_{k,t+1} \delta P_{t,t+1}) +
\]

\[
+ \xi_{t+1} \left[ (1 - \delta) - \frac{1}{2} \psi \left( \frac{I_{t+1}(h)}{K_{t}(h)} - \delta \right)^2 + \psi \left( \frac{I_{t+1}(h)}{K_{t}(h)} - \delta \right) \frac{I_{t+1}(h)}{K_{t}(h)} \right] \}
\]

Since all households face the same stochastic discount factor, and if we assume they start off with the same level of wealth, their \( \lambda_{t}(h) \) and \( C_{t}(h) \) will be the same in equilibrium.

Similar equations hold for the foreign households with appropriate variables denoted by asterisks.

International risk-sharing conditions and households’ first order conditions imply that

\[
q_{t} = \Gamma \frac{C_{t}^{\phi}(h)(1 + \tau_{c,t})}{C_{t}^{\phi}(h)(1 + \tau_{c,t})}, \quad (1.10)
\]

where \( q_{t} \equiv \frac{P_{t}}{\tilde{P}_{t}} \) is the relative CPI index (price of foreign consumption bundle in terms of home consumption bundle, not equal to one due to the presence of the home bias in the two countries) and constant \( \Gamma \) is determined by the initial consumption conditions at home and abroad\(^{6}\).

\(^{6}\)While the existence of the internationally traded bond simplifies aggregation by equalizing marginal utilities
1.2.3 The Government

The instrument of the common monetary authority is the short-term nominal interest rate $i_t$. The conduct of monetary policy has important implications for national fiscal governments because the value of the interest rate affects interest payments on national debt. In addition, price and wage stickiness offer a channel for monetary policy to affect real activity in both economies.

The monetary authority sets the interest rate in response to the aggregate monetary union inflation $\pi_t$ (change in the union-wide price level):

$$
i_t = (1 - \rho_i)\bar{i} + \rho_i i_{t-1} + \varpi(1 - \rho_i)\pi_t + \varepsilon_{i,t}$$

$$
\pi_t = \kappa H_t + \kappa^* F_t 
$$

(1.11)

Here $\bar{i} = -\log(\beta)$ is the steady state level of the interest rate. $\kappa \equiv \frac{M_M}{M_T+M_H}$ and $\kappa^* \equiv \frac{M_T^*}{M_T+M_H}$ measure the relative size of the two regions, and $\varpi$ indicates the relative weight on inflation targeting.

We chose to exclude the output gap from the Taylor rule for several reasons. First, the existing empirical literature does not seem to agree on the value of the coefficient on the gap term: estimates (based on monthly data) range from statistically insignificant to 1.03. Additionally, the focus of this paper is on the differences of welfare costs among countries of different size, rather than on the costs themselves (the latter clearly would be strongly influenced by the value of the gap term coefficient). We perform a robustness check (further discussed below) by adding the output gap term to the monetary policy rule, and the main conclusion of the paper does not change: nominal costs of inertia are almost identical in small and large members of the monetary union. Finally, the moments of the model fit the data much better without the gap term.

of wealth in the two countries, it has an uncomfortable implication that the variation in the relative consumption growth rates is directly linked to the volatility of the inflation dispersion. To see this, divide (1.10) by its lag to get

$$\frac{C_t}{C_{t-1}} = \left[\frac{F_{t-1}}{F_{t-1}} \right]^{\theta} \left[\frac{F_t}{F_{t-1}}\right]^{-\omega} \left[\frac{H_{t-1}}{H_t}\right].$$

In the data, the correlation between these two variables is rather low [see, for example, Chari et al (2002)].

7 A non-exhaustive list of such studies includes Andrés, López-Salido and Vallés (2006), Hayo and Hofman (forthcoming), Gerlach-Kristen (2003), and Gerdesmeier and Roffia (2004).
The home country fiscal authority has the following budget constraint (expressed in per capita terms):

\[
B_{t+1}^s + \tau_{o,t} P_t C_t + \tau_{w,t} W_t N_t + \tau_{k,t} K_{t-1} (R_t - \delta P_{t,t}) =
\]

\[
= (1 + i_{t-1}) B_t^s + P_{G,t} G_t/M + P_t T_t
\]

Here \( G_t \) denotes aggregate government purchases, which may have a different composition (and therefore a different price) than the consumption or investment goods:

\[
G_t = \left[ \frac{1}{\zeta} G_{H,t}^{\eta-1} + (1 - \zeta) \frac{1}{\zeta} G_{F,t}^{\eta-1} \right]^{\frac{1}{\eta}}
\]

\[
P_{G,t} = \left[ \frac{1}{\zeta} P_{H,t}^{1-\eta} + (1 - \zeta) P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}
\]

\[
G_{H,t}^d = \zeta_t \left[ \frac{P_{H,t}}{P_t} \right]^{-\eta} G_t, \quad G_{F,t}^d = (1 - \zeta_t) \left[ \frac{P_{F,t}}{P_t} \right]^{-\eta} G_t
\]

In our model, government purchases do not yield utility to households. Analogous equations (with starred variables) describe the fiscal policy in the foreign country.

Each government has five tools at its disposal to affect the functioning of its domestic economy: three taxes, transfers, and purchases. In this paper we will let government purchases be described by an autoregressive process as follows:

\[
\log G_t = (1 - \rho_g) \log G + \rho_g \log G_{t-1 IS} + \varepsilon_{g,t}
\]

Taxes are assumed to be constant; transfers react to the debt-to-GDP ratio to ensure long-run fiscal solvency:

\[
\log T_t = (1 - \rho_{t,r}) \log T + \rho_{t,r} \log T_{t-1} + \rho_b (\log B - \log B_{t-1})
\]

In the above equations bars denote steady state variables; shocks to government variables are uncorrelated white noise processes.
1.2.4 Aggregation and Equilibrium

Since the measure of households in the two countries is not one, aggregate and per-capita quantities will be different in equilibrium: aggregate consumption is a multiple of the per-capita value

\[ C_t = \int_0^M C_t(h) dh = MC_t(h) \]

Similar expressions link aggregate and per-capita investment and capital.

Given the above expressions for the aggregate price and wage levels, the aggregate output can be rewritten in terms of aggregate capital stock \( K_{t-1} \equiv \int_0^1 K_{t-1}(f) df \) and the economy-wide labor input \( N_t \equiv \int_0^1 N_t(f) df \):

\[
Y_{H,t} = Z_t K_{t-1}^{\nu} N_t^{1-\nu} DP_t^{-1}
\]

\[
DP_t = \int_0^1 \left[ \frac{P_{H,t}(f)}{P_{H,t}} \right]^{-\sigma} df = \left( \frac{P_{H,t}}{P_{H,t-1}} \right)^\sigma (1 - \alpha) + \alpha \left( \frac{P_{H,t}}{P_{H,t-1}} \right)^\sigma DP_{t-1},
\]

where the last term is a measure of the aggregate price dispersion in the economy.

An equilibrium in this economy is a collection of:

1. allocations and wages for home consumers \( C_t(h), W_t(h), I_t(h), B_t^d(h) \) and \( D_t(h) \);
2. allocations and wages for foreign consumers \( C_t^*(h), W_t^*(h), I_t^*(h), B_t^{d*}(h) \) and \( D_t^*(h) \);
3. allocations and prices for home firms \( K_{t-1}(f), N_t(f) \) and \( P_{H,t}(f) \);
4. allocations and prices for foreign firms \( K_t^*(f), N_t^*(f) \) and \( P_{H,t}^*(f) \);
5. rental rate of capital \( R_t \); and
6. monetary and fiscal policies, such that

(a) given the government policies, \( R_t \), and exogenous shock processes, household allocations solve the households’ problem;
(b) given the government policies, $R_t$, and exogenous shock processes, firm allocations solve the firms’ problem;

(c) goods markets clear:

$$Y_{H,t} = MC_{H,t} + M^*C^*_{H,t} + MI_{H,t} + M^*I^*_{H,t} + G_{H,t} + G^*_{H,t}$$

$$Y_{F,t} = MC_{F,t} + M^*C^*_{F,t} + MI_{F,t} + M^*I^*_{F,t} + G_{F,t} + G^*_{F,t}$$

(Notice that in the above two expressions, output and government purchases are not in per-capita units.)

(d) labor markets clear:

$$N_t = \int_0^1 N_t(f)df = N_t(f) = \left[ \int_0^1 L_t(h,f) \frac{\phi^{-1}}{\phi^{-1}} dh \right]^{\phi^{-1}} = L_t(h,f)$$

$$N^*_t = \int_0^1 N^*_t(f)df = N^*_t(f) = \left[ \int_0^1 L^*_t(h,f) \frac{\phi^{-1}}{\phi^{-1}} dh \right]^{\phi^{-1}} = L^*_t(h,f)$$

(e) Asset markets clear:

$$\int_0^M D_{t+1}(h)dh = - \int_0^M D^*_{t+1}(h)dh$$

$$\int_0^M B^d_{t+1}(h)dh = B^d_{t+1}$$

$$\int_0^M B^s_{t+1}(h)dh = B^s_{t+1}$$

The last two conditions ensure that the households are willing to hold all the bonds issued by their domestic governments.

1.2.5 Incomplete Asset Markets

The assumption of complete markets is not innocuous in that it tightly links volatility of consumption to inflation fluctuations and through these, to the interest rate movements (see equation (1.10)). Such high correlations are not observed in the data. Additionally, if households do not have access to
contingent securities markets, their welfare losses from market frictions may be higher. For these reasons, we reproduce our welfare calculations in the incomplete asset markets setting.

The modeling of incomplete markets is borrowed from Benigno (2001) and Andrés et al (2006). Households can allocate their wealth between domestically traded state-contingent claims ($D_t$ in the home country, $D_t^*$ in the foreign country) and domestic government bonds, denoted by $B_t^d$ in the home country and $B_t^{d*}$ abroad.

Additionally, all households can buy an internationally traded one period risk-free bond $A_t$ with the gross nominal rate of return $1 + i_t$. However, home and foreign consumers pay different prices for this bond: foreign households pay the usual price $(1 + i_t)^{-1}$, while home consumers’ price $[1 + i_t - p(e^{a_t} - 1)]^{-1}$ depends on their position in the international asset market. Here $a_t = \frac{A_t}{r_t C_t}$ is the ratio of the aggregate asset holdings by home consumers to their consumption, and the parameter $p$ captures transaction costs. As lenders, domestic households pay a higher price for the bond, and as borrowers they must offer a rate of return higher than $(1 + i_t)$. Thus, markets are complete within each country, but consumers cannot perfectly share risk internationally.

This specification breaks down the international risk sharing relationship (1.10). Now, following a shock that reduces real income in the home country, home consumers must borrow in the international markets to maintain their consumption level; however, the increasing cost of borrowing will drive a wedge between home and foreign marginal utilities of consumption (and in our model, between the consumption levels) and thus potentially increase the costs of nominal inertia in small countries.

---

Footnote: Equilibrium dynamics of a small open economy with incomplete asset markets generally include a random walk component; the transaction cost modification guarantees stationary of the model. See Schmitt-Grohé and Uribe (2003) for explicit treatment of the problem.
More specifically, the representative home consumer’s budget constraint becomes

\[ E_t[\Delta_{t,t+1}D_{t+1}(h)] + B^d_{t+1}(h) + A_{t+1}(h) + P_t(1 + \tau_{c,t})C_t(h) + P_{t,I_t}(h) + P_{I_t}(h) = \]

\[ = (1 - \tau_{w,t})W_t(h)l_t^d(h) + D_t(h) + (1 + i_t)B^d_t(h) + [1 + i_{t-1} - p(e^{\alpha_t} - 1)] A_t(h) + \]

\[ + (1 - \tau_{k,t})R_tK_{t-1}(h) + \tau_{k,t}\delta P_{t,J_k}K_{t-1}(h) + \Pi_t(h) \]

An additional first-order condition of the home households with respect to the international bond holdings is

\[ \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \right] = \frac{1}{1 + i_t - p(e^{\alpha_{t+1}}) - 1} \]

Condition (1.12) now becomes

\[ \int_0^M D_{t+1}(h)dh = 0, \quad \int_0^{M^*} D_{t+1}^*(h)dh = 0 \]

and two additional market clearing conditions for the international bond are

\[ A_{t+1} = \int_0^M A_{t+1}(h)dh = -\int_0^{M^*} A_{t+1}^*(h)dh = -A_{t+1}^* \]

\[ A_{t+1} = [1 + i_{t-1} - p(e^{\alpha_t} - 1)] A_t + P_{H,t}^*M^tC_{H,t}^* + P_{G,t}^*G_{H,t}^* + P_{I,t}^*M^tI_{H,t}^* - \]

\[ -P_{F,t}^*MC_{F,t} - P_{G,t}^*G_{F,t} - P_{I,t}^*MI_{F,t} \]

1.2.6 National Welfare Measure

A natural (and commonly used) measure of the national welfare is the aggregate utility of all the domestic households:

\[ U_t = \int_0^M E_t \sum_{j=1}^{\infty} \beta^{j-t} \left\{ \frac{(C_j(h))^{1-\Theta}}{1-\Theta} - \frac{1}{1+\chi} L_j(h)^{1+\chi} \right\} dh, \quad (1.13) \]

Since there is complete risk sharing in the benchmark specification of the model, all households (and in particular those in the home country) consume the same amount of the aggregate good; therefore, the first term on the right-hand side of (1.13) simplifies to \((1 - \Theta)^{-1}MC_t^{1-\Theta}\). The
aggregation of hours (the second term on the right-hand side) is slightly more complicated, since
households supply different amount of labor due to differently set wage rates. The aggregate disutility
of work $AL_j \equiv \int_0^M L_j(h)^{1+\chi} dh$ can be written as

$$AL_j = \left( \frac{N_j}{M} \right)^{1+\chi} DW_j$$

$$DW_j = (1 - \omega) M \left( \frac{\bar{W}_j}{W_j} \right)^{-\phi(1+\chi)} + \omega \left( \frac{W_j}{W_j} \right)^{-\phi(1+\chi)} DW_{j-1}$$

where $N_j$ is given by (1.6). The term $DW_j$ measures the amount of wage dispersion in the economy,
and so captures the inefficiency in household labor allocations outside of the steady state.

We define the value function that measures national welfare as

$$V_t = \max E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \frac{MC_j^{1-\Theta}}{1 - \Theta} - \frac{1}{1 + \chi} AL_j \right\}$$

This value function will allow us to make quantifiable comparisons of consumer welfare across
different specifications of the two economies. To see how this can be done, let $\tilde{V}_t$ correspond to some
benchmark specification of the domestic economy (for example, with no wage or price rigidities,
so $\alpha = \omega = 0$) and let $V_t$ be the value function from a different specification (to follow the above
example, let $\alpha = \omega = 0.5$). Denote by $\tilde{C}_t$ and $C_t$ household consumption, and by $\tilde{AL}_t$ and $AL_t$
aggregate labor effort in these two economies, respectively. Finally, let $\epsilon$ be a number such that
consumers are indifferent between receiving consumption streams $\tilde{C}_j$ and $(1 + \epsilon)C_j$ for all $j \in (t; \infty)$:

$$\tilde{V}_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \frac{M\tilde{C}_j^{1-\Theta}}{1 - \Theta} - \frac{1}{1 + \chi} \tilde{AL}_j \right\} =$$

$$= E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \frac{M ((1 + \epsilon)C_j)^{1-\Theta}}{1 - \Theta} - \frac{1}{1 + \chi} AL_j \right\}$$

Notice that we’re equalizing the two value functions by adjusting only the stream of consumption,
holding labor efforts unchanged. Therefore, $\epsilon$ can be interpreted as the fraction of consumption each
household is willing to give up in order to move to the benchmark economy, keeping its labor supply
stream fixed.
In the case of log utility, the expression for $\epsilon$ simplifies to the difference between the two value functions:

$$\epsilon = V_t - \tilde{V}_t$$  \hspace{1cm} (1.14)

Below we will interpret $\epsilon$ as cost, expressed as percent of consumption, of moving away from the benchmark specification to (in the above example) the economy with nominal rigidities.

Notice that in general country size enters explicitly in the expression for the welfare function. Clearly, national welfare of a large country is greater than that of a small country, which makes it difficult to make comparisons between two asymmetric members of the currency union. However, for the purposes of quantifying the welfare effects of country size, we will keep the size of the home country constant at one ($M = 1$) while increasing the size of the foreign country ($M^* > 1$). If no other parameters of the model change, then (1.14) can be used to study the welfare of the home country as it becomes progressively smaller relative to the foreign country.

1.3 Calibration

We consider three different calibration schemes in this paper. As a benchmark case, we assume that both economies are symmetric with measure one of households in each country, since this will allow for easy comparisons of our results with the existing studies. In the other two specifications, we increase the size of the foreign country: $M^* = 4$ and $M^* = 19$, so that the home country makes up 20 and 5 percent of the union, respectively (corresponding to France and the Netherlands, for example).

1.3.1 Production and Preferences

Each time period in the model corresponds to one quarter. We follow Canzoneri, Cumby and Diba (2004) and Pappa (2004) in setting most of the parameters of the model; Table 2 summarizes their values. Many of these values are common in the literature on the European currency union; a few
other parameters merit further description.

Empirical estimates put the value of Frisch labor elasticity anywhere between 0.05 and 0.35. It has been noted by Canzoneri et al (2007), among others, that welfare costs are very sensitive to this parameter; therefore, we chose a conservative value \( \chi^{-1} = 0.33 \) as our benchmark; robustness checks (not reported) indicate that this parameter has no impact on the differences in welfare costs of large and small countries.

In the benchmark calibration of the model, we set the home bias parameter \( \mu = \mu^* = 0.75 \), which, in accordance with Table 1, is a lower bound on the share of imports in consumption in the EMU. We allow investors to have lower biases for the domestically produced good: \( \vartheta = \vartheta^* = 0.5 \); finally, since governments consume mostly domestic goods, we let \( \zeta = \zeta^* = 0.9 \).

As the relative size of the home country decreases, its share of imports still remains the same: \( (1 - \mu) = 0.25 \). However, foreign home bias \( \mu^* \) is adjusted upward, which means that imports constitute a smaller share of foreign consumption. For example, when \( M^* = 4 \), \( \mu^* = 0.94 \). In the absence of this adjustment, foreign country would demand a disproportionately large share of home country output, which would raise home employment and wages and make it difficult to compare home welfare across the three relative size specifications.

The cost of participating in the international asset markets, \( p \), is set to \( 10^{-3} \) following Benigno (2001); this value implies a spread of 10 basis points between home and foreign yields.

The productivity process was approximated as a quasi-Solow residual, holding capital stock constant (a reasonable assumption for the estimation of short-run correlations): 
\[
\ln(Z_t) \cong \ln(Y_t) - \frac{1}{\nu}(1 - \nu)\ln(L_t).
\]
We estimated several bivariate VARs of individual country productivities and the aggregate EMU productivity \( Z_t^* \) (calculated using the aggregate EMU real output and employment)\(^9\). Thus, the foreign productivity shock \( z_t \) in our model can be interpreted as the average technological

\(^9\)Estimations presented below were carried out using 1980-2004 quarterly data; see appendix A for data description and sources.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta$</td>
<td>Relative risk aversion</td>
<td>1.0</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Inverse of Frisch labor elasticity</td>
<td>3.0</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Consumption home bias (domestic good)</td>
<td>0.75</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of substitution between home and foreign goods</td>
<td>1.5</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Capital share in the production function</td>
<td>0.33</td>
</tr>
<tr>
<td>$A$</td>
<td>Matrix of technology coefficients</td>
<td>$\begin{pmatrix} 0.83 &amp; 0.39 \ 0.03 &amp; 0.72 \end{pmatrix}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Investment adjustment cost</td>
<td>8.0</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Elasticity of substitution between goods varieties</td>
<td>8.0</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Elasticity of substitution between labor varieties</td>
<td>7.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Price stickiness</td>
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</tr>
<tr>
<td>$\omega$</td>
<td>Wage stickiness</td>
<td>0.80</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>Consumption tax</td>
<td>0.17</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>Labor income tax</td>
<td>0.36</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>Tax on capital (net of depreciation)</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Table 2: Benchmark parameter values
improvement in the rest of the EMU vis-a-vis the home country. Variances of these shocks were estimated to lie in the following ranges: $\text{Var}(\varepsilon_z) \in [0.000021, 0.000128]$ with Portugal exhibiting the highest variance of productivity; $\text{Var}(\varepsilon_z^*) = 0.000015$ for the Euro aggregate; and $\text{Cov}(\varepsilon_z, \varepsilon_z^*) \in [0.000008, 0.000056]$. In the simulations, we set $\text{Var}(\varepsilon_z)$ to its lowest estimated value, so that our calculated costs of nominal inertia can be interpreted as the lower bounds for the true costs.

