ESSAYS IN QUANTITATIVE MACROECONOMICS

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By

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ABSTRACT

What might the increasing labor market risk imply for fertility and the timing of births? And what were the contributions of financial innovations and low interest rates to the run up in U.S. housing prices in the decade prior to the global financial crisis? This dissertation uses quantitative macroeconomic tools to answer these questions.

Chapter 1 studies the changes in the U.S. fertility patterns over the last several decades. This chapter offers the first quantitative theoretical exploration of the link between earnings risk and fertility patterns. Empirically, I combine the estimates of occupational risk with the Decennial Census data to document the negative relationship between labor market risk and household fertility. Next, I develop a calibrated structural model of household fertility, consumption, and savings, and show that realistic increases in the persistent labor market risk generate quantitatively large increases in the mean age at the first and second births, and are associated a decline in the total number of births.

Chapter 2 studies the joint dynamics of real house prices and rents over the past decade. In this chapter, which is a joint work with Paul Sullivan and Randal Verbrugge, we build a dynamic general equilibrium stochastic life cycle model of housing tenure choice with a fully specified rental market and a market for homeownership, and endogenous house prices and rents. Lower interest rates, relaxed lending standards, and higher incomes are shown to account for roughly 50 percent of the increase in the U.S. house price-rent ratio between 1995 and 2005, and
generate the observed pattern of rapidly growing house prices, sluggish rents, and increasing homeownership and household indebtedness.
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INTRODUCTION

This dissertation is a collection of two essays which apply quantitatively-rich heterogeneous-agent models and other quantitative macroeconomic tools to study policy-relevant questions in macroeconomics and labor economics. The unifying theme of my research agenda is the variation in individual labor market outcomes (such as unemployment spells or income fluctuations) that feed into a broad variety of household choices ranging from a decision to have a child to a decision to purchase a home. These decisions and their interactions with realistic assumptions about the tax system or financial frictions in turn affect macroeconomic aggregates, for example the population size or house prices.

In Chapter 1, I study the changes in the U.S. fertility patterns over the recent decades. My interest in this question was motivated by the dramatic fall of fertility rates in advanced economies over the past three decades, which has underpinned wide-ranging policy debates about future potential growth, sustainability of social security systems, and immigration policies. While the literatures in demography, sociology, and