Finally, wage stickiness parameter $\omega$ has been set to 0.80 (which is slightly higher than the commonly used value of 0.75) to bring wage volatility in range of the data.

### 1.3.2 Government Policies

Fiscal policy was estimated using data from Finland, France, Ireland and the Netherlands; the choice of countries is based on the availability of fiscal data.

The ratio of government debt to GDP has to be stationary in the model; fiscal policy, therefore, must respond in some way to either deficit or debt. The reactions of government spending and (separately) transfers to the debt-to-GDP ratio are estimated to be significant in all four countries; responses of both variables to the deficit ratio are much smaller and not statistically significant in two of the four cases. (Estimation results are presented in Appendix B.) Therefore, in the model we let transfers respond to the deviation of debt ratio from its steady-state value. The latter is set at 60 percent of GDP in both economies, which corresponds to the Maastricht debt criterion and is roughly representative of the above four countries during the 1999-2004 period. Our model does not capture all sources of revenues and outlays available to the European governments, and so cannot match all features of the observed fiscal policies. To achieve the 60 percent debt-to-GDP ratio, we set transfers equal to 10 percent of GDP (compared with the EMU average of 17 percent). Of all the fiscal instruments, transfers play the least distortionary role in our model (they act as a lump-sum tax on consumers), and so adjusting their value has minimum impact on the rest of the model. As
a first pass, all taxes are assumed to be constant\footnote{Source: Carey and Rabesona (2002). Tax rates in Table 2 represent the average of the corresponding values in the eleven EMU countries (Luxembourg not reported) during 1990-2000.}.

The parameters of the government policy functions are estimated to be as follows: \(\rho_g = \rho_{tr} = 0.9\) and \(\text{Var}(\varepsilon_g) = 0.0082\). Responsiveness of transfers to the level of debt \(\rho_b\) was set to 0.18 in order to satisfy the Blanchard-Kahn conditions.

The estimates of the Taylor rule parameters (1.11) are \(\rho_i = 0.95\), \(\varpi = 1.4\) and \(\text{Var}(\varepsilon_i) = 10^{-6}\) using the data from ten EMU countries. Our findings are consistent with the existing literature: Duarte and Wolman (2002) use \(\rho_i = 0.91\) and \(\varpi = 1.31\). Coenen and Straub (2005) report \(\rho_i\) to be as high as 0.96 using Bayesian estimation methods.

### 1.3.3 Demand Shocks

It has been noted (see, for example, Canzoneri et al (2006)) that the standard NNS models are unable to match the observed positive correlation between output and inflation because they do not properly capture aggregate demand shocks. For this reason, we introduce a disturbance to consumer preferences when we study the incomplete asset market setting\footnote{With complete asset markets, preference shocks cause wealth transfers between the two countries and so have no impact on relative consumption streams.}. We assume that a positive shock increases demand in both countries for Home good \((Y_H)\) by influencing the home bias coefficients:

\[
\mu_t = \mu \exp(\eta_t), \quad \mu^*_t = \frac{\mu^*}{\exp(\eta_t)}
\]

\[
\eta_t = 0.9\eta_{t-1} + \varepsilon_{t,1}
\]

The same adjustment is made to the investment and government spending home biases. Here \(E(\eta) = 0\) and \(\text{Var}(\varepsilon_\eta) = 0.01\). Since this shock cannot be measured empirically, its volatility was chosen to match the standard deviation of output in the model and in (the mid-range of) the data. As a matter of robustness check, we also report simulation results in the absence of this disturbance.
The model was solved numerically using Dynare (see Collard and Juillard (2003)). First order approximations were used to compute moments, variance decompositions and impulse response functions presented below; value functions were calculated using second-order approximations to the model.

1.4 Simulation Results

In this section we discuss the computation of welfare costs predicted by our model for different country sizes, as well as the sensitivity of these predictions to several key parameters that govern the propagation of shocks between the two countries. We calculate the responses of prices to each shock and elucidate the cross-country linkages that help small member states to mitigate the effects of economic disturbances originating in bigger countries.

1.4.1 Matching the Data

All specifications of the model result in the following decomposition of steady state GDP (in both countries): consumption share of 63%, investment share of 17% and government spending share of 20%; the corresponding EMU averages for the period 2002-2006 are 57%, 21% and 20% (source: Eurostat).

Table 3 shows that the model approximates the volatility of the actual data\textsuperscript{12} reasonably well, except in the case of employment, which is almost twice as volatile in the model as it is in the data. Adjusting the value of the Frisch labor elasticity (between 0.05 and 0.35, the empirically estimated range) produces virtually no effect on the standard deviation of employment.

On the correlations front, the most noticeable inconsistencies arise with respect to inflation and interest rate. As mentioned above, the sign of output-inflation and output-interest rate correlations

\textsuperscript{12}The ranges presented in the table refer to the dispersion of corresponding moments across Finland, France, Ireland and Netherlands (countries for which all relevant data are available).
<table>
<thead>
<tr>
<th></th>
<th>Standard Deviations</th>
<th>Correlation w/Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual Data</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$Y_H$</td>
<td>0.010-0.028</td>
<td>0.022</td>
</tr>
<tr>
<td>$C_H$</td>
<td>0.014-0.029</td>
<td>0.017</td>
</tr>
<tr>
<td>$I_H$</td>
<td>0.035-0.078</td>
<td>0.064</td>
</tr>
<tr>
<td>$N_H$</td>
<td>0.006-0.019</td>
<td>0.032</td>
</tr>
<tr>
<td>$W_H$</td>
<td>0.006-0.016</td>
<td>0.006</td>
</tr>
<tr>
<td>$\pi_H$</td>
<td>0.003-0.006</td>
<td>0.004</td>
</tr>
<tr>
<td>$i$</td>
<td>0.002-0.005</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 3: Kappa is the relative size of the home economy. Moments of the actual and simulated data under benchmark calibration (complete markets, no preference shocks, ECB-calibrated policy rule).

has been a rather controversial issue in New-Keynesian models. Both correlations are positive in the data; our model, however, is incapable of correctly matching the output-interest rate correlation, and overshoots on the size of output-inflation correlation.

A standard one-sector New-Keynesian model predicts that a positive supply shock creates an increase in output and a drop in price level; interest rates then move down in response to lower inflation (countercyclical movement). Demand shocks, on the other hand, put an upward pressure on output, prices and interest rates and create procyclical movements in these variables. Finally, a monetary policy shock causes a reduction in output and inflation; it is another source of countercyclical movement of the interest rate. The sign and magnitude of the resulting correlations between output, inflation and the interest rate, therefore, depends on the relative importance of the three types of shocks.

Historically, New-Keynesian models have been unable to properly capture demand-side shocks;

---

13See Canzoneri et al (2006) for a detailed study of the signs of these correlations in New-Keynesian models.
it is also true of our model. Variance decompositions, presented in the next subsection, show that government (demand) shocks have a relatively insignificant impact on the key variables of the model; the latter are driven primarily by monetary policy disturbances and (to a lesser extent) by technological shocks.

Monetary policy is given additional importance by the omission of the output gap term; this form of Taylor rule creates a strong negative correlation between interest rate and output. We will address this issue again below when we present the variance decompositions of several key variables following the shocks modeled in our paper.

1.4.2 Welfare Implications

Having described the specifications and workings of the model, the first question to be answered is: what are the resulting costs of nominal (wage and price) inertia, and do they depend on the country size?

Canzoneri et al (2004) have studied the interactions of twelve separate fiscal policies and the common monetary rule in the EMU. They found that large EMU member states are better off than smaller ones because inflation rates in the former are more highly correlated with the aggregate Euro area inflation to which the ECB reacts. Since prices, wages and fiscal policy cannot move instantly to counteract country-specific shocks, the EMU members rely on the central bank policy to offset their high-frequency economic disturbances. However, a German inflation shock would elicit a stronger interest rate response than a similar shock in Belgium, resulting in relatively high welfare costs of nominal rigidities in the latter country. The authors calculate that the welfare costs of business cycles can be up to four times higher in an "average-size" EMU country than in a "large" one.

There are two reasons why the above findings may exaggerate welfare losses of the small EMU countries. First, the authors present a partial-equilibrium model, which potentially ignores cross-country links that can offset idiosyncratic shocks even before the ECB steps in. Indeed, as will be
shown below, there exists a strong positive correlation between home and foreign inflations that causes the central bank to respond to home country shocks even when the country is very small. Second, if technological innovations are positively correlated across member states (as we found to be the case in the OECD data), then, as in the previous example, German shock would spill over into Belgium almost instantaneously, and the resulting central bank response would be helpful to both countries.

Duarte and Wolman (2002) consider a two-country DSGE model in order to explain the observed inflation differentials in the EMU and their possible dependence on country size. The authors find that smaller countries experience higher volatility of inflation following a shock to productivity. The authors do not calculate the resulting welfare costs of nominal rigidities, but at least within our framework, their results would imply bigger losses for small EMU member states: the same arguments used in the preceding paragraphs would suggest that the response of the ECB would be skewed towards larger countries.

In our paper, inflation volatility is uncorrelated with country size (this result is discussed further below). The reason is that our model lacks two potential sources of inflation variability which are present in Duarte and Wolman (2002): non-traded goods sectors (which allow for the Balassa-Samuelson effect) and segmented markets for traded goods (with the potential failure of the Law of One Price). We dispense with these features for two reasons: first, Altissimo et al. (2005) note that the standard two country models cannot properly capture the Balassa-Samuelson effect because they don’t possess many necessary features (such as international capital mobility and homogeneous traded goods markets). The authors also note that the magnitude of the terms-of-trade effect usually overshadows the Balassa-Samuelson effect. Secondly, Duarte and Wolman find that their model does not generate significant deviations from the LoOP and that it overestimates the volatility of inflation gaps across countries.

We now turn to our main results. Table 4 presents the costs of wage and price rigidities for
the three specifications of the relative country size. Our calculations show that when asset markets are complete, welfare costs are independent of country size\textsuperscript{14}. Variance decompositions for the benchmark calibration (reported in the same table), shed some light on this rather surprising result.

Our measure of welfare (1.13) is computed using the expected streams of consumption and work hours. As was mentioned above, complete international risk sharing equates marginal utilities (and, in our model, levels) of per-capita consumption across the two countries at all times, and so serves as a buffer against idiosyncratic shocks. Additionally, as can be seen from Table 4, volatilities of consumption and hours are influenced mostly by the interest rate, which, being common to both countries, elicits identical responses of consumption and hours at home and abroad (see Figure 1).

Consequently, the monetary policy shock does not favor the larger country and does not create a dispersion in welfare costs.

\textsuperscript{14}Our calculations of welfare costs are well above those reported in Lucas (2003); however, they are within the range of costs presented in recent studies. Canzoneri et al (2004) have estimated the welfare costs of nominal rigidities in the U.S. to be between one and three percent of consumption.

Table 4: Variance decomposition for the benchmark calibration with nominal rigidities, infinite horizon (percent). Percentages in the table may not add up to 100 due to rounding errors.
The interest rate shock plays the dominant role in our model; this result contrasts with the findings of the above two papers, in which productivity shocks are the main drivers of output volatility. This is partly due to the fact that our estimations show less persistence in the EMU productivity processes: the AR(1) coefficients lie between 0.7 and 0.8, compared with over 0.9 in Canzoneri et al (2004).

1.4.3 The Feedback Channel

The case of technological shocks is a more interesting one because it highlights the feedback channel between the two economies that magnifies the interest rate response to the small country’s inflation. In what follows, we will consider the middle-of-the-road "20-80" case, since it is applicable to the
largest EMU countries (Germany, France and Italy); however, the same arguments hold for the other two specifications.

At first glance, the sequence of events should be as follows: when the (small) home country is hit with a positive productivity shock, the price of its product declines, causing domestic deflation. The consequent wealth effect increases demand for home and foreign goods in both countries. As a result, the foreign producers effectively experience an increase in demand and raise prices in response. The weighted area-wide deflation will be smaller than in the home country; if the latter is very small, the area-wide consumer price index may even increase, prompting an upended (from the perspective of the home country) response from the central bank. Thus, it would seem that small countries would not get any help from the ECB following a domestic productivity disturbance.

Upon more careful examination, however, it turns out that the foreign country also experiences deflation, and consequently the interest rate unambiguously declines (Figure 2). To see this, it is helpful to consider what happens to the two PPI inflations (equation (1.2)). The Philips curve for the home country (derived by log-linearizing equations (1.3) and (1.4)) takes the form

\[
\pi_{H,t} = \frac{(1 - \alpha \beta)(1 - \alpha)}{\alpha} \frac{MC_t}{P_{H,t}} + \beta E_t \pi_{H,t+1},
\]

where \(\pi_{H,t}\) refers to the home country’s PPI inflation at date \(t\); an analogous equation holds for the foreign country. In other words, today’s PPI inflation depends on the discounted expectations of future real marginal costs. Because of the technological spillovers between the two countries, a positive productivity shock in the home country almost immediately translates into higher productivity abroad, which reduces the foreign marginal cost. Moreover, price and wage stickiness both slow down the rise in foreign factor prices, which would normally follow an increase in demand for the foreign good. Together, these forces make the present value of foreign future marginal costs negative, resulting in the foreign PPI deflation (Figure 3). Consequently, both CPIs, which are weighted averages of the two PPIs, also fall and therefore cause the central bank to lower the interest rate.

The same argument holds in the other direction: productivity shocks originating abroad very
quickly affect home technology, causing the two CPI inflations to move together and consequently prompting an appropriate (for both countries) response from the central bank.

However, a word of caution is in order. Our model overstates the importance of technology in smoothing inflation differentials. Canzoneri et al. (2006) estimate the correlation between EMU-wide and individual countries’ inflations:

\[ \pi_k^{\text{eur ex}} = \theta_i \pi_{i,t} + \varepsilon_{i,t}. \]

The authors find that this correlation is much larger for countries like France and Germany (\( \theta_i = 0.7 \)) than for small EMU members like Ireland and Portugal (\( \theta_i = 0.2 \)). In contrast to this, calculations based on our model simulations indicate that \( \theta_i \) does not fall below 0.9 even when the country is
only 5 percent of the size of the union. Insofar as our estimates of correlations in technological processes differ from the above study, we should in fact find higher correlations between national and euro-wide inflations. However, if our model is failing to capture some country-specific shocks that are present in the data, our estimates of the differences in welfare costs should be interpreted as the lower bounds for their true values.

Productivity typically plays a dominant role in NNS models; as mentioned above, attempts to beef up the demand shocks haven’t been entirely satisfactory. It is then entirely plausible that the standard New-Keynesian models are incapable of properly capturing differences in welfare costs of different countries. Further research in this direction is needed to shed more light on the influence
of relative country size on welfare costs in a monetary union.

1.4.4 Incomplete Asset Markets and Robustness

We now turn to a more realistic scenario of incomplete international asset markets. The moments and variance decompositions of all variables are identical to the ones reported above.

Column A of Table 5 shows that this specification does not alter our previous findings - welfare costs of nominal rigidities are the same for both countries regardless of their relative size. This is not surprising, since the two most important shocks in our model (to monetary policy and technology) are highly correlated across the two countries, and therefore do not require a high level of international risk sharing.

Following the addition of the demand shock, which is illustrated in Figure 4, smaller countries do incur higher welfare costs, but the difference in these costs across the three sets of country sizes is negligible at 0.05 percent of consumption (column B of Table 5). The resulting increase in home output gradually disappears over the period of 20 quarters. An increase in home inflation more than offsets a (ten times smaller) decline in the foreign price level, prompting the central bank to increase the interest rate. Even though the monetary authority does respond to the disturbance in the home economy, the additional fluctuations in consumption and hours caused by demand shocks (and reported in Table 6) coupled with non-zero costs of lending and borrowing in international markets add almost 0.2 percentage points to welfare costs of home residents.

These results are in line with the findings of Benigno (2001): he reports that incomplete markets cost between 0.07 and 0.7 percent of consumption. In our model, the costs of going from complete to incomplete markets do not exceed 0.2 percent of consumption for large or small countries.

We next turn to the impact of trade openness on the differences in the costs of welfare between the two monetary union members. As Table 1 shows, the shares of exports and imports vary significantly across European countries. In the fifth column of Table 5, we increase the share of
Figure 4: Impulse response functions of several key variables following an increase in international demand for the home good in the "20-80" incomplete markets scenario.

Imports in the home country to 40 percent while maintaining the incomplete markets and demand shocks assumptions. The welfare costs decrease slightly for all country sizes and become more similar: if a country is relatively more open to trade, its CPI inflation is more correlated with foreign inflation. A reduction in inflation differentials makes the job of the common monetary policy easier and increases the degree of implicit international risk-sharing.

Finally, we perform a sensitivity exercise to calculate consumption costs of nominal rigidities under a different specification of the ECB Taylor rule, which includes the output gap term (measured as the deviation of output from its steady state value):

\[ i_t = (1 - 0.95) \log(\beta) + 0.95 \kappa_{t-1} + 1.4 \times 0.05 \pi_t + 1.4 \times 0.05 (y_t - y^{ss}) \]
## Table 5: Welfare costs of nominal inertia, percent of consumption.

In column (A), international asset markets are incomplete; column (B) adds demand shocks. In the next column, home economy is more open, with imports share of 40 percent. The last column presents welfare calculations when the monetary policy also responds to output gap.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>A</th>
<th>B</th>
<th>$\mu = 0.6$</th>
<th>$Y_{gap}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa = 0.5$</td>
<td>1.81</td>
<td>1.81</td>
<td>1.97</td>
<td>1.94</td>
<td>0.96</td>
</tr>
<tr>
<td>$\kappa = 0.2$</td>
<td>1.80</td>
<td>1.81</td>
<td>2.00</td>
<td>1.95</td>
<td>0.99</td>
</tr>
<tr>
<td>$\kappa = 0.05$</td>
<td>1.80</td>
<td>1.81</td>
<td>2.02</td>
<td>1.96</td>
<td>1.01</td>
</tr>
</tbody>
</table>

## Table 6: Variance decomposition for the "20-80" scenario when asset markets are incomplete and demand shocks are present, infinite horizon (percent).

Percentages in the table may not add up to 100 due to rounding errors.

<table>
<thead>
<tr>
<th></th>
<th>&quot;20-80&quot;</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_z + \varepsilon_i^*$</td>
<td>$\varepsilon_i$</td>
<td>$\varepsilon_\theta$</td>
<td>$\varepsilon_\theta + \varepsilon_i^*$</td>
</tr>
<tr>
<td>$C_H$</td>
<td>10.4</td>
<td>87.7</td>
<td>1.7</td>
<td>0.2</td>
</tr>
<tr>
<td>$N_H$</td>
<td>1.1</td>
<td>79.3</td>
<td>19.1</td>
<td>0.5</td>
</tr>
<tr>
<td>$W_H$</td>
<td>48.2</td>
<td>46.7</td>
<td>4.9</td>
<td>0.2</td>
</tr>
<tr>
<td>$\pi_H$</td>
<td>17.6</td>
<td>74.2</td>
<td>7.9</td>
<td>0.3</td>
</tr>
<tr>
<td>$i$</td>
<td>2.9</td>
<td>97.0</td>
<td>0.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>
We continue assuming that asset markets are incomplete, and there are country-specific demand shocks. We would like to note that it is not the goal of this paper to address the issue of optimal monetary policy in a currency union; therefore, the results of this section should be viewed as a robustness exercise.

The last column in Table 5 reports the estimated welfare costs under this new specification of the monetary policy, which are similar across country sizes, but are significantly lower than in other robustness exercises. In our model, interest rate responds to current as opposed to expected inflation, and so output gap term adds new information about future price level. Monetary policy is more effective in stabilizing inflation and output fluctuations, which lowers costs of nominal stickiness.

We are not aiming at estimating welfare costs of price and wage inertia per se; clearly, the numbers are very sensitive not only to policy rule and asset market specifications, but also to the values of parameters used in the model. The robustness checks, however, show that the our main finding - that difference in welfare costs across small and large EMU countries are negligible - are independent of the modeling assumptions.

1.5 Conclusion

In this paper, we have employed a two-country general equilibrium model of a currency union to answer the question of how (if at all) country size affects welfare costs of nominal rigidities in the EMU member states. The main result - that there are virtually no differences in these costs between small and large countries - is primarily driven by two features of the model.

First, idiosyncratic productivity shocks, which have the potential to drive a wedge between welfare measures of large and small countries, are mitigated by cross-country productivity links. EMU members have experienced an increase in the degree of alignment of their business cycles, as is reported in Lane (2006) and is seen in our estimates of productivity processes. Insofar as productivity shocks are among the main causes of business cycles, our model is able to explain
why welfare costs of nominal rigidities are so close across countries of different sizes: technology
spillovers bring national output and price movements closer in sync and so help to make the interest
rate response appropriate for both member countries.

Secondly, our model indicates that monetary policy shocks (which are symmetric across all union
countries) account for most of the variability of output, consumption and hours and so affect welfare
measures of all countries equally.

Some caution needs to be taken when interpreting the results of our study. As is common in New-
Keynesian literature, our model does not properly capture the observed cyclical correlation between
output and interest rate, and overestimates the output-inflation correlation due to the dominance
of interest rate movements. If asymmetric demand shocks play a more prominent role in influencing
the variability of inflation, interest rates and output than our model suggests, then the common
monetary policy may be less effective in smoothing country-specific business cycles and welfare costs
of nominal rigidities may be higher than the ones reported in this paper.
References


1.A Data Sources and Description

All data are taken from OECD Economic Outlook No. 77.

- $P_t$: Harmonized consumer price index
- $Y_t$: GDP at market prices, deflated by the HICP
- $C_t$: Private consumption, volume, deflated by the HICP
- $I_t$: Private fixed investment (excluding stockbuilding), volume, deflated by the HICP
- $G_t$: Government consumption (including wages), deflated by the HICP
- $T_t$: Government transfers (the sum of Subsidies, Social benefits paid by Government, and Other current transfers paid by government), deflated by the HICP
- $B_t$: Gross government debt, % of GDP
- $N_t$: Average hours per employee times the Total employment
- $W_t$: Real total compensation per employee
- $\pi_t$: Inflation rate, calculated as $\log\left(\frac{P_t}{P_{t-1}}\right)$
- $i_t$: Short-term interest rate, approximated as $\log(1 + i_t)$

Deficit: Government net lending, % of GDP
1.B Estimation

1.B.1 Productivity

We assume that in the short run, nation-wide capital stock is fixed; this allows us to approximate total factor productivity (TFP) as $\ln Z_t \cong \ln Y_t - (1 - \alpha) \ln L_t$. (Details on the variable definitions can be found in Appendix A.) We use 1980:1-2004:4 data on real output and employment to compute the TFP series for individual EMU members (where data are available) and for the monetary union as a whole. We then estimate VARs of the form

$$
\begin{bmatrix}
  z_t \\
  z_t^*
\end{bmatrix} =
\begin{bmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
  z_{t-1} \\
  z_{t-1}^*
\end{bmatrix} +
\begin{bmatrix}
  \varepsilon_{z,t} \\
  \varepsilon_{z,*t}
\end{bmatrix},
$$

where $z_t$ is H-P filtered natural log of TFP for an individual country, and $z_t^*$ is the analogous value for the EMU area. Table 7 presents our estimation results.

Clearly, there is a lot of heterogeneity in our estimates; for the simulations, we pick Portugal as the middle-of-the-road result (that is also statistically significant).

1.B.2 Fiscal policy

We compute the series of H-P filtered natural logs of real government purchases and transfers and estimate two autoregressions of the form

$$
\begin{align*}
g_t &= \alpha_0 + \alpha_1 g_{t-1} + \varepsilon_{g,t} \\
tr_t &= \beta_0 + \beta_1 tr_{t-1} + \varepsilon_{tr,t}
\end{align*}
$$

Using data for 1980:1-2004:4, we get the results reported in Table 8:

In our simulations, we use Ireland’s values, since they fall in the mid-range of the estimates. We choose the value of $\alpha_0$ so that the steady state share of government purchases $\frac{G}{Y}$ is equal to 20 percent; $\beta_0$ is set to achieve 60 percent debt-to-GDP ratio in the steady state.
<table>
<thead>
<tr>
<th>Country</th>
<th>$A_{11}$</th>
<th>$A_{12}$</th>
<th>$A_{21}$</th>
<th>$A_{22}$</th>
<th>$\sigma^2(z_t)$</th>
<th>$\sigma^2(z_t^*)$</th>
<th>Cov($z_t, z_t^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>0.69</td>
<td>0.02</td>
<td>0.11</td>
<td>0.64</td>
<td>4.8x10$^{-5}$</td>
<td>1.5x10$^{-5}$</td>
<td>1.6x10$^{-5}$</td>
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<tr>
<td></td>
<td>(6.62)</td>
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<td>(1.93)</td>
<td>(7.21)</td>
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<tr>
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<td>0.16</td>
<td>0.05</td>
<td>0.66</td>
<td>1.1x10$^{-4}$</td>
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<td>1.4x10$^{-5}$</td>
</tr>
<tr>
<td></td>
<td>(12.00)</td>
<td>(0.74)</td>
<td>(2.08)</td>
<td>(8.39)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
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<td>0.49</td>
<td>-0.22</td>
<td>0.99</td>
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<td>1.5x10$^{-5}$</td>
</tr>
<tr>
<td></td>
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<td>(-1.72)</td>
<td>(7.12)</td>
<td></td>
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<tr>
<td>Ireland</td>
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<td>0.76</td>
<td>1.9x10$^{-4}$</td>
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<td>1.6x10$^{-5}$</td>
</tr>
<tr>
<td></td>
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<td>(0.56)</td>
<td>(11.18)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>0.36</td>
<td>0.47</td>
<td>-0.05</td>
<td>0.83</td>
<td>5.2x10$^{-5}$</td>
<td>1.6x10$^{-5}$</td>
<td>2.1x10$^{-5}$</td>
</tr>
<tr>
<td></td>
<td>(2.84)</td>
<td>(2.63)</td>
<td>(-0.66)</td>
<td>(8.46)</td>
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<tr>
<td>Luxem.</td>
<td>0.89</td>
<td>0.01</td>
<td>0.03</td>
<td>0.73</td>
<td>7.0x10$^{-5}$</td>
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<td>1.2x10$^{-5}$</td>
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<tr>
<td></td>
<td>(16.28)</td>
<td>(0.01)</td>
<td>(1.08)</td>
<td>(10.33)</td>
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</tr>
<tr>
<td>Nether.</td>
<td>0.51</td>
<td>0.14</td>
<td>0.02</td>
<td>0.76</td>
<td>6.0x10$^{-5}$</td>
<td>1.6x10$^{-5}$</td>
<td>1.1x10$^{-5}$</td>
</tr>
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<td></td>
<td>(5.45)</td>
<td>(1.07)</td>
<td>(0.39)</td>
<td>(11.09)</td>
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<tr>
<td>Portugal</td>
<td>0.83</td>
<td>0.39</td>
<td>0.03</td>
<td>0.72</td>
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<td>1.5x10$^{-5}$</td>
<td>5.6x10$^{-5}$</td>
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<tr>
<td></td>
<td>(16.00)</td>
<td>(2.06)</td>
<td>(1.78)</td>
<td>(11.07)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.37</td>
<td>0.07</td>
<td>0.00</td>
<td>0.77</td>
<td>4.4x10$^{-5}$</td>
<td>1.6x10$^{-5}$</td>
<td>0.8x10$^{-5}$</td>
</tr>
<tr>
<td></td>
<td>(3.77)</td>
<td>(0.68)</td>
<td>(0.03)</td>
<td>(12.07)</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Table 7: Estimates of the productivity processes (t-stats in parentheses).
<table>
<thead>
<tr>
<th>Country</th>
<th>$\alpha_1$</th>
<th>$\beta_1$</th>
<th>$\text{Var}(g_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finland</td>
<td>0.80</td>
<td>0.92</td>
<td>$1.7 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>(12.92)</td>
<td>(26.06)</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>0.87</td>
<td>0.85</td>
<td>$2.0 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>(17.10)</td>
<td>(15.93)</td>
<td></td>
</tr>
<tr>
<td>Ireland</td>
<td>0.92</td>
<td>0.87</td>
<td>$8.9 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>(21.87)</td>
<td>(17.71)</td>
<td></td>
</tr>
<tr>
<td>Nether.</td>
<td>0.69</td>
<td>0.94</td>
<td>$9.7 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>(9.02)</td>
<td>(27.30)</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Estimates of the government policy processes (t-stats in parentheses).

1.B.3 Interest rate rule

We assume that the monetary policy of the European Central Bank can be described by the Taylor rule of the form

$$i_t = \alpha_0 + \alpha_1 i_{t-1} + (1 - \alpha_1) \alpha_2 \pi_t + \varepsilon_{i,t}$$

We estimated the coefficients over two different periods: 1991:3-2004:4 (during the economic convergence process) and 1999:4-2004:4 (after the introduction of the Euro). The results of least squares estimations are presented in Table 9:

In the simulations, we use the average value for the weight on inflation: $\alpha_2 = 1.4$. 

43
Table 9: Estimates of the Taylor Rule (t-stats in parentheses).

<table>
<thead>
<tr>
<th>Period</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\sigma_\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991:3-2004:4</td>
<td>-0.00</td>
<td>0.95</td>
<td>1.27</td>
<td>1.06$x10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>(-0.27)</td>
<td>(32.01)</td>
<td>(1.04)</td>
<td></td>
</tr>
<tr>
<td>1999:1-2004:4</td>
<td>-0.00</td>
<td>0.94</td>
<td>1.73</td>
<td>1.01$x10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>(-0.28)</td>
<td>(11.41)</td>
<td>(1.4)</td>
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</tr>
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2 New Keynesian Explanations of Cyclical Movements in Aggregate Inflation and Regional Inflation Differentials

2.1 Introduction

Traditional Keynesian explanations of the cyclical movements of inflation focused primarily on demand shocks. An increase in say government spending was thought to have multiplier effects on consumption and output, and the increase in aggregate demand would eventually create inflation (via the Phillips Curve) and an increase in the interest rate (via the central bank's monetary policy rule). U.S. data appear to be consistent with this view: recent VAR studies suggest that consumption rises in response to a government spending shock, and that the Federal Funds Rate rises in response to the increase in output and inflation\footnote{Fatas and Mihov (2001a,b), Blanchard and Perotti (2002) and Canzoneri, Cumby and Diba (2002) find that an increase in government spending increases consumption and output; Canzoneri, Cumby and Diba (2002) also find that the Federal Funds rate reacts in a manner that is consistent with standard Henderson-McKibbin-Taylor rules. On the other hand, Perotti (2004) suggests that the effect on consumption may have diminished in recent years, and he questions whether it exists at all in some European countries.}; moreover, the unconditional correlation between inflation and output is positive (0.33), and so is the correlation between nominal interest rates and output (0.35)\footnote{For HP-filtered quarterly U.S. data from 1960:1 to 2003:2, the correlation between CPI inflation and the log of GDP is 0.33; the correlation between the Federal Funds Rate and the log of GDP is 0.35. These correlations do not}. Recent data from the Euro Area also appear to be consistent with the traditional Keynesian explanations of the cyclical movements of inflation focused primarily on demand shocks. An increase in say government spending was thought to have multiplier effects on consumption and output, and the increase in aggregate demand would eventually create inflation (via the Phillips Curve) and an increase in the interest rate (via the central bank’s monetary policy rule). U.S. data appear to be consistent with this view: recent VAR studies suggest that consumption rises in response to a government spending shock, and that the Federal Funds Rate rises in response to the increase in output and inflation\footnote{Fatas and Mihov (2001a,b), Blanchard and Perotti (2002) and Canzoneri, Cumby and Diba (2002) find that an increase in government spending increases consumption and output; Canzoneri, Cumby and Diba (2002) also find that the Federal Funds rate reacts in a manner that is consistent with standard Henderson-McKibbin-Taylor rules. On the other hand, Perotti (2004) suggests that the effect on consumption may have diminished in recent years, and he questions whether it exists at all in some European countries.}; moreover, the unconditional correlation between inflation and output is positive (0.33), and so is the correlation between nominal interest rates and output (0.35)}.
sian view: national inflation differentials are positively correlated with national growth differentials (a fact we will document below).

The Real Business Cycle (RBC) model that followed focused primarily on productivity shocks. In the RBC view, productivity shocks drive fluctuations in output, while the cyclical behavior of interest rates and inflation is simply the manifestation of a monetary policy that is otherwise irrelevant. More recently, a New Neoclassical Synthesis (NNS) adds monopolistic competition and nominal inertia to the RBC model to create a new Keynesian model in which both productivity shocks and demand shocks play a role in the cyclical movements of interest rates and inflation\(^ \text{17} \). In NNS models, demand shocks tend to produce procyclical movements in interest rates and inflation, while productivity shocks tend to produce countercyclical movements.

In this paper, we analyze standard NNS models to see if they are capable of generating the procyclical movements of interest rates and inflation that are observed in the data. We begin with a model developed in Canzoneri, Cumby and Diba (CC&D) (2007). The CC&D model captures some key features of the U.S. business cycle, but as we shall see it generates strongly negative correlations between interest rates and output, and between inflation and output. We attribute this model failure to the fact that – despite the presence of shocks to government purchases and the interest rate rule – productivity shocks play a dominant role in the determination of inflation: variance decompositions indicate that productivity shocks explain 95\% of the fluctuations in inflation in the change sign for leads or lags of one quarter. See the appendix of CC&D for data sources.

\(^ {17} \)Goodfriend and King (1997) outlined the New Neoclassical Synthesis, and gave it the name. Woodford (2003) provides a masterful introduction to this class of models. NNS models are now being used widely in the academia and at policy making institutions. Important early contributions to the study of monetary policy include Rotemberg and Woodford (1997), King and Wolman (1999), and Erceg, Henderson and Levin (2000). Recent extensions to include fiscal policy include Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004). Larger institutional models include the Bank of England’s BEQM (see Bank of England (2004)), the IMF’s GEM (see Bayoumi et al (2004)), and the FRB’s SIGMA (see Erceg et al (2004)); similar models are being developed at the ECB and a number of other central banks.
CC&D model. We suspect that some demand side shocks are either absent or incorrectly modeled, and we investigate both possibilities in this paper.

We begin by augmenting the CC&D model with a private spending shock that has been suggested by Ireland (2004), Gali and Rabanal (2004) and others. Private spending shocks – like government spending shocks – produce procyclical movements in interest rates and inflation, and this increases the unconditional correlations of interest rates and inflation with output. However, private spending shocks are modeled as shocks to preferences, and – unlike government spending shocks – they are not directly observable: it is unclear how large we can plausibly make them. In the CC&D model, for standard deviations consistent with the existing literature, the unconditional correlations of interest rates and inflation with output remain negative.

For this reason, we go on to investigate the possibility that the propagation of fiscal shocks is incorrectly modeled in the typical NNS model. The CC&D model is Ricardian in the sense that households respond to an increase in government spending (and the implied increased tax burden) by working more and spending less, in apparent contradiction to the VAR studies cited earlier. This raises the possibility that a government spending shock has less effect on aggregate demand in the model than it does in the U.S. economy. Galí, López-Salido and Vallés (2004) have shown that adding “rule of thumbers” – households that just consume their income each period – can make aggregate consumption rise in response to an increase in government spending\(^\dagger\). Here, we add rule of thumbers to the CC&D model to see if we can generate the procyclical movements in inflation and interest rates that are observed in the data.

The CC&D model describes a single country with a single aggregate production sector. In this paper, we extend the CC&D model to a two country currency union, and we investigate its explanation of the cyclical behavior of the national inflation differentials.

\(^\dagger\) Some of the larger institutional models employ a similar device.

The early experience of the Euro has generated interest in explanations of national inflation...
differentials. Differences between national inflation rates and the Euro area average are proving to be larger than many had anticipated\(^\text{19}\). Figure 5 illustrates the average inflation differentials since the Euro’s inception\(^\text{20}\); they range from a high of 1.8% p.a. in Ireland to a low of -0.6% p.a. in Germany. These inflation differentials are also quite volatile; for example, the standard deviation of the inflation differential between France and Germany is 1.6% p.a.

\[\text{Figure 5: Inflation and growth differentials 1999-2004, correlation = 0.69}\]

We are not aware of any rigorous analysis of the welfare consequences of these inflation differentials. ECB (2003) documents these inflation differentials. Altissimo, Benigno and Rodriguez Palenzuela (2004) provide an interesting statistical decomposition of the inflation differentials. See also Duarte (2003) and Angeloni and Ehrmann (2004).


\(^\text{20}\)Quarterly inflation differentials for country \(J\) are computed as \(4 \log \left( \frac{P_{J,t}}{P_{J,t-1}} \right) - \log \left( \frac{P_{E,t}}{P_{E,t-1}} \right)\), where \(P_{J,t}\) is the average over the three months of quarter \(t\) of the HIPC for country \(J\) and \(P_{E,t}\) is similarly defined for the Euro Area. Real growth differentials are computed similarly by taking annualized averages of quarterly growth rates of real GDP and subtracting the annualized average quarterly growth rate for the Euro Area. The source for both the HIPC and real GDP data is Eurostat.
entials, but the way in which they are being viewed seems to depend upon what is thought to be generating them. When the differentials are thought to be driven by unstable fiscal policies, then the presumption seems to be that the Stability and Growth Pact may be useful in controlling them. When the differentials are thought to be driven by other national or regional demand disturbances, then the presumption seems to be that the Stability and Pact is getting in the way of automatic stabilizers embodied in national fiscal policies. And finally, when the differentials are thought to be driven by asymmetric productivity shocks, the presumption seems to be that the differentials reflect relative price movements that do not need to be corrected. While a rigorous welfare analysis is well beyond the scope of this paper, it is clearly of interest to ask what is driving the inflation differentials, both in the data and in the NNS models that are currently being used to evaluate policy.

Figure 5 illustrates what might be described as a cross-sectional Phillips Curve for the Euro area: average HICP inflation differentials are positively correlated with average GDP growth differentials; the correlation is 0.69. This positive correlation has in fact been rather well documented in the recent literature. Similar graphs appear in Angeloni and Ehrmann (2004) and Duarte (2003), and Chart 16 in ECB (2003) illustrates a positive correlation between average HICP inflation and cumulative output gaps. In addition, the time series correlations appear to be consistent with the cross-sectional correlations; for example, the correlation between French and German inflation and growth differentials is 0.58. All of these correlations seem to suggest that the inflation differentials are being driven by demand shocks of some sort, and not by productivity shocks.

What drives regional inflation differentials in the new NNS models? There is not yet a large literature on this, but initial results suggest that regional inflation differentials in the Euro area are driven by productivity shocks. Duarte and Wolman (2002) and Altissimo, Benigno and Rodriguez-Palenzuela (2004) developed small two-country NNS models to study inflation differentials in a
monetary union\textsuperscript{21}. Duarte and Wolman found that productivity shocks alone were enough to explain the observed volatility in the French-German inflation differential, and that the volatility of the model’s inflation differential was little affected by the addition of government spending shocks. Altissimo, Benigno and Rodriguez-Palenzuela found that fiscal shocks played a very minor role in their model’s variance decomposition for national inflation. It is unclear however that either of these NNS models would be capable of generating the positive correlations illustrated in Figure 5 or by the French and German time series data.

Here, we extend the original CC&D model to a two-country model of a currency union, loosely calibrated to French and German data. We find that it is not capable of generating the correlations illustrated in Figure 5 or by the French and German time series data. Once again, we attribute this failure to the dominant role played by productivity shocks in the determination of inflation. And once again, we add private spending shocks and “rule of thumbs” to see if we can make the model generate the positive correlations found in the data.

The rest of the paper proceeds as follows: Section 2.2 outlines the basic framework we use in all of our models. Section 2.3 discusses the failure of the closed economy model to explain the procyclical movements in interest rates and inflation that are observed in the U.S. data. Section 2.4 adds non-Ricardian elements to the closed economy model in an attempt to make the model more consistent with the data. Section 2.5 discusses the failure of the currency union model to explain the correlations shown in Figure 5 and the French and German time series data. Section 2.6 adds non-Ricardian elements to the currency union model in an attempt to make the model consistent with those correlations. Section 2.7 concludes.

\textsuperscript{21}Their models are more elaborate than the CC&D model in that they incorporate service and manufacturing sectors; they are less elaborate than the CC&D model in that they do not incorporate endogenous capital formation or Calvo-style wage setting.
2.2 A Framework that Encompasses All Four Models

In this paper we analyze four models: Ricardian and non-Ricardian versions of a closed economy calibrated to the U.S., and Ricardian and non-Ricardian versions of a two-country currency union (very roughly) calibrated to the larger countries in the Euro area. In this section, we develop a general framework that encompasses all four models.

The general framework includes a home country (designated by $H$) and a foreign country (designated by $F$). In each country, monopolistically competitive firms and workers set their prices and wages in standard Calvo contracts; and in each country, CES aggregators show how the differentiated products and labor services are valued. At the top of the pyramid, CES aggregators show how the two aggregate national products are valued for consumption and investment. Households own the capital stocks in each country and rent them to firms in their own country; so capital is freely mobile within a country, but immobile across countries. Governments in each country levy taxes on sales, capital and labor, and governments make lump sum transfers (which may be negative) to households. The two countries share a common currency.

More specifically, home and foreign firms, indexed by $f_H \in [0,1]$ and $f_F \in [0,1]$, produce differentiated goods that are aggregated into national products:

$$Y_J = \left[ \int_0^1 Y_J(f_J)^{\frac{\sigma-1}{\sigma}} df_J \right]^{\frac{\sigma}{\sigma-1}}, \quad J = H, F$$

where $\sigma > 1$. (Time subscripts will be suppressed when there is little chance for confusion.) The producer price indices are

$$P_J = \left[ \int_0^1 P_J(f_J)^{1-\sigma} df_J \right]^{\frac{1}{1-\sigma}}, \quad J = H, F$$

and the demands for the individual firms’ products are

$$Y_J^d(f_J) = \left[ \frac{P_J}{P_J(f_J)} \right]^\sigma Y_J, \quad J = H, F$$
where \( Y_J \) is aggregate demand for the national product\(^{22}\).

Home and foreign consumption goods are CES aggregates of the two national products:

\[
C = \left[ \mu \frac{\eta - 1}{\eta} C_H^{\frac{\eta - 1}{\eta}} + (1 - \mu) \frac{\eta - 1}{\eta} C_F^{\frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}} \\
C^* = \left[ \mu^* \frac{\eta - 1}{\eta} C_F^{\frac{\eta - 1}{\eta}} + (1 - \mu^*) \frac{\eta - 1}{\eta} C_H^{\frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}}
\]

where \( \eta > 1 \), \( C_H \) \((C_H^*)\) is home (foreign) consumption of home output, \( C_F \) \((C_F^*)\) is home (foreign) consumption of foreign output, and \( \mu \) and \( \mu^* \in [\frac{1}{2}, 1] \) measure the degree of home bias in consumption. The national CPI’s are

\[
P = \left[ \mu P_H^{1 - \eta} + (1 - \mu) P_F^{1 - \eta} \right]^{\frac{1}{1 - \eta}} \\
P^* = \left[ (1 - \mu^*) P_H^{1 - \eta} + \mu^* P_F^{1 - \eta} \right]^{\frac{1}{1 - \eta}}
\]

Note that if we eliminate the home bias (by setting \( \mu = \mu^* = \frac{1}{2} \)), then the home and foreign consumption goods will be identical, and there will be no CPI inflation differentials\(^{23}\).

Home and foreign investment goods are also CES aggregates of the two national products:

\[
I = \left[ \mu_1 \frac{\eta - 1}{\eta} I_H^{\frac{\eta - 1}{\eta}} + (1 - \mu_1) \frac{\eta - 1}{\eta} I_F^{\frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}} \\
I^* = \left[ \mu_1^* \frac{\eta - 1}{\eta} I_F^{\frac{\eta - 1}{\eta}} + (1 - \mu_1^*) \frac{\eta - 1}{\eta} I_H^{\frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}}
\]

where \( I_H \) \((I_H^*)\) is home (foreign) investment demand for home output, \( I_F \) \((I_F^*)\) is home (foreign) investment demand for foreign output, and \( \mu_1 \) and \( \mu_1^* \in [\frac{1}{2}, 1] \) measure the degree of home bias in

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\(^{22}\)The modeling of monopolistic competition in NNS models is now standard; for a more detailed discussion, see Canzoneri, Cumby and Diba (2003).

\(^{23}\)The alternative (and more cumbersome) way of modeling inflation differentials would be to introduce non-traded goods and the familiar Balassa-Samuelson effect, whereby productivity gains in the traded sector cause a real appreciation, or an increase in the CPI inflation differential in a currency union. We doubt, however, that adding a non-traded sector would change our main point that productivity gains have precisely the opposite effect. As Altissimo, et al. (2004) point out, the Balassa-Samuelson appreciation is smaller than the real depreciation resulting from the terms of trade effect in reasonably calibrated models.
investment. Home and foreign investment good prices are

\[ P_l = \left[ \mu_H P_H^{1-\eta} + (1-\mu_H) P_F^{1-\eta} \right]^{\frac{1}{1-\eta}} \]
\[ P_l^* = \left[ (1-\mu_H^*) P_H^{1-\eta} + \mu_H^* P_F^{1-\eta} \right]^{\frac{1}{1-\eta}} \]

Home and foreign households, indexed by \( h_H \in [0,1] \) and \( h_F \in [0,1] \), supply differentiated labor services that are aggregated into national labor services:

\[ N_J = \left[ \int_0^1 L(h_J) \frac{1}{\phi} dh_J \right]^{\frac{\phi}{\phi-1}}, \quad J = H, F \quad (2.4) \]

where \( \phi > 1 \). Labor, like capital, is mobile within a country, but immobile across countries. The price of this composite labor service to the firms of country \( J \) is:

\[ W_J = \left[ (1-\phi) \int_0^1 W(h_J) \frac{1}{\phi} dh_J \right]^{\frac{1}{1-\phi}}, \quad J = H, F \quad (2.5) \]

and the demand for the individual household’s labor service is

\[ L(h_J) = \left[ \frac{W_J}{W(h_J)} \right]^\phi N_J, \quad J = H, F \quad (2.6) \]

where \( N_J \) is the aggregate demand for the national labor service.

The production technology in each country is Cobb-Douglas:

\[ Y_J(f_J) = Z_J K^{\nu} N^{1-\nu}, \quad J = H, F \quad (2.7) \]

where total factor productivity, \( Z_J \), is common to all of the firms in country \( J \); \( Z_J \) is governed by a stochastic process that will be described below. As is well known (and discussed in CC&D), aggregate production functions can be written as

\[ Y_J \left( \equiv \int_0^1 Y_J(f_J) \frac{1}{\phi} df_J \right) = \int_0^1 Y_J(f_J) df_J DP_J^{-1} = Z_J K^{\nu} N^{1-\nu} DP_J^{-1}, \quad J = H, F \]

\[ DP_J = \int_0^1 \left[ \frac{P_J(f_J)}{P_J} \right]^{-\sigma} df_J, \quad J = H, F \]

52
where the $DP_J$ are measures of price dispersion in the two countries.

The staggered price setting in each country follows the familiar Calvo pattern. In any given quarter, each firm in country $J (= H, F)$ gets to reset its price with probability $1 - \alpha_J$. The first order conditions for a firm that gets to reset its price, and resulting equations for aggregate price dynamics, are now well known in the literature; we do not need to repeat them here\textsuperscript{24}.

Utility in period $t$ for a Home household is\textsuperscript{25}

$$U_t(h_H) = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \log \left[ C_s(h_H) \right] - \frac{L_s(h_H)^{1+\chi}}{1+\chi} \right\}$$

(2.8)

Households have access to a complete contingent claims market. A Home household’s budget constraints are

$$E_s[\Delta_{s,s+1} B_{s+1}(h_H)] + (1 + \tau_c)P_s C_s(h_H) + P_{t,s}I_s(h_H) = (1 - \tau_w)W_s(h_H)L_t^H(h_H)$$

$$+ (1 - \tau_k)R_{H,s}K_{H,s-1}(h_H) + \delta \tau_k P_{t,s} K_{H,s-1}(h_H) + TR_{H,s} + B_s(h_H) + \Pi_s(h_H)$$

(2.9)

where (using Woodford’s compact notation) $E_s[\Delta_{s,s+1} B_{s+1}(h_H)]$ is the price of a portfolio of state contingent claims and $B_s(h_H)$ is the payoff in period $s$; $\Pi_s(h_H)$ are dividends. The first two terms on the RHS of (2.9) are the household’s after tax labor and rental income; $R_{H,s}$ is the rental rate on capital and $K_{H,s-1}(h_H)$ is the household’s capital stock at the beginning of the period. The next term represents a simple rendition (following Erceg et al. (2004)) of depreciation allowances for the tax on capital. $TR_{H,s}$ is a lump sum transfer (or tax, if negative); the distortionary tax rates, $\tau_c$, $\tau_w$, and $\tau_k$, are assumed to be constant. The household’s capital accumulation constraint is

$$K_s(h_H) = (1 - \delta)K_{s-1}(h_H) + I_s(h_H) - \frac{1}{2} \psi \left[ \frac{I_s(h_H)}{K_{s-1}(h_H)} - \delta \right]^2 K_{s-1}(h_H)$$

(2.10)

where the last term is a capital adjustment cost, and $\delta$ is the rate of depreciation. The household chooses $C_t(h_H)$, $L_t(h_H)$, $W_t(h_H)$, $I_t(h_H)$ and $B_{t+1}(h_H)$ to maximize (2.8) subject to the demand for its labor services, (2.6), and the constraints (2.9) and (2.10).

\textsuperscript{24}They are described in some detail in CC&D.

\textsuperscript{25}Foreign households are modeled symmetrically.
Our assumption of complete contingent claims markets has the implication that the marginal utility of nominal wealth will equalize across households in both countries. This means that all households in a given country will make the same decisions about consumption and investment, and that the aggregate and individual values of these variables will be identical in equilibrium\(^{26}\). In addition, there will be complete consumption risk sharing across countries\(^{27}\):

\[
(1 + \tau_c) P_t C_t = \xi (1 + \tau_c^*) P^*_t C^*_t
\]

The market clearing conditions for the two national products are

\[
Y_{H,t} = C_{H,t} + C^*_{H,t} + I_{H,t} + G_t
\]

\[
Y_{F,t} = C_{F,t} + C^*_{F,t} + I_{F,t} + I^*_{FH,t} + G^*_t
\]

where \(G_t\) and \(G^*_t\) are home and foreign government purchases; we assume complete home bias in government consumption.

### 2.3 A Closed Economy Model with Ricardian Equivalence

In this section and the next, we explore the implications of our closed economy models for the cyclical behavior of the interest rate and inflation. The Ricardian Closed Economy (RCE) model is identical to the CC&D model.

#### 2.3.1 The RCE model

The RCE model emerges from the general framework outlined in Section 2.2 if we eliminate the foreign country, set the home bias parameters \((\mu_H, \mu_F)\) equal to one, and set the distortionary tax

\(^{26}\)For example, \(C = \int_0^1 C(h_H) dh_H = C(h_H)\int_0^1 dh_H = C(h_H)\). Canzoneri, Cumby and Diba (2003, 2004) discuss the household’s first order conditions and the implications of complete contingent claims market in some detail.

\(^{27}\)The constant \(\xi\) depends on initial conditions, and upon factors like relative tax rates, if the rates are known when the contingent claims market meets. The value of \(\xi\) plays no role in our analysis since we use first order approximations around a non-stochastic steady state.
rates \((\tau_c, \tau_w, \text{and } \tau_k)\) equal to zero. Lump sum taxes finance government spending, and the model exhibits Ricardian equivalence.

The RCE model is calibrated to U.S. data, and estimated stochastic processes explain the behavior of monetary policy, government spending, and total factor productivity. A detailed discussion of the calibration process, estimation procedures and data sources can be found in Appendix B of CC&D.

A Henderson-McKibbin-Taylor rule describes monetary policy; our estimate of the rule is:

\[
i_t = 0.222 + 0.824i_{t-1} + 0.3552\pi_t + 0.032384(\text{output gap})_t + \varepsilon_{i,t},
\]

where \(\pi_t = \log \left( \frac{P_t}{P_{t-1}} \right)\) and the standard deviation of the interest rate shock, \(\varepsilon_{i,t}\), is 0.00245. CC&D used nonlinear least squares to estimate this rule over the Volcker and Greenspan years (1979.3 - 2003.2). For estimation purposes, CC&D defined the output gap to be actual GDP minus the Congressional Budget Office’s ‘potential’ GDP (both in logs); in numerical solutions of the model, we replace potential output with the steady state output.\(^{28}\)

CC&D used a longer sample period (1960:1 - 2003:2) to estimate the productivity process:

\[
\log (Z_{H,t}) = 0.923 \log (Z_{H,t-1}) + \varepsilon_{p,t},
\]

where \(\log (Z_t)\) is the deviation of total factor productivity from an estimated linear trend, and the standard deviation of the productivity shock, \(\varepsilon_{p,t}\), is about 0.009.

CC&D also estimated an auto regressive process for government spending:

\[
\log (G_t) = \Gamma + 0.973 \log (G_{t-1}) + \varepsilon_{g,t},
\]

where the standard error of the fiscal shock, \(\varepsilon_{g,t}\), is about 0.01. Estimates over the two sample periods (1979:3 - 2003:2 and 1960:1 - 2003:2) were quite similar. In our model simulations, we choose the intercept term, \(\Gamma\), to make \(\frac{\hat{Q}}{Q} = 0.20\) in the steady state.

\(^{28}\)A natural alternative would be to use the flexible wage/price level of output. However, CC&D found that the model’s simulated output gap more closely resembled the output gap in the data with the specification we use here.
The other parameters used to calibrate the RCE model are given in Table 10. The Frisch labor supply elasticity, $\chi^{-1}$, is low by RBC standards, but high by the standards of the empirical labor literature; CC&D find that its value does not matter much for the ability of the model to fit the data. The values for $\alpha$ and $\omega$ (the analogous parameter governing the degree of wage rigidity) imply that prices are fixed for three quarters on average, and wages are fixed for four quarters on average. The values for $\sigma$ and $\phi$ imply that price and wage markups are about 17%. The value of $\psi$ is chosen to make the volatility of investment match that in the data.

![Table 10](image)

Table 10: Parameters for the benchmark calibration of the RCE

### 2.3.2 Inflation and interest rates in the RCE Model

CC&D showed that the RCE model is capable of explaining several key characteristics of the U.S. business cycle\(^{29}\).

Table 11 compares results from the calibrated model with quarterly data from the U.S. economy\(^{30}\). The model’s variables are expressed as log deviations from a deterministic steady state. The

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29\(^{The model is not capable of capturing the persistence found in U.S. data; it does not include the elements

Christiano, Eichenbaum and Evans (2005) find necessary to do so.

30\(^{Notes:

(i) Model data and actual data are in logarithms, and have been HP-filtered.

(ii) Model data was generated by Dynare, using 1st order approximations.

(iii) Actual data are computed using a sample of 1960:1 to 2003:2.

(iv) Standard deviations for the GDP row are the first number in each cell. For other rows standard deviations relative to standard deviation of output are the first numbers in each cell.

(v) As both hours and the real wage are for the nonfarm business sector, we normalize their standard deviations by the standard deviation of real GDP of the nonfarm business sector.

(vi) Correlations with GDP are the second number in each cell.
U.S. data are also in logs, and both the model data and the actual data have been HP-filtered. We used Dynare (see Juillard (2003)) to calculate the model’s steady state, to find a first order approximation, and to calculate the moments reported in Table 11. Beginning with the row for GDP, 0.014 is the model’s standard deviation of output, which is slightly smaller than the standard deviation in the data, 0.016. Proceeding to the row for consumption, 0.839 is the ratio of the standard deviation of consumption to the standard deviation of output in the model, and 0.962 is the correlation between consumption and output. These are close to the corresponding statistics in the data. The next three rows provide the same statistics (standard deviations relative to the standard deviation of output and correlations with output) for investment, hours worked and real wages.

The RCE model comes fairly close to matching the data for all these variables, though real wages and output are more positively correlated in the model than they are in the data. Impulse response functions from the model (not pictured) show that productivity shocks make the real wage move procyclically, while the other shocks make them move countercyclically. This is our first indication that productivity shocks may be playing an inordinate role in the RCE model, or equivalently that some demand side shocks may be either absent or incorrectly modeled.

The rows for inflation and the nominal interest rate do alert us to some weaknesses in the RCE model. The volatility of inflation in the model is less than it is in the data. But even more alarming is the fact that both the interest rate and inflation are negatively correlated with output in the model, while they are positively correlated in the data.

Where are these model failures coming from? Figure 6 reports the model’s impulse response functions (IRFs) for output, inflation and the nominal interest rate. These IRFs suggest that productivity shocks make inflation and the interest rate move countercyclically, while government spending shocks make inflation and the interest rate move procyclically. The interest rate shock makes inflation move procyclically, but of course the interest rate itself moves countercyclically. The model’s variance decompositions – reported in Table 12 – show which of these shocks matter the
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<thead>
<tr>
<th>STD,COR</th>
<th>RCE Model</th>
<th>Actual Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.014</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.839</td>
<td>0.799</td>
</tr>
<tr>
<td></td>
<td>0.962</td>
<td>0.871</td>
</tr>
<tr>
<td>Investment</td>
<td>3.133</td>
<td>3.122</td>
</tr>
<tr>
<td></td>
<td>0.992</td>
<td>0.893</td>
</tr>
<tr>
<td>Hours</td>
<td>0.972</td>
<td>0.894</td>
</tr>
<tr>
<td></td>
<td>0.635</td>
<td>0.857</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.497</td>
<td>0.470</td>
</tr>
<tr>
<td></td>
<td>0.553</td>
<td>0.243</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.259</td>
<td>0.357</td>
</tr>
<tr>
<td></td>
<td>-0.389</td>
<td>0.330</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.203</td>
<td>0.253</td>
</tr>
<tr>
<td></td>
<td>-0.998</td>
<td>0.333</td>
</tr>
</tbody>
</table>

Table 11: Benchmark Calibration of the RCE Model
Figure 6: Impulse responses from the RCE model
most. Productivity shocks explain more than 90% of the variation in inflation and about 50% of the variation in output; interest rate shocks do move output, but they have little effect on inflation; and government spending shocks do almost nothing to either variable. Productivity shocks are clearly the most important factor in the cyclical behavior of inflation, and this would appear to account for the model’s counterfactual negative correlations.

<table>
<thead>
<tr>
<th></th>
<th>( \varepsilon_{p,t} )</th>
<th>( \varepsilon_{i,t} )</th>
<th>( \varepsilon_{g,t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>52.30</td>
<td>47.60</td>
<td>0.10</td>
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<tr>
<td>Inflation</td>
<td>94.94</td>
<td>5.06</td>
<td>0.00</td>
</tr>
<tr>
<td>Interest rate</td>
<td>49.59</td>
<td>50.38</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 12: Variance decompositions for the benchmark RCE model (infinite horizon, in percent) with shocks to productivity, interest rate and government spending.

This suggests that the RCE model may be missing some demand shocks, or that the demand shocks that have been included may not have been modeled correctly. The IRFs in Figure 6 suggest that the government spending shocks may not have been modeled correctly. An increase in government spending crowds out consumption as well as investment. This is a familiar result from the RBC literature: the increased tax burden causes optimizing households to work more and consume less. Adding nominal inertia does not change this Ricardian type of response. But, as Fatas and Mihov (2001a,b) have noted, this response in consumption is at odds with several recent VAR studies\(^{31}\).

This suggests that government spending shocks may not have as much effect on aggregate demand in the model as they do in the U.S. economy, and that this may be why the model fails to predict procyclical movements in inflation. We explore this possibility in the next section. But before going on, we check the robustness of our results to different assumptions about nominal rigidity, and we investigate the possibility that an alternative demand shock – one suggested by Ireland (2004)

\(^{31}\)See the first footnote in this chapter.
and Gali and Rabanal (2004) – may help the model explain the cyclical behavior interest rates and inflation.

2.3.3 The Importance of Nominal Inertia

Some NNS models assume wages are flexible; indeed, Goodfriend and King (2001) and others have argued that the observed rigidity of nominal wages may not even be allocative. And of course, the RBC model did not have any nominal inertia. For these reasons, we test the robustness of our results to different assumptions about the type and degree of nominal inertia.

If we let wages be flexible ($\omega = 0, \alpha = 0.67$) in the RCE model, the correlation of inflation and output improves slightly (compared to the benchmark case reported in Table 2): it rises from -0.389 to 0.204. If we let both wages and prices be flexible ($\omega = \alpha = 0$), the correlation is slightly worse: it falls from -0.389 to -0.429. The correlation between the interest rate and output is virtually the same, -0.998, in all of these cases. So, no matter what we assume about the type or degree of nominal inertia, the benchmark RCE model seems quite incapable of generating the positive correlations that are observed in the U.S. data.

2.3.4 Adding a Private Spending Shock to the RCE Model

Ireland (2004), Gali and Rabanal (2004) and others have added what might be viewed as a private spending shock to NNS models; the household utility function becomes:

$$U_t(h_H) = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ a_s \log |C_s(h_H)| - \frac{L_s(h_H)^{1+\chi}}{1+\chi} \right\}$$

where $a_s$ is a preference shock. Both Ireland (2004) and Gali and Rabanal (2004) model a highly persistent shock; we let:

$$\log (a_t) = 0.9 \log (a_{t-1}) + \varepsilon_{a,t}$$

where the standard deviation of the innovation, $\varepsilon_{a,t}$, is 0.03. We chose this value to be large enough to make the standard deviation of output in the model match the standard deviation of output in
the data; our choice happens to coincide with Ireland’s (post 1980) estimate.

Figure 6 shows the model’s IRFs for the private spending shock. As expected, the shock makes inflation and the interest rate move procyclically, and this might be expected to help with the model’s unconditional correlations. Table 13 reports the model’s variance decompositions. The private spending shock moves both output and the interest rate, but it has little effect on inflation. The model’s unconditional correlations do rise, but not to the positive levels observed in the data: the correlation between inflation and output rises from -0.389 (as reported in Table 11) to 0.218, and the correlation between interest rate and output rises from -0.998 to 0.686.

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon_{p,t}$</th>
<th>$\varepsilon_{i,t}$</th>
<th>$\varepsilon_{g,t}$</th>
<th>$\varepsilon_{a,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>39.67</td>
<td>36.11</td>
<td>0.07</td>
<td>24.15</td>
</tr>
<tr>
<td>Inflation</td>
<td>89.92</td>
<td>4.79</td>
<td>0.00</td>
<td>5.29</td>
</tr>
<tr>
<td>Interest rate</td>
<td>41.76</td>
<td>42.43</td>
<td>0.02</td>
<td>15.79</td>
</tr>
</tbody>
</table>

Table 13: Variance decompositions for the benchmark RCE model with preference shocks (infinite horizon, in percent) with shocks to productivity, interest rate, government spending and private spending

We could increase the unconditional correlations further by raising the standard deviation of the private spending shock. We should note, however, that this is already a very volatile shock: its standard deviation is three times the standard deviation of the government spending shock. The private spending shock – unlike the government spending shock – is not directly observable, since it is modeled as a shock to household preferences. So, we have no direct way of measuring its volatility. We have followed standard practice in choosing the parameter to help the model match second moments in the data – here the standard deviation of output\textsuperscript{32}. And, as stated earlier, our choice coincides with Ireland’s (2004) estimate. We conclude that the private spending shock

\textsuperscript{32}Estimating the full model – say with Bayesian methods – would not resolve the issue. This would just be a more formal way of choosing the standard deviation of the shock to match the moments in the data.
suggested by Ireland (2004) and Gali and Rabanal (2004) is a step in the right direction, but it does
not resolve the problem fully.

2.4 A Closed Economy Model with Departures from Ricardian Equiva-

lence

In this section, we investigate the possibility that the government spending shocks do not propagate
correctly in the RCE model, and that this might be the source of the counterfactual correlations the
model exhibits. We add several non-Ricardian elements to the model, some of which are designed to
augment the effect of a government spending shock on private consumption, and aggregate demand;
our discussion of the IRFs in Figure 6 suggested that this was an experiment worth trying. First, we
describe our modifications of the RCE model; then, we explore their implications for the correlations
in question.

2.4.1 The NRCE Model

We add two types of non-Ricardian elements to the RCE model to arrive at what we will call the Non-
Ricardian Closed Economy (NRCE) Model. First, we add distortionary taxation: taking average
tax rates for the U.S. from Table 2 of Carey and Rabesona (2002), we set $\tau_c = 0.064$, $\tau_w = 0.234$,
and $\tau_k = 0.273$. And second, we add what Gali, López-Salido and Vallés (2003) call “rule of thumb
consumers”, or what we will think of as “liquidity constrained” households.

More specifically, the NRCE model has two types of households. Optimizing households (denoted
by an $O$) are like the households we have already modeled; we do not need to change the equations
that describe their behavior (except to add an $O$ subscript to the relevant variables). Liquidity
constrained households (denoted by an $L$) hold no assets; they simply consume their disposable
incomes each period.

We have a number of choices to make in our modeling of $L$ households, and these choices do affect
the way in which aggregate household consumption responds to an increase in government spending. Generally, our choices will err on the side of making the response of aggregate consumption large. This gives the model the best chance of explaining the procyclical movements in interest rates and inflation.

The first set of choices has to do with the importance of \( L \) households in the economy. We assume that the population of \( L \) households is equal to the population of \( O \) households (each having a unit mass), but we let the \( L \) households be less productive than \( O \) households. \( L \) households supply a homogeneous labor service, and the composite labor input entering the production functions \((2.7)\) becomes

\[
N_t = \left[ \zeta N_{O,t}^{\frac{\theta-1}{\theta}} + (1 - \zeta) N_{L,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{1-\theta}}
\]

where \( \theta > 0 \) and \( 0.5 < \zeta < 1 \). \( N_{O,t} \) is the aggregate labor input (defined by \((2.4)\)) of \( O \) households and \( N_{L,t} \) is the labor input of \( L \) households. The aggregate wage rate for this composite labor input is

\[
W_t = \left[ \zeta^\theta W_{O,t}^{1-\theta} + (1 - \zeta)^\theta W_{L,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}
\]

where \( W_{O,t} \) is given by \((2.5)\). We follow Erceg et. al. (2004) in assuming that the wages of \( L \) households are proportional to the aggregate wage of \( O \) households, but we make the constant of proportionality less than one (since \( L \) households are less productive). Specifically, we let

\[
W_{L,t} = \frac{1 - \zeta}{\zeta} W_{O,t}
\]

Then, firms’ cost minimization implies that \( N_{L,t} = N_{O,t} \).

In our simulations, we set \( \zeta = 0.6 \), making \( W_{L,t} = \frac{2}{3} W_{O,t} \); the steady state share of aggregate consumption going to \( L \) households about 40 percent. Campbell and Mankiw (1989) estimated that the rule of thumbers’ share of consumption is between 40 and 50 percent, but our value of 40 percent is quite high when compared to more recent estimates reported by Coenen and Straub (2004), Heathcote (2005) and Reis (2004). Clearly, our choice of a large consumption share for \( L \)
households enhances the non-Ricardian effects on consumption that we are trying to model.

$L$ households consume their disposable incomes:

$$(1 + \tau_c)C_{L,t} = (1 - \tau_w)\frac{W_{L,t}}{F_t} N_{L,t} + TR_{L,t}$$

where $TR_{L,t}$ are government transfers. Since we assume that both types of households have unit mass, aggregate consumption in the NRCE model is

$$C_t = C_{O,t} + C_{L,t}$$

The stock of real government debt, $D_t$, evolves according to the budget constraint

$$D_t = \frac{(1 + i_{t-1})}{\pi_t} D_{t-1} + G_t + TR_{O,t} + TR_{L,t}$$

$$-\tau_c C_t - \tau_w (W_{O,t} N_{O,t} + W_{L,t} N_{L,t}) \frac{1}{F_t} - \tau_k \frac{R_{H,t} - \delta P_{H,t}}{F_t} K_{H,t-1}$$

where $TR_{O,t}$ is a lump sum transfer to (or tax on) $O$ households. Letting $\frac{D}{Y}$ represent the debt to GDP ratio in the steady state, the government’s spending and transfer policies are

$$\log (G_t) = \Gamma_g + 0.973 \log (G_{t-1}) - \rho_g \left[ \log \frac{D_{t-1}}{Y_t} - \log \frac{D}{Y} \right] + \varepsilon_{g,t}$$

$$\log (TR_{L,t}) = \Gamma_{trl} + 0.9 \log (TR_{L,t-1}) - \rho_{trl} \left[ \log \frac{D_{t-1}}{Y_t} - \log \frac{D}{Y} \right]$$

$$\log (TR_{O,t}) = \Gamma_{tro}$$

where $\rho_g > 0$ and $\rho_{trl} > 0$. The responses of government spending and transfers to the national debt – measured by $\rho_g$ and $\rho_{trl}$ – stabilize public debt dynamics. We set the intercept terms – $\Gamma_g$, $\Gamma_{trl}$ and $\Gamma_{tro}$ – so that $\frac{G}{Y} = 0.20$, $\frac{C}{Y} = 0.67$ and $\frac{D}{Y} = 0.34$ in the steady state; these steady state ratios seem appropriate for the U.S. economy.

The next set of choices we have to make has to do with the strength of the fiscal response to a change in the level of debt: big values of $\rho_g$ and $\rho_{trl}$ enhance the non-Ricardian effects on consumption that we are trying to model. A bigger value of $\rho_{trl}$ shifts the tax burden associated with an increase in government spending away from $O$ households and onto $L$ households; this
limits the Ricardian consumption response of $O$ households, and magnifies the effect of an increase in government spending on aggregate consumption. Similarly, a larger value of $\rho_g$ implies lower government spending in the future, and this lowers the tax burden on $O$ households.

In our simulations, we set $\rho_g = \rho_{trt} = 0.125$, and here again we may have erred on the side of making the non-Ricardian effect on consumption large. Using annual data from 1975 - 2001, we regressed the HP-filtered log of real transfers and government purchases on a constant, a lagged dependent variable, and the lagged HP-filtered log of the debt-to-GDP ratio. The values for $\rho_g$ and $\rho_{trt}$ computed from U.S. data are -0.075 (0.028) and -0.050 (0.059). The corresponding values computed by pooling the Euro Area countries are -0.054 (0.030) and -0.087 (0.043). Thus, our values of 0.125 are about one standard error above our estimates for the U.S. data.

2.4.2 Inflation and interest rates in the NRCE Model

Figure 7 shows IRFs for government spending shocks in both of the models. As noted in the last section, government spending shocks produce procyclical movements in inflation and the interest rate in the RCE model. But, they crowd out both consumption and investment and have a relatively weak effect on aggregate demand; as mentioned above, the effect on consumption runs counter to some recent VAR studies. In the NRCE model, government spending shocks still crowd out both consumption and investment; in fact, we have not been able to make consumption rise for reasonable parameterizations of the NRCE model. However, the crowding out of consumption is an order of magnitude smaller here, and the increase in aggregate demand (or output) is about 3.5 times larger.

---

33 Similar regressions for taxes (in the U.S. or in Europe) fail to find any significant response of taxes to the level of the debt. All data are taken from the OECD.

34 Eliminating the gap term in the interest rate rule (2.11) makes monetary policy less restrictive and allows consumption to rise. But we see little justification in this, given that the coefficient on the output gap in our estimated rule is highly significant; moreover, the gap term is well established in the empirical literature. Similarly, larger values of $\rho_g$ and $\rho_{trt}$ allow consumption to rise; given the uncertainty about these parameters, this may be a more promising avenue to pursue.
So, the non-Ricardian modifications of the RCE model do seem to be having the intended effect.

Variance decompositions are reported in Table 14, and they seem somewhat encouraging. Comparing Table 14 with Table 12, government spending shocks now appear to have a measurable effect on movements in output, inflation and the interest rate; however, the numbers are still quite modest. The basic message from Table 12 seems to carry over to the NRCE model: productivity shocks still explain more than 90% of the variation in inflation, and roughly half the variation in output and the interest rate; and interest rate shocks still explain rather little of the variation in inflation. Productivity shocks would still be expected to play the dominant role in determining the cyclical behavior of inflation and the interest rate; and indeed, the NRCE model’s unconditional correlation between inflation and output is 0.281, and the unconditional correlation between the interest rate and output is 0.707. The correlations in the NRCE model are less negative than they were in the RCE model (see Table 11), but they are nowhere near the positive correlations observed in the U.S. data. This suggest that the modeling of demand shocks may still be inadequate.

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon_{p,t}$</th>
<th>$\varepsilon_{i,t}$</th>
<th>$\varepsilon_{g,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>50.2</td>
<td>45.2</td>
<td>4.6</td>
</tr>
<tr>
<td>Inflation</td>
<td>92.4</td>
<td>5.4</td>
<td>2.2</td>
</tr>
<tr>
<td>Interest</td>
<td>42.5</td>
<td>48.5</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Table 14: Variance decompositions for the NRCE model (in percent) with shocks to productivity, interest rate and government spending

2.5 A Currency Union Model with Ricardian Equivalence

In this section and the next, we explore two country extensions of our NNS models; the basic question is whether our models can explain the positive correlation between inflation differentials and growth differentials illustrated in Figure 5. We begin with a Ricardian model in this section, and proceed
Figure 7: Government spending shocks in the RCE and NRCE models
to a non-Ricardian model in the next section. First, we explain how the Ricardian Currency Union (RCU) model emerges from the basic framework developed in Section 2.2.

2.5.1 The RCU Model

The RCU model emerges naturally from the general framework outlined in Section 2.2. We set the home bias parameters in consumption and investment ($\mu$ and $\mu_I$) equal to 0.75 and 0.50 respectively. These parameter values make the steady state imports about 25% of GDP, which is roughly in line with the import shares of France and Germany. And in the Ricardian version of the model, we set the distortionary tax rates ($\tau_c$, $\tau_w$, and $\tau_k$) equal to zero; lump sum taxes finance government spending.

A serious modeling of the Euro Area is well beyond the scope of the present paper, but we do calibrate the two symmetric countries in our currency union with countries like France and Germany in mind. The correlation between inflation differentials and growth differentials in France and Germany from 1999 through 2004 is 0.58, which is very close the cross sectional correlation illustrated in Figure 5\(^{35}\); the standard deviation of the inflation differential is 0.0038. These statistics appear to be representative for the Euro Area.

Our calibration of the productivity process is (roughly) based on Collard and Dellas (2002), who estimated a bivariate process using French and German data. We assume

$$
\begin{bmatrix}
\ln Z_{H,t} \\
\ln Z_{F,t}
\end{bmatrix} =
\begin{bmatrix}
0.76 & 0.10 \\
0.10 & 0.76
\end{bmatrix}
\begin{bmatrix}
\ln Z_{H,t-1} \\
\ln Z_{F,t-1}
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_{H_{P,t}} + \varepsilon_{cp,t} \\
\varepsilon_{F_{P,t}} + \varepsilon_{cp,t}
\end{bmatrix}
$$

where $\varepsilon_{cp}$ is a common shock (with standard deviation 0.0050) and $\varepsilon_{H_{P,t}}$ and $\varepsilon_{F_{P,t}}$ are country specific shocks (with standard deviation 0.0082)\(^{36}\). Our calibration of the government spending processes is

\(^{35}\)The (HIPC) inflation and growth differentials are quarterly, and they were HP filtered.

\(^{36}\)The autoregressive coefficients are averages of the Collard and Dellas estimates; the eigenvalues of the coefficient matrix are nearly identical to those computed from their estimates. The volatilities of the country specific productivity shocks are the averages of the two estimated by Collard and Dellas. The volatility of the common productivity shock
also based on Collard and Dellas’ (2000) estimates for France and Germany. We assume
\[
\log (G_{J,t}) = \Gamma + 0.94 \log (G_{J,t-1}) + \varepsilon_{Jg,t}, \quad J = H, F
\]
where \(\Gamma\) is chosen to make the steady state \(\frac{G}{Y}\) ratio equal to 0.22, and the standard deviation of the innovation term set at 0.02\(^{37}\). It should be noted that this standard deviation is twice the size of the standard deviation for the U.S. government spending process; so, the currency union models have relatively large demand shocks, even without any private spending shocks.

We assume that the common monetary policy can be described by an interest rate rule without an output gap term\(^{38}\):
\[
i_t = -\log \beta (1 - 0.8) + 0.8i_{t-1} + 2(1 - 0.8)\pi_t
\]
where \(\pi_t = 0.5\pi_{H,t} + 0.5\pi_{F,t}\) is the aggregate inflation rate, and \(\pi_{H,t}\) and \(\pi_{F,t}\) are the rates of growth in the national CPIs (defined by (2.2)). Equal weights are used to define aggregate inflation since the Home and Foreign countries are symmetric.

We have omitted an interest rate shock in (2.12) since it would not affect either the inflation differential or the growth differential. Only asymmetric shocks affect the variables of interest in our symmetric two-country model. They include the asymmetric productivity shocks, the government spending shocks (since government spending falls exclusively on the national product), and the asymmetric private spending shocks (which will be discussed later in this section).

\(^{37}\)The autoregressive coefficient and the standard deviation are averages of the Collard and Dellas estimates for France and Germany.

\(^{38}\)We have not tried to estimate an interest rate rule for the ECB since there are fewer than six years of data. Given the primacy of inflation in the ECB’s mandate, we have omitted the gap term, and we have used standard values for the response to the lagged interest rate and inflation. We also note that eliminating the gap term enhances the effect of government spending shocks on aggregate demand; see footnote 18.
2.5.2 Inflation Differentials and Growth Differentials in the RCU Model

Figure 8 shows the IRFs from the RCU model. Inflation differentials are defined as Home CPI inflation minus Foreign CPI inflation; growth differentials are defined as Home output growth minus Foreign output growth. As might be expected, asymmetric productivity shocks produce a negative correlation between inflation and growth differentials. An increase in Home productivity lowers Home marginal cost and inflation, and raises home output; the interest rate falls since aggregate inflation falls. In fact, productivity shocks produce a perfect negative correlation between relative prices and relative output\(^{39}\). So, asymmetric productivity shocks will make it difficult for the RCU model to explain the positive correlation observed in the French and German data. On the other hand, an increase in Home government spending creates a positive inflation differential and a positive growth differential. And the central bank raises the interest rate in response to the increase in aggregate inflation.

So, once again, demand shocks (represented here by government spending shocks) appear to work in the right direction for explaining the positive correlations observed in the French and German data (and in Figure 5), while productivity shocks appear to work in the other direction. And, once again, the problem is that the variance decompositions reported in Table 15 show that productivity shocks are more important than government spending shocks in explaining the movements of these two variables. Asymmetric productivity shocks explain more than 90% of the movements in the inflation differential, and almost 25% of the movements in the growth differential. Government spending

---

\(^{39}\)It should be noted that our assumption of complete international risk sharing does not play a direct role in generating this correlation. Individual consumers and firms in the model minimize expenditures on the CES indexes (2.1). Expenditure minimization induces an inverse relationship between relative prices and relative quantities at the individual level, and aggregation does not alter this perfect negative correlation unless we let government purchases fall entirely on home goods or add shocks to (2.1) and (2.3), as we will in Section 2.5.4. The correlation we highlight here is closely related to the correlation between the terms of trade and output discussed in the RBC literature [e.g., Backus, Kehoe, and Kydland (1995)].
Figure 8: Impulse responses from the RCU model
shocks do move the growth differential, but they have very little effect on the inflation differential. So, productivity shocks would be expected to play the dominant role in the correlation between the inflation differential and the growth differential. And indeed, the RCU model’s unconditional correlation is -0.32; it is far from the positive correlations observed in the French and German data (0.58) or in the cross country data in Figure 5 (0.69).

<table>
<thead>
<tr>
<th></th>
<th>Combined asymmetric productivity shocks</th>
<th>Combined government spending shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth differential</td>
<td>23.6</td>
<td>76.4</td>
</tr>
<tr>
<td>Inflation differential</td>
<td>92.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Table 15: Variance decompositions for the RCU model (in percent)

One might speculate that demand shocks are either missing or incorrectly specified in the RCU model. Another indication of this is that the standard deviation of the inflation differential in the RCU model is only 0.0016; the standard deviation of the inflation differential between France and Germany is 0.0038. In other words, the RCU model explains only half of the volatility that is observed in the data.\(^{40}\)

IRFs for a government spending shock are shown in Figure 9. An increase in Home government spending raises Home output, but crowds out Home consumption.\(^{41}\) In the next section, we add features to the model that accentuate the effect of an increase in government spending on private consumption. But before going on, we check the robustness of our results to different assumptions.

\(^{40}\) Duarte and Wolman (2002) found that productivity shocks alone were able to explain the volatility of the French and German inflation differential. We do not know what accounts for the difference in our results, but our model is different than theirs in a number of ways. Our model is more elaborate than theirs in that we have Calvo wage setting and endogenous capital formation; theirs is more elaborate than ours in that they have non-traded sectors.

\(^{41}\) All variables in Figure 9 are Home variables. The rise in Home investment is curious, and may be counterfactual as well. The increase in government spending raises demand for Home goods, and causes a real appreciation that makes foreign investment goods less expensive.
about nominal rigidity and the elasticity of demand for the Home and Foreign goods, and we investigate the possibility that an asymmetric private demand shock may help the model explain the positive correlation between inflation and growth differentials that is observed in the data.

2.5.3 The Importance of Nominal Inertia and the Elasticity of Substitution

In the RCU model, the type and degree of nominal inertia do seem to matter. If we let wages be flexible \((\omega = 0, \alpha = 0.67)\), the correlation between inflation and growth differentials falls from -0.33 (in the benchmark case) to -0.58. And if we let both prices and wages be flexible \((\omega = \alpha = 0)\), the correlations falls even further to -0.88. In terms of matching the correlations in the data, the RCU model performs better (albeit not very well) with both price and wage rigidity.

The elasticity of substitution between Home and Foreign goods is given by \(\eta\) in the final consumption good aggregator (2.1). We have been setting \(\eta = 1.5\), which is consistent with what is found in both the RBC and NNS literatures. The newer trade literature has been using much higher values. But if we let \(\eta = 5\) in the RCU model, the correlation between inflation and growth differentials falls from -0.33 (in the benchmark case) to -0.73. Increasing the elasticity of substitution means that larger relative price movements are required in response to relative supply shocks, and this does not help the RCU model explain the positive correlations in the data.

2.5.4 Adding Private Preference Shocks to the RCU Model

In Section 2.3, we added the private spending shock suggested by Ireland (2004) and Galì and Rabanal (2004) to the RCE model. Here, we model the private spending shocks as shocks to the home bias preference parameters:

\[
\mu_t = 0.75 s_{H,t}, \quad \mu_{I,t} = 0.5 s_{H,t}, \quad \mu^*_t = 0.75 s_{F,t}, \quad \mu^*_{I,t} = 0.5 s_{F,t}
\]

where

\[
\log (s_{J,t}) = 0.9 \log (s_{J,t-1}) + \varepsilon_{s,t}, \quad J = H, F
\]
RCU Model –

NRCU Model –

Figure 9: Government spending shocks in the RCU and NRCU models
In the steady state (where $s_{J,t} = 1$), the home biases remain at their benchmark values: $\mu = \mu^* = 0.75$ and $\mu_I = \mu^*_I = 0.5$. A positive Home private spending shock ($s_{H,t} > 1$) raises the biases in the Home country. Figure 8 shows the IRFs for this shock; as expected, it produces a positive correlation between inflation and growth differentials.

We choose the standard deviation of the innovations, $\varepsilon_{J,s,t}$, to make the standard deviation of the inflation differential in the RCU model equal to the standard deviation the inflation differential for France and Germany. Since this requires a doubling of the standard deviation of the inflation differential, the required shocks are very big: the standard deviation of the $\varepsilon_{J,s,t}$ is about 1.75 times the standard deviation of the $\varepsilon_{J,g,t}$, the innovations in government spending processes, and the European government spending processes were already quite volatile in comparison with the U.S.

Since the private spending shocks are so large, they play an absolutely dominant role in the variance decompositions reported in Table 16. The private spending shocks explain more over 80% of the variation in the inflation differential, and virtually all the variation in the growth differential. So, not surprisingly, the model’s unconditional correlation between inflation and growth differentials rises dramatically. In fact, the correlation in the model rises to 0.57, which is almost identical to the correlation observed for France and Germany.

<table>
<thead>
<tr>
<th></th>
<th>Asymmetric productivity shocks</th>
<th>Government spending shocks</th>
<th>Private spending shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth differential</td>
<td>0.4</td>
<td>1.2</td>
<td>98.4</td>
</tr>
<tr>
<td>Inflation differential</td>
<td>16.2</td>
<td>1.3</td>
<td>82.5</td>
</tr>
</tbody>
</table>

Table 16: Variance decompositions of the RCU model with preference shocks (in percent)

This might be viewed as a modeling success, but the variance decompositions in Table 16 may raise questions. The positive correlation between inflation and growth differentials was achieved by modeling a highly volatile demand shock. As noted in Section 2.3, we have no direct way of
measuring the standard deviation of an unobserved preference shock like this. We have followed standard practice in choosing the parameter to help the model match a standard deviation in the data – here the standard deviation of the inflation differential for France and Germany. But since the shock is not observable, it is hard to know exactly what it represents, or how to gauge its empirical realism. We suspect that the shock is standing in for a number of structural shocks that have not been modeled, and for the way they propagate through aggregate demand to inflation. Once again, we conclude that the private spending shock we have modeled is a step in the right direction, but we think that more work needs to be done to identify the missing demand shocks.

2.6 A Currency Union Model with Departures from Ricardian Equivalence

In this section, we investigate the possibility that the government sector has not been modeled correctly in the RCU model, and that this might be the source of its (counterfactual) negative correlation between inflation and output differentials. We modify the model to include Non-Ricardian elements that are designed to enhance the effect of a government spending shock on consumption, and thus aggregate demand. First, we describe the modifications that are needed; then, we discuss their implications for the correlation between inflation and growth differentials.

2.6.1 The NRCU Model

Our description here can be very brief, as the modifications are essentially the same as those discussed in Section 2.4.1. We add two types of non-Ricardian elements to the RCU model to arrive at what we will call the Non-Ricardian Currency Union (NRCU) Model. First, we add distortionary taxes: taking average tax rates for the EU-15 from Table 2 in Carey and Rabesona (2002), we set $\tau_c = 0.178$, $\tau_w = 0.380$, and $\tau_k = 0.287$. And, second, we add “liquidity constrained” households. This is done in the much same way as in Section 2.4.1. However, we choose parameters to make $G_Y = 0.23$ and
$\frac{D}{Y} = 0.60$ in the steady state; these ratios are more representative of countries in the Euro Area.

### 2.6.2 Inflation Differentials and Growth Differentials in the NRCU Model

Figure 9 shows IRFs for government spending shocks in both the Ricardian model and the Non-Ricardian model. As noted in the last section, government spending shocks produce a positive correlation between inflation and growth differentials. But, in the RCU model, they crowd out consumption and have a relatively weak effect on aggregate demand. In the NRCE model, government spending shocks increase consumption and have a bigger impact on aggregate demand\(^\text{42}\). So, once again, the modifications of the Ricardian model seem to be having the intended effect.

<table>
<thead>
<tr>
<th></th>
<th>Combined asymmetric productivity shocks</th>
<th>Combined government spending shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth differential</td>
<td>13.5</td>
<td>86.5</td>
</tr>
<tr>
<td>Inflation differential</td>
<td>88.3</td>
<td>11.7</td>
</tr>
</tbody>
</table>

Table 17: Variance decompositions for the NRCU model (in percent)

Variance decompositions for the NRCU model are reported in Table 17, and they are somewhat encouraging. Comparing Table 17 with Table 15, government spending shocks and productivity shocks play a more balanced role, with government spending shocks have a big effect on growth differentials and productivity shocks having a big effect on inflation differentials. But in the end, productivity shocks still play the dominant role; the correlation between inflation differentials and growth differentials is -0.13, about what it was in the original RCU model.

\(^{42}\)Adding an output gap term to the interest rate rule attenuates the effect on consumption. For example, the rule in equation (2.11) implies the increase in consumption is about half the size, and only lasts 3 quarters.
2.7 Conclusion

In this paper, we investigated the ability of simple NNS models to capture stylized facts about the cyclical behavior of inflation and nominal interest rates. All of our models include monopolistic wage and price setting, Calvo style nominal inertia, and endogenous capital formation. In that sense, they are representative of the NNS paradigm.

The first set of stylized facts come from the U.S. data: interest rates and inflation are positively correlated with output. However, inflation and interest rates are negatively correlated with output in our Ricardian Closed Economy Model. We blamed this model failure on the dominance of productivity shocks, as evidenced by the model's variance decompositions. We tried adding private spending shocks, and we tried adding “rule of thumb” households to enhance the effect of government spending shocks on private consumption. Both of these experiments seemed to be steps in the right direction, but neither innovation seemed to resolve the problem fully.

The second set of stylized facts come from the early Euro experience: national inflation differentials are positively correlated with national growth differentials; this is seen in the cross sectional data presented in Figure 1 and in the French and German time series data. However, inflation differentials are negatively correlated with output differentials in our Ricardian Currency Union Model. Once again, we blamed this model failure on the dominance of productivity shocks. We tried adding private spending shocks, and we tried adding “rule of thumb” households to enhance the effect of government spending shocks on private consumption. And once again, both of these experiments seemed to be steps in the right direction, but neither innovation seemed to resolve the problem fully.

In the introduction, we noted that it is a matter of some concern in the Euro Area whether national inflation differentials are being driven by productivity shocks or by uncoordinated fiscal policies. If they are being driven by fiscal policies, then there may be a case for constraints like the Stability and Growth Pact; if they are being driven by productivity shocks, then there may be no need for such constraints. Initial results from the new NNS models seem to indicate that the
inflation differentials are being driven by productivity shocks. However, our analysis shows that it may be difficult for a model driven by productivity shocks to explain the positive correlation between inflation and growth differentials observed in the data. In our view, this promising new paradigm needs more work before it can give useful advice on such matters.
References


3 Optimal Monetary Policy with Non-Zero Net Foreign Wealth

3.1 Introduction

Interaction between exchange rates and the choice of monetary policy rules has been the focus of the recent open economy macro literature. Recent developments in the global financial flows (currency crises of the late 1990’s, rapid accumulation of U.S. government debt by East Asian countries) have also sparked interest in the choice of optimal monetary policy regime for developing countries. Calvo and Reinhart (2002) have documented the fact that many emerging economies exhibit "fear of floating," insofar as their exchange rates, quite independently of the official stance of the central banks, appear to fluctuate within a very narrow band. This observation has motivated many economists to study the question of whether exchange rate targeting could be an optimal policy choice for emerging markets.

This paper utilizes a standard two-country DSGE modeling framework to assess the importance of exchange rate targeting for developing economies, which in our model are characterized by an exogenous interest rate premium on their liabilities (denominated in foreign currency) and non-zero net foreign wealth holdings in the steady state. We abstract from the issues of central bank credibility and from any effects of the choice of monetary rules on the risk premium. The central question addressed is the choice of monetary policy that maximizes welfare of the emerging economy, given its structural features, the behavior of the foreign country, and exogenous disturbances.

The fact that strict inflation targeting is optimal only under a very restrictive set of parameters has been well-documented in the literature\textsuperscript{13}. Similarly, the majority of recent papers analyze monetary policy of a small open economy, a setup that carries with it two important implications: the real exchange rate is constant because consumption baskets of the two countries (or home country and the rest of the world) are identical, and foreign country is unaffected by the choice of monetary

\textsuperscript{13}See, for example, Faia and Monacelli (2007), Benigno and Benigno (2003a), and De Paoli (2004).
rule of the small economy. Therefore, the (often clear-cut) results of these papers may not be valid in a more realistic case of a small country having non-negligible impact on its trading partner.

The standard approach to answering the question of optimal monetary policy has been to rank the following three rules in terms of their implications for welfare and economic stability: CPI inflation targeting, PPI inflation targeting, and exchange rate peg44. We extend the analysis by including "hybrid" policies that target a weighted average of inflation and exchange rates. After calibrating the model based on a consensual set of parameter values, we find that our results are quite different from conventional wisdom in the literature that the policy of CPI inflation targeting welfare-dominates other options.

We show that central banks can exploit the risk-sharing value of international assets (more specifically, the degree of cyclicality of interest payments or receipts) by adjusting their monetary policy stance in response to the level of net foreign wealth. Thus, indebted countries can increase their interest payments to foreigners in good times (following, for example, a local productivity shock) by allowing for a sharper depreciation of their currency; analogously, net creditor countries optimally put relatively more weight on exchange rate targeting to reduce income receipts when domestic marginal utility is already low. These results are partly driven by the relative importance of various disturbances in our calibration: technology shocks tend to dominate the New-Keynesian models. Our conclusions, however, seem to be robust to the inclusion of other common sources of uncertainty (government and demand shocks).

The results of this paper add to the current debate on the optimal monetary policy in the open economy setting. Studies like Obstfeld and Rogoff (1995, 2002) have sparked interest in the ability and advisability of monetary policy influencing exchange rates in the context of fully articulated DSGE models. Galí and Monacelli (2005) lay out the theoretical framework to analyze the choice

of monetary policy for a small open economy and present a special case for which domestic inflation targeting constitutes the optimal regime. Cova and Søndergaard (2005), Benigno and Benigno (2003), and Faia and Monacelli (2007), among others, contemplate the terms of trade channel through which monetary policy can increase welfare of its domestic residents.

The rest of the paper is organized as follows. Section 3.2 introduces the model and discusses the monetary policy rules under consideration. Section 3.3 presents the values, used in simulations, of the model parameters and discusses implications associated with particular choices of these values on the results of the paper. Section 3.4 analyzes the impact of non-zero steady-state international debt on the choice of welfare-maximizing policy rules. Section 3.5 summarizes our findings and lists several extensions for future research.

3.2 The Model

The model belongs to the class of DSGE models that are commonly used for evaluating effects of different monetary policies in both closed and open economy settings.

The model is composed of two countries, home (H) and foreign (F). Both countries are populated by infinitely lived households of measure $M$ in the home country and $M^*$ in the foreign$^{45}$; there is no migration. Households consume all varieties of home and foreign goods and have access to international markets where they can trade a nominal bond. Each country has measure one of firms that use country-specific labor and capital to produce a continuum of domestic goods, which are then traded internationally. Firms are monopolistically competitive, and the prices they set for their products are sticky à la Calvo (1983). International goods markets are segmented because firms set prices in local currency; therefore, the Law of One Price (LoOP) fails.

The key features of the model are (1) the requirement that foreign liabilities of Home (developing) country are denominated in foreign currency; (2) non-zero steady state net foreign wealth of the

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$^{45}$See Mykhaylova (2008) for a detailed analysis of the explicit modeling of relative country size.
two countries; and (3) local currency pricing.

The first two assumptions are driven by the data. Local currency pricing (which results in incomplete exchange rate pass-through) is motivated by two observations. On the empirical side, the LoOP has been shown to fail when applied to tradable goods (for example, Rogoff (1996)); on the theoretical side, it adds another channel through which the central bank can affect the real exchange rate and, consequently, relative consumptions of the two countries (see section 3.4.1 for details.)

For modeling simplicity, we assume that all goods are traded. As a matter of notation, subscripts \(H\) and \(F\) will refer to a good’s country of origin; asterisks will indicate that it is consumed in country \(F\). For example, \(C^*_H\) denotes consumption of country \(H\)’s good in country \(F\). The two economies have a similar structure; therefore, most of the equations will be presented only for the Home country.

3.2.1 Firms

In this paper, we explicitly model international price discrimination by firms, which may optimally choose to charge different prices at home and abroad, depending on their expectations of the future nominal exchange rates.

Each country has a continuum of firms indexed by \(f\) on the unit interval. At time \(t\), each Home firm rents capital \(K_{t-1}(f)\) from the domestic households at the rate \(R_t\), hires a labor bundle \(N_t(f)\) at the rate \(W_t\) and produces one of the varieties of the domestic good. Each firm is free to set its own price level \(P_{H,t}(f)\) (denominated in local currency) at home and \(P^*_{H,t}(f)\) (denominated in foreign currency) abroad.

Home firms use the following Cobb-Douglas technology to produce output:

\[
Y^*_H(f) = Z_t K_{t-1}(f)^\nu N_t(f)^{1-\nu},
\]

where \(0 < \nu < 1\), and \(Z_t\) denotes the level of productivity enjoyed by all the home firms at time \(t\).
Productivity in the two countries evolves according to the following autoregressive process:

\[
\begin{bmatrix}
\ln Z_t \\
\ln Z^*_t
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
\ln Z_{t-1} \\
\ln Z^*_{t-1}
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{z,t} \\
\varepsilon^*_{z,t}
\end{bmatrix}
\]

All goods varieties are then bundled into a composite home and foreign goods using the Dixit-Stiglitz aggregator:

\[
Y^*_H, t = \left[ \int_0^1 Y^*_H(f)^{\frac{1-\sigma}{-\sigma}} df \right]^{-\frac{1}{\sigma}}
\]

where \( \sigma > 1 \). This composite good can then be used for public and private consumption or private investment.

Imposing the zero-profit condition, the Home and Foreign prices of the bundle are given by

\[
P_H, t = \left[ \int_0^1 P_H(f)^{1-\sigma} df \right]^{-\frac{1}{\sigma}}
\]

and therefore demand for firm \( f \)'s product is

\[
Y^d_H, t(f) = \left[ \frac{P_H(f)}{P_H^*} \right]^{-\sigma} Y_H, t
\]

\[
Y^{*d}_H, t(f) = \left[ \frac{P_H^*(f)}{P_H} \right]^{-\sigma} Y_H^*, t
\]

Here \( Y_H, t \) and \( Y^*_H, t \) are the aggregate demands for home and foreign goods bundles from households, investors and governments.

As in Calvo (1983), home and foreign firms reset their prices each period with a constant probability \( (1 - \alpha) \) and \( (1 - \alpha^*) \), respectively; otherwise, the old prices remain in effect. If a (home) firm \( f \) gets to announce a new price in period \( t \), it chooses \( \hat{P}_H, t(f) \) and \( \hat{P}_H^*(f) \) to maximize its expected discounted future profits

\[
E_t \sum_{j=t}^{\infty} \Omega_{t,j} \alpha^{j-t} \left\{ \hat{P}_H, t(f) Y_H, j, t(f) + S_j \hat{P}_H^*(f) Y^{*d}_H, j, t(f) - TC_j \left[ Y_H, j, t(f) + Y^{*d}_H, j, t(f) \right] \right\}
\]

\( \Omega_{t,j} \) is the home households’ stochastic discount factor: \( \Omega_{t,j} = \beta^{j-t}(\frac{\lambda}{\lambda_j}) \), where \( \lambda \) is the marginal utility of nominal wealth (defined in section 3.2.2). Notice that in equilibrium, \( Y^d_H, t(f) + Y^{*d}_H, t(f) = Y^*_H, t(f) \).
The optimal price is given by

\[ \hat{P}_{H,t} = \frac{P_{B,H,t}}{P_{A,H,t}} \]

\[ \varphi_p = \frac{\sigma}{\sigma - 1} \]

\[ PB_{H,t} = \alpha \beta E_t [PB_{H,t+1}] + \lambda_t P^\sigma_{H,t} Y_{H,t} MC_{H,t} \]

\[ PA_{H,t} = \alpha \beta E_t [PA_{H,t+1}] + \lambda_t P^\sigma_{H,t} Y_{H,t} \]

and

\[ \bar{P}^*_{H,t} = \frac{P^*_{B,H,t}}{P^*_{A,H,t}} \]

\[ PB^*_{H,t} = \alpha \beta E_t [PB^*_{H,t+1}] + \lambda_t P^*\sigma_{H,t} Y^*_{H,t} MC_{H,t} \]

\[ PA^*_{H,t} = \alpha \beta E_t [PA^*_{H,t+1}] + \lambda_t P^*\sigma_{H,t} Y^*_{H,t} S_t \]

The nominal marginal cost of production is

\[ MC_t = \frac{R^\nu_{t} W_t^{1-\nu}}{Z_t \nu (1-\nu)^{1-\nu}} \]

Notice that in the presence of price stickiness, Home firms optimally take into account the entire path of future expected nominal exchange rates \( S_t \) when setting foreign prices for their products; the LOOP need not hold.

We consider a symmetric equilibrium in which every firm that gets a chance to reset its prices in period \( t \) will set it to the same value; therefore, optimal prices are not denoted by a firm subscript \( (f) \). As \( \alpha \to 0 \) and prices become perfectly flexible, each period firms set two new prices

\[ \hat{P}_{H,t} = S_t \hat{P}^*_{H,t} \to \varphi_p MC_{H,t} \]

Note that LOOP holds in this case. Since the monopolistic markup \( \varphi_p > 1 \), prices are set above the marginal cost and so output is inefficiently low.

Given the price-setting behavior of individual firms, the aggregate price indices of the Home
goods at home and abroad can be written as

\[ P_{H,t}^{1-\sigma} = (1 - \alpha) \hat{P}_{H,t}^{1-\sigma} + \alpha P_{H,t-1}^{1-\sigma} \]

\[ P_{H,t}^{*1(1-\sigma)} = (1 - \alpha) \hat{P}_{H,t}^{*1(1-\sigma)} + \alpha P_{H,t-1}^{*1(1-\sigma)} \]

### 3.2.2 Households

There is a continuum of households in the home country, indexed by \( i \) on the interval \([0, M]\). A representative household maximizes expected lifetime utility \( 46 \)

\[ U_t(h) = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \frac{C_j(h)^{1-\sigma}}{1-\sigma} - \frac{L_j(h)^{1+\chi}}{1+\chi} \right\} \]

(3.1)

Here \( C_t(h) \) denotes the household’s consumption of the composite good, which is aggregated from home and foreign goods using the CES aggregator:

\[ C_t(h) = \left[ \mu_t^{\frac{1}{\eta}} C_{H,t}(h)^{\frac{n-1}{\eta}} + (1 - \mu_t)^{\frac{1}{\eta}} C_{F,t}(h)^{\frac{n-1}{\eta}} \right]^{\frac{\eta}{n-\eta}} \]

(3.2)

\( \eta \) is the elasticity of substitution between home and foreign goods, and \( 0 < \mu_t < 1 \) determines the degree of home bias in consumption. \( C_t^*(h) \) denotes consumption of the aggregate good (with a different home bias \( \mu_t^* \)) in the foreign country.

The specification of consumption aggregator (3.2) breaks away from the standard simplification in the open economy literature of equating relative country size and share of domestic good in the consumption basket (i.e., the assumption that small countries consume mostly imported goods) for several reasons. On the empirical side, home bias in consumption has been well documented in the trade literature (for example, Obstfeld and Rogoff (2000)). Moreover, by allowing for a different composition of Home and Foreign baskets, we decouple movements in the two countries’ CPI levels and thus introduce richer dynamics of the real exchange rate, which has non-trivial implications for the choice of optimal monetary policy, as noted in Faia and Monacelli (2007) and is discussed in

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\( ^{46} \)Following Woodford (2003), Chapter 3, we model monetary policy as directly targeting interest rates, inflation or exchange rates, and therefore drop real balances from the consumer utility function.
detail in Section 3.4.1. Finally, introducing a stochastic process for the home bias parameter $\mu$ will allow us to model demand (preference) shocks.

The prices of the two final goods, which also represent the countries’ CPIs, are given by

\[
\begin{align*}
P_t &= \left[ \mu_t P_{H,t}^{1-\eta} + (1 - \mu_t) P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \\
P_t^* &= \left[ \mu_t^* P_{F,t}^{\phi(1-\eta)} + (1 - \mu_t^*) P_{H,t}^{\phi(1-\eta)} \right]^{\frac{1}{\phi(1-\eta)}}
\end{align*}
\] (3.3) (3.4)

Given these prices, the household $(h)$ demands the following quantities of the composite goods:

\[
C_{H,t}^d(h) = \mu_t \left[ \frac{P_{H,t}}{P_t} \right]^{-\eta} C_t(h), \quad C_{F,t}^d(h) = (1 - \mu_t) \left[ \frac{P_{F,t}}{P_t} \right]^{-\eta} C_t(h)
\]

Households supply differentiated labor services to all the firms in their country. The composite labor bundle used in production by any given home firm is given by$^{47}$

\[
N_t(f) = M^{\frac{1}{1-\phi}} \left[ \int_0^M L_t(h, f) \left( \frac{W_t}{W} \right)^{\frac{\phi-1}{\phi}} dh \right]^{\frac{\phi}{\phi-1}}
\]

Each household enjoys a degree of monopolistic power in setting its wage $W_t(h)$. The demand for household $(h)$’s labor services given its wage is

\[
L_t(h) = \left[ \frac{W_t(h)}{W} \right]^{-\phi} \frac{N_t}{M}
\] (3.5)

Here $N_t = \int_0^1 N_t(f) df$ is the aggregate demand for the household’s labor services from all firms in the economy, and $W_t$ is the aggregate real wage:

\[
W_t = M^{\frac{1}{1-\phi}} \left[ \int_0^M W_t(h)^{1-\phi} dh \right]^{\frac{1}{1-\phi}}
\]

$^{47}$The scaling factor $M^{\frac{1}{1-\phi}}$ is necessary to maintain the aggregate relationship $N_t = \int_0^1 N_t(f) df = ML_t(h)$. Together with the expression $K_t = MK_t(h)$ this will ensure that the production function exhibits constant returns to scale. Additionally, in the steady state the aggregate wage $W$ will equal the individual wage $W(h)$.
Each household faces the following budget constraint:

\[ E_t[\Delta t_{t+1} D_t(h)] + B^d_t(h) + S_t A_t(h) + P_t [C_t(h) + I_t(h) - T_t(h)] = \]

\[ = W_t(h) L^d_t(h) + D_{t-1}(h) + (1 + i_{t-1}) B^d_{t-1}(h) + [1 + i^*_t - p(e^{a_t - 1} - 1)] S_t A_{t-1}(h) + \]

\[ R_t K_{t-1}(h) + \Pi_t(h) \]

The first term on the left-hand side is the price of a portfolio of state-contingent bonds traded domestically, and \( D_{t-1} \) is the payoff of such portfolio in period \( t \). \( B^d_t(h) \) represents household’s demand for the riskless one-period nominal domestic government bond\(^{48}\). Households receive transfers \( P_t T_t(h) \) from their government (which can be negative in the event of lump-sum taxation). \( \Pi_t(h) \) represent household’s dividend income.

The modeling of incomplete markets is borrowed from Benigno (2001) and Andrés et al (2006).

Home households can buy the foreign bond \( A_t(h) \), but at a different price than foreign households. Home consumers’ price \( S_t [1 + i^*_t - p(e^{a_t - \bar{a}} - 1)]^{-1} \) depends on their position in the international asset market. Here \( a_t \equiv M \frac{S_t A_t}{L^d_t Y_t} \) is the ratio of the aggregate real foreign asset holdings by home consumers to domestic output, \( \bar{a} \) is the steady state level of \( a_t \), and the parameter \( p \) captures transaction costs\(^{49}\). As lenders \( (a_t > 0) \), domestic households pay a higher price for the bond, and as borrowers they must offer a rate of return higher than \( (1 + i^*_t) \). Thus, consumers cannot perfectly share consumption risk internationally. Since the bond is denominated in foreign currency, its domestic value depends on the nominal exchange rate \( S_t \).

The household’s capital accumulation is given by

\[ K_t(h) = (1 - \delta) K_{t-1}(h) + I_t(h) \quad (3.7) \]

\(^{48}\)In the presence of the complete set of Arrow securities \( D_t \), government bonds are redundant for the purposes of risk-sharing; we introduce them to model the dynamics of national debt.

\(^{49}\)Equilibrium dynamics of a small open economy with incomplete asset markets generally include a random walk component; the transaction cost modification guarantees stationary of the model. See Schmitt-Grohé and Uribe (2003) for explicit treatment of the problem.
Here, the investment good \( I_t(h) \) has the same composition as consumption in (3.2), and demands for Home and Foreign bundles are given by

\[
I^d_{H,t}(h) = \mu_t \left[ \frac{P_{H,t}}{P_t} \right]^{-\eta} I_t(h), \quad I^d_{F,t}(h) = (1 - \mu_t) \left[ \frac{P_{F,t}}{P_t} \right]^{-\eta} I_t(h)
\]

Households maximize utility (3.1) subject to the budget constraint (3.6), labor demand (3.5) and capital accumulation constraint (3.7) by choosing wage rate \( W_t(h) \), consumption \( C_t(h) \), portfolio holdings \( D_t(h), B^d_t(h) \) and \( A_t(h) \), and investment \( I_t(h) \).

Wages are sticky, and in any given period a household gets to reset its wage with probability \( (1 - \omega) \) \(((1 - \omega^*) \text{ abroad})\). The optimal new wage satisfies

\[
\dot{W}_t = W B_t \frac{W B_t}{W A_t} \\
\hat{\varphi}_w = \frac{\phi}{1 - \phi} \\
W B_t = W_t^{\phi(1+\lambda)} \left( \frac{N_t}{M} \right)^{1+\lambda} + (\omega \beta) E_t[W B_{t+1}] \\
W A_t = \lambda_t W_t^{\phi} \left( \frac{N_t}{M} \right) + (\omega \beta) E_t[W A_{t+1}]
\]

Assuming that every household that chooses its wage in period \( t \) sets it to the same new value, we drop the subscript \((h)\) on the optimal wage rule. If \( \omega = 0 \) (flexible wages), it follows that \( \lambda_t \dot{W}_t = \varphi_w L_t(\chi) \): households do not set their marginal disutility of work equal to the marginal product of labor, which creates an inefficiency in their (too low) labor allocation.

Similar to the derivations of the aggregate price level given firms’ first-order conditions, the aggregate wage level is given by

\[
W_t^{1-\phi} = (1 - \omega) \dot{W}_t^{1-\phi} + \omega W_{t-1}^{1-\phi}
\]

The rest of the first order conditions (with respect to consumption, domestic and foreign bond
holdings, investment, and capital stock, in that order) for the household problem are as follows:

\[
\frac{1}{C_t^P P_t} = \lambda_t \quad (3.8)
\]

\[
\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \right] = \frac{1}{1 + i_t} \quad (3.9)
\]

\[
1 = \beta E_t \left[ 1 + i_t^* - p (e^{\sigma_t} - 1) \right] \frac{\lambda_{t+1} S_{t+1}}{\lambda_t S_t} \quad (3.10)
\]

\[
\lambda_t P_t = \xi_t \quad (3.11)
\]

\[
\xi_t = \beta E_t \left[ \lambda_{t+1} R_{t+1} + \xi_{t+1} (1 - \delta) \right] \quad (3.12)
\]

Equations (3.8) and (3.9) describe intertemporal consumption choice of the household, while equation (3.10) gives the demand for the international bond. Equations (3.11) and (3.12) result from the optimal investment decision.

In a symmetric equilibrium (and assuming all households in a country start off with the same level of wealth), their \( \lambda_t(h) \) and \( C_t(h) \) will be equalized.

### 3.2.3 The Government

The aim of this paper is to analyze the effects of various monetary policy rules on consumer welfare. We do not address the question of monetary policy cooperation; instead, we assume that the central bank of the Foreign country credibly targets domestic CPI inflation (so that \( \pi_t^{CPI} = 0 \) at all times), and look for the welfare-maximizing rule that can be adopted by the Home monetary authority.

The first three monetary policies we consider (and which are analyzed in most of the related literature) can be generally written as

\[
X_t = 0, \text{ for } X_t \in \{ \pi_t^{CPI}, \pi_t^{PPI}, \Delta s_t \}
\]

Here \( \pi_t^{CPI} \) and \( \pi_t^{PPI} \) denote Home country’s CPI and PPI inflations, respectively, and \( \Delta s_t \) measures (log) depreciation of the domestic currency. Alternatively, the Home central bank can adopt a
variation of the Taylor rule commonly used in monetary literature:

\[ i_t = (1 - \rho_1) \bar{i} + \rho_1 i_{t-1} + (1 - \rho_i) \left[ \varpi \pi_t^{CPI} + (1 - \varpi) \Delta s_t \right] \]  

(3.13)

Here \( \bar{i} = \frac{1}{\beta} - 1 \) is the steady state level of the interest rate, and \( \varpi \) indicates the relative weight on inflation vs. exchange rate targeting. We study a range of values for this parameter: \( \varpi \in [0, 1] \). Section 3.3 provides more detail on the calibration of the model.

The Home country fiscal authority has the following budget constraint:

\[ B_t^* = (1 + i_{t-1}) B_{t-1}^* + P_t G_t + P_t T_t \]

Here \( G_t \) denotes per-capita government purchases, which have the same composition as consumption in (3.2).

Each government can affect the functioning of its domestic economy through transfers and purchases. We let government purchases be described by an autoregressive process

\[ \ln G_t = (1 - \rho_g) \ln G_{t-1} + \rho_g \ln G_{t-1} + \varepsilon_{g,t}, \]

and shocks to government purchases are uncorrelated white noise processes\(^{50}\). Transfers react to the debt-to-GDP ratio to ensure long-run fiscal solvency:

\[ \ln T_t = (1 - \rho_{tr}) \ln T_{t-1} + \rho_{tr} \ln T_{t-1} + \rho_b (\ln B - \ln B_{t-1}) \]

In the above equations bars denote steady state values of variables.

### 3.2.4 Aggregation and Equilibrium

Since the measure of households in the two countries is not one, aggregate and per-capita quantities will be different in equilibrium: aggregate consumption is a multiple of the per-capita value

\[ C_t = \int_0^M C_t(h) dh = MC_t(h) \]

\(^{50}\)Adding government spending shocks moves the economy farther away from the complete markets allocation and makes monetary policy more relevant for consumer welfare.
Similar expressions link aggregate and per-capita investment and capital.

Given the above expressions for the aggregate price and wage levels, the aggregate output can be rewritten in terms of aggregate capital stock $MK_{t-1} \equiv \int_0^1 K_{t-1}(f)df$ and the economy-wide labor input $ML_t \equiv \int_0^1 N_t(f)df$:

\[
MZ_tK_{t-1}^{1-\nu} = Y_{H,t}DP_{H,t} + Y_{H,t}^*DP_{H,t}^*
\]

\[
DP_{H,t} = \int_0^1 \left[ \frac{P_{H,t}(f)}{P_{H,t}} \right]^{-\sigma} df = (1 - \alpha) \left[ \frac{\hat{P}_{H,t}}{P_{H,t}} \right]^{-\sigma} + \alpha \left[ \frac{P_{H,t}}{P_{H,t-1}} \right]^\sigma DP_{H,t-1}
\]

\[
DP_{H,t}^* = (1 - \alpha) \left[ \frac{\hat{P}_{H,t}^*}{P_{H,t}} \right]^{-\sigma} + \alpha \left[ \frac{P_{H,t}}{P_{H,t-1}} \right]^\sigma DP_{H,t-1}^*
\]

where the last two equations measure the aggregate dispersion of prices in the economy.

Goods markets must clear in equilibrium:

\[
Y_{H,t} + Y_{H,t}^* = M \left( C_{H,t} + I_{H,t} + G_{H,t} \right) + M^* \left( C_{H,t}^* + I_{H,t}^* + G_{H,t}^* \right)
\]

\[
Y_{F,t} + Y_{F,t}^* = M \left( C_{F,t} + I_{F,t} + G_{F,t} \right) + M^* \left( C_{F,t}^* + I_{F,t}^* + G_{F,t}^* \right)
\]

Equilibrium in the asset markets requires that

\[
\int_0^M D_t(h)dh = \int_0^{M^*} D_t^*(h)dh = 0
\]

\[
\int_0^M B_t^d(h)dh = MB_t^*
\]

\[
\int_0^M A_t(h)dh + \int_0^{M^*} B_t^d(h)dh = M^*B_t^{**}
\]

By combining budget constraints of all Home households with aggregate Home firms’ profits and Home government’s budget constraint we obtain the expression for the Home balance of payments:

\[
\int_0^M A_t(h)dh = MA_t = \left[ 1 + \xi_{t-1} - p(e^{\alpha_{t-1} - \pi} - 1) \right] MA_{t-1} + P_{H,t}^*M^* \left[ C_{H,t}^* + G_{H,t}^* + I_{H,t}^* \right] -
\]

\[
-\frac{P_{F,t}}{S_t}M \left[ C_{F,t} + G_{F,t} + I_{F,t} \right]
\]
3.2.5 Measure of National Welfare

We define the value function that measures national welfare as

\[ V_t = \max E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \frac{MC_j^{1-\Theta}}{1-\Theta} - \frac{1}{1+\chi} AL_j \right\}, \]

where \( AL_j = \int_0^M L_j(h)^{1+\chi}dh \) measures the aggregate disutility of work. This function will allow us to make quantifiable comparisons of consumer welfare across different specifications of the two economies (for example, \( V_t \) measures welfare of a flexible price economy, and \( \tilde{V}_t \) captures the case of sticky prices). In the case of log utility, the difference between the two value functions,

\[ \epsilon = V_t - \tilde{V}_t \]

can be interpreted as cost, expressed as percent of consumption, of moving away from the flexible price specification to (in the above example) the economy with nominal rigidities.

3.3 Parametrization

Each time period in the model corresponds to one quarter. The benchmark parameter values are described in Table 18. Our goal was to choose the most non-controversial set of parameter values to illustrate our results. Therefore, most of the reported values are standard in the literature; a few others merit further description. Unless otherwise indicated, all parameters describing the Foreign economy are identical to the ones in the Home country.

We assume log utility of consumption, and set \( \chi = 3 \), which implies a labor supply elasticity of \( \frac{1}{3} \). We set \( \beta = 0.99 \), which produces a steady state riskless annual return of 4 percent. Cobb-Douglas parameter \( \nu \) in the production function is set to 0.33. Calibration of productivity processes is borrowed from Backus, Kehoe and Kydland (1993), with \( Var(\varepsilon_z) = Var(\varepsilon_z^*) = 0.000074 \), and \( Cov(\varepsilon_z, \varepsilon_z^*) = 0.0000185 \). Elasticities of substitution between goods varieties, \( \sigma \), and labor varieties, \( \phi \), result in price markup of 14 percent and wage markup of 17 percent, respectively.

\(^{51}\)A more detailed explanation and derivations of this value function can be found in Mykhaylova (2008).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta$</td>
<td>Relative risk aversion</td>
<td>1.0</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Inverse of Frisch labor elasticity</td>
<td>3.0</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Consumption home bias (domestic good)</td>
<td>0.75</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of substitution between home and foreign goods</td>
<td>1.5</td>
</tr>
<tr>
<td>$A$</td>
<td>Matrix of technology coefficients</td>
<td>$\begin{pmatrix} 0.90 &amp; 0.09 \ 0.09 &amp; 0.90 \end{pmatrix}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Investment adjustment cost</td>
<td>8.0</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Elasticity of substitution between goods varieties</td>
<td>8.0</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Elasticity of substitution between labor varieties</td>
<td>7.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Price stickiness</td>
<td>0.67</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Wage stickiness</td>
<td>0.75</td>
</tr>
<tr>
<td>$p$</td>
<td>Risk premium on int’l borrowing/lending</td>
<td>0.001</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Interest rate inertia in the Taylor Rule</td>
<td>0.9</td>
</tr>
<tr>
<td>$\varpi$</td>
<td>Relative weight on CPI inflation</td>
<td>[0; 1]</td>
</tr>
</tbody>
</table>

Table 18: Benchmark parameter values
We follow Canzoneri et al (2007) in setting the value of nominal price and wage rigidities: \( \alpha = 0.67 \) and \( \omega = 0.75 \), resulting in price and wage contracts that on average last three and four quarters, respectively.

We set the home bias parameter \( \mu = 0.75 \) for all specifications of the relative country sizes. However, we must adjust the parameter \( \mu^* \) as the Foreign country becomes progressively larger, to avoid disproportionate demand on either country’s output; otherwise, discrepancy in steady state unemployment and wages would complicate welfare comparisons\(^{52}\).

There is no agreement in the literature on the appropriate value for the elasticity of substitution between Home and Foreign goods, \( \eta \). However, recent studies report the value of \( \eta \) between 1.5 and 2; see Faia and Monacelli (2007) and papers cited therein. We follow Monacelli (2003) and Backus, Kehoe and Kydland (1993) in setting this parameter equal to 1.5. The non-trivial implication of this choice of \( \eta \) (in conjunction with log-utility of consumption) is that Home and Foreign bundles in (3.2) are substitutes. We perform a robustness check by running the simulations with \( \eta = 0.9 \) (complements), and find that our conclusions are robust to this change in the parameter value.

The cost of participating in the international asset markets, \( p \), is set to \( 10^{-3} \), as in Benigno (2001).

3.3.1 Government Policies

It is a well-known and documented fact that many countries have non-zero net holdings of foreign assets\(^{53}\). In such cases, the policy of producer or consumer price stability may no longer be optimal, since nominal exchange rate fluctuations may have significant impact on the interest rate spread and the amount of interest paid on international loans.

The ratio of government debt to GDP has to be stationary in the model; fiscal policy, therefore,\(^{52}\) More specifically, we require that the following relationship holds in all specifications of the model: \( M \mu + M^* (1 - \mu^*) = M \) (and, similarly, \( M(1 - \mu) + M^* \mu^* = M^* \)).

\(^{53}\)Lane and Milesi-Ferretti (2001, 2005) present evidence on NFW positions of developing and industrial countries.
must respond in some way to either deficit or debt. In the model we let transfers respond to the deviation of debt ratio from its steady-state value, which we set at 20 and 50 percent of GDP. We adjust the level of transfers as necessary to achieve this debt-to-GDP ratio; transfers play the least distortionary role in our model (they act as a lump-sum tax on consumers), and so adjusting their value has minimum impact on the rest of the model.

The parameters of the government policy functions are set as follows: \( \rho_g = \rho_{tr} = 0.9 \) and \( \text{Var}(\varepsilon_g) = 0.0082 \); see Mykhaylova (2008) for details of estimation. Responsiveness of transfers to the level of debt \( \rho_b \) was set to 0.1 in order to satisfy the Blanchard-Kahn conditions.

Solution to the model is found using perturbation methods described in Schmitt-Grohé and Uribe (2004) and Collard and Juillard (2001); computer code is written in Dynare (Collard and Juillard (2003)). Moments, variance decompositions and impulse response functions presented below were computed using first-order approximations; value functions were computed using second order approximations.

### 3.4 Welfare Results

We consider three different specifications of the relative country size: symmetric case, and Foreign country being four and then nineteen times the size of the Home economy. Recall, however, that in all specifications a large portion \((\mu = 0.75)\) of Home consumer basket consists of domestically produced goods, which makes our analysis different from the standard small open economy setup. All numbers are reported for the Home country, since the policy of the Foreign central bank is taken to be exogenous.

Before presenting the results of our simulations, it is worth noting the forces behind welfare numbers. The first two economic inefficiencies characterizing our model are nominal rigidities, which lead to inefficient allocation of goods produced with the same technology and call for at least

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54 As a matter of reference, the U.S. national debt is close to 70% of GDP as of October 2008.
some degree of price stabilization; and monopolistic competition, which results in suboptimal levels of output and work effort and introduces an incentive for the central bank to pursue expansionary policy. In addition, the failure of the aggregate PPP introduces a channel through which monetary authority can affect domestic consumption by manipulating the real exchange rate and terms of trade.

For each relative size specification, we study the impact of international asset holdings on the welfare-maximizing choice of monetary policy rule. As a benchmark (and for easier comparison with the existing literature), we consider the no-debt steady state. We then discuss welfare implications of non-zero holdings of foreign assets or liabilities on the optimal monetary regime.

3.4.1 Benchmark Zero-Debt Case

We find that in all three size specifications, welfare-maximizing policy (among the class of policies we study in the paper) takes the form of the Taylor Rule with a lagged interest-rate term and a weighted average of CPI inflation and exchange rate depreciation, as in equation (3.13).

The first (non-controversial) finding is that adding a lagged interest rate term to the monetary policy rule improves welfare numbers. Thus, pursuing the policy described by \( \pi_t^{CPI} = 0 \) is welfare-dominated by the rule of the form \( i_t = (1 - \rho_t)^i + \rho_i i_{t-1} + (1 - \rho_t) \pi_t^{CPI} \); the same holds true for pure exchange rate targeting. In a forward-looking model, increasing interest rate persistence means that stabilization (of inflation, output, exchange rate or any other target of the central bank) requires a much smaller movement of the time-\( t \) interest rate, since such movement is expected to prevail far into the future. Lower volatility of interest rates, in turn, implies lower volatility of consumption, output, and labor effort, and thus higher consumer welfare. This result has been discussed in great detail in Woodford (1999), and has also been reported more recently by Senay (2007).

Our second result is that the optimal policy in the majority of considered specifications is a mix of CPI and exchange rate targeting; more specifically, in equation (3.13) \( \varpi \) belongs to the open interval
(0, 1). There are several reasons (documented in literature) why central bank in a New-Keynesian model should pay some attention to the exchange rate movements. As noted in Faia and Monacelli (2007) and De Paoli (2004), the presence of home bias in the international consumption aggregator (3.2) generates endogenous movements of the real exchange rate in response to the actions of the monetary authority aimed at manipulating terms of trade; real exchange rate, in turn, affects relative consumptions in the two countries\textsuperscript{55}. To see this, we substitute equations (3.3) and (3.4) into the definition of the real exchange rate $Q_t = \frac{S_t P^*_t}{P_t}$ and linearize around $P^*_t = P^*$ and $P_{F,t} = P_{H,t}$:

$$q_t = [\mu_t^* + \mu_t - 1] \tau_t + \mu_t \varepsilon_t^H + \mu_t \varepsilon_t^F,$$

(3.14)

where $\tau_t \equiv p_{F,t} - (p_{H,t}^* + s_t)$ is the terms of trade, and we’ve let $\varepsilon_t^H$ and $\varepsilon_t^F$ measure deviations from the LoOP arising due to local currency pricing coupled with nominal rigidities: $\varepsilon_t^F = p_{F,t}^* + s_t - p_{F,t}$ and $\varepsilon_t^H = p_{H,t}^* + s_t - p_{H,t}$.

As can be seen from equation (3.14), these deviations create an additional channel through which nominal exchange rates can affect consumer welfare (again, through its impact on relative consumptions in the two countries). As discussed in detail in Monacelli (2003), the presence of imperfect exchange rate pass-through calls for an optimal management of nominal exchange rates by the central bank.

3.4.2 Non-Zero NFW and the Timing of Payments

Having established the fact that some degree of nominal inertia is beneficial, we now turn to the main question of this paper: the optimal weight on CPI targeting vs. exchange rate targeting in the policy rule of the central bank (the parameter $\varpi$ in equation (3.13)). One of the contributions

\textsuperscript{55}International asset markets are incomplete, so we do not get the usual link between home and foreign consumption levels, $C_t = Q_t C_t^*$. However, the degree of international risk-sharing remains very high: $\text{Corr}(c_t - c_t^*, q_t)$ remains above 0.95 in most cases and dropping down to 0.50-0.60 when Home country holds claims on Foreign country in the steady state.
The main results of our simulations are presented in Table 19. The second column presents the optimal (consumer welfare-maximizing) weight on CPI targeting for the three size specifications; columns III and V show how the stance of the monetary policy should be changed when countries accumulate foreign liabilities (in the amount of 50% of their GDP) or foreign assets, respectively. Finally, columns IV and VI show the welfare gain that can be achieved by shifting from the benchmark policy rule (column II) towards the new optimal policy (column III or V, respectively).

Rather provocatively, our welfare calculations suggest that a country that carries a large amount of dollarized foreign debt should pay more attention to targeting CPI inflation than a country with no foreign debt. Countries that have large holdings of foreign assets, on the other hand, can maximize their consumers’ welfare by shifting towards exchange rate targeting. This finding goes contrary to the usual wisdom that it is beneficial to highly indebted countries to lower their exchange rate volatility and therefore stabilize the value of foreign payments. Calvo and Rainhart (2002), for example, present a partial equilibrium model in which the tendency of countries to target the exchange rate is driven by shocks to the international risk premia, an ad-hoc objective function.
of the central bank that in quadratic in inflation deviations, and high pass-through of exchange rates into local prices. Our study, in contrast, explicitly models the trade and financial interactions of the two economies and encompasses a wider array of disturbances, with productivity shocks accounting for most of the variance decomposition of variables such as domestic consumption and labor, the two inputs of the micro-based consumer welfare function\textsuperscript{56}.

Intuition for our results can be gained by considering the risk-sharing service offered by the international asset. Any asset whose payoff covaries negatively with consumption offers insurance to Home consumers, and is therefore more desirable\textsuperscript{57}.

In order to understand how a shift in the monetary policy stance in our model can achieve lower correlation between Home consumption and the payoff of the international bond, consider the expression for real income receipts (or payments) on net foreign wealth:

\[
\text{Receipts} = NFW_{t-1} \left[ 1 + i^*_t - p \left( e^{a_{t-1}} - 1 \right) \right] \frac{\Delta S_t}{\Pi_t} - 1
\]

Here \(NFW_t = \frac{MA_t S_t}{P_t}\) denotes real net foreign wealth of the Home country, expressed in Home consumption units.

We start by considering the effects of productivity shocks on the Home economy; Figures 10 and 11 show the impulse response functions of several key variables to this shock for a net creditor country (China, for example) and a net debtor country (Argentina) under different monetary policy rules. To take advantage of a positive technological shock, which is accompanied by a drop in the Home marginal utility of consumption, a debtor nation \((NFW < 0)\) would prefer to temporarily increase its interest payments to foreigners. This can be done immediately by allowing a sharp depreciation of the nominal exchange rate, i.e., by shifting the focus of monetary policy away from pegging and towards inflation targeting. Conversely, a country with positive holdings of foreign assets \((NFW > 0)\) would prefer to dampen its currency depreciation following an increase in home productivity so as to lower

\textsuperscript{57}See Cochrane (2001), Chapter 1 for an in-depth discussion.
Table 20: Correlation between Home consumption and debt service for debtor (1st row) and creditor (2nd row) countries: benchmark (zero-debt) vs. optimal policy.

interest income from foreigners when Home marginal utility of consumption is already low. Table 20 offers further evidence in support of this claim: optimally deviating from the benchmark policy (of the no-debt scenario) decreases correlation between domestic consumption and debt service for both debtor and creditor nations.

The strength of this channel (which affects the timing of payments and thus enhances the risk-sharing feature of the international bond) depends on the relative importance of the various shocks present in our model. In the next section, we discuss the effects of adding a shock to preferences for Home good and increasing the risk premium on the optimal policy and consumer welfare. The main conclusion, however, remains unchanged: indebted countries tend to benefit from shifting some of their attention to CPI targeting.

The last thing to note about the results of Table 19 is the increasing preference for inflation targeting of small countries (as can be seen in column II, for example). As the Home country becomes progressively smaller, the share of its output in the Foreign country’s consumption, $C^*_t$, decreases; consequently, changes in the stance of the Home central bank have an ever-smaller impact on $E[C^*_t]$. Through (albeit imperfect) risk-sharing, Home consumption becomes effectively shielded from its local monetary policy. The latter can then be focused more on stabilizing inflation, à la the...
closed-economy setting.

3.4.3 Other Shocks and Robustness Checks

Welfare gains reported in the previous section, although admittedly very small, represent the lower bound for the true benefits from reoptimizing the monetary rule based on net foreign asset position of a country. Most papers studying the question of optimal monetary policy in the open economy setting do not report welfare calculations. Several papers that do perform such calculations report results very similar to ours: welfare gains from optimally adjusting monetary policy are on the order of 0.001-0.01 percent of consumption. See, for example, Benigno (2007), Cova and Søndergaard (2005), and Galí and Monacelli (2005). Below we consider two realistic scenarios that could result in higher welfare gains of reoptimizing monetary policy stance for developing economies than the levels reported in Table 19.

Several related papers have examined the impact of demand shocks on the optimal behavior of central banks in open economy settings. Here we assume that a positive shock increases demand in both countries for Home good \(Y_H\) by influencing the home bias coefficients:

\[
\mu_t = \overline{\mu} \exp(\varrho_t), \quad \mu_t^* = \overline{\mu}^* \exp(\varrho_t),
\]

\[
\varrho_t = 0.9 \varrho_{t-1} + \varepsilon_{\varrho,t},
\]

and \(E(\varrho) = 0\). The same adjustment is made to the investment and government spending home biases. Since this shock cannot be measured empirically, its variance was set equal to that of productivity shocks. Adding demand shocks does not change the main conclusion of this paper: debtor nations can increase the welfare of their domestic residents by putting a relatively heavier weight on CPI inflation in the Taylor Rule of their central banks\(^{59}\). Adding this extra source of uncertainty doubles welfare gains relative to those in Table 19.

\(^{59}\)Naturally, the optimal monetary policy is different from the case of no demand shocks, but the relative shifts in the optimal central bank stance following addition of international liabilities or assets remain the same.
Developing countries with significant external asset positions often find themselves subject to very high interest rate premia, well above our rather conservative benchmark 10 basis point spread. For example, Bouvatier (2007) estimates risk premia in several Asian countries to exceed 5 percent during the mid-1990s. Therefore, we repeat our simulations for a higher value of the risk premium, $p = 0.01$. Unsurprisingly, welfare gains from optimally adjusting monetary policy increase nearly tenfold.

Put together, these two effects imply that readjusting monetary policy in response to changes in a country’s NFW position could improve consumer welfare by 0.1% of consumption.

3.5 Conclusion

We have analyzed the impact of non-zero steady-state net foreign wealth on the optimal monetary policy in a dynamic forward-looking model of monetary policy. Our framework accounts for the presence of home bias in consumption (which causes deviations from PPP), nominal rigidities in price and wage setting, and incomplete international asset markets. We have shown that international debt changes the welfare-maximizing choice of monetary policy by allowing it to take advantage of the risk-sharing nature of the international bond. Our main finding is that the optimal monetary policy, which is always a mix of CPI and exchange rate targeting, shifts more towards stabilizing inflation as countries accumulate foreign debt, and leans closer to pegging in countries that hold foreign assets.

Based on this study, it seems that central banks of developing countries are too concerned with their external position (maintaining the stability of their exchange rate) and are not paying enough attention to internal business cycles (the state of productivity), to the detriment of consumer welfare. However, our model is not well suited to understand such a preference for external stability since it assumes away the link between international risk premia, local monetary policy and central bank credibility. Therefore, we may be missing an additional channel through which exchange rates affect
the behavior of domestic interest rates, output and consumption. In our future work, we hope to pursue this question further by endogenizing the risk premium and studying the impact of policy regime on the behavior of international capital flows.

The model developed in this paper lends itself to several other possible extensions. In order to better understand the interplay of relative country size, NFW position and optimal policy, we would like to derive the welfare loss function of the households. In our future work, we also hope to explore the game-theoretic dimension of the international monetary policy coordination. Finally, it would be particularly interesting to put our model into the framework of endogenous dynamic portfolio choice à la Devereux and Sutherland (2006) and Evans and Hnatkovska (2007) to gain insight into the interplay between monetary policy rules and individual asset market behavior.
References


Figure 10: Response of several Home variables to a Home productivity shock for a net lender nation.
Figure 11: Response of several Home variables to a Home productivity shock for a net debtor nation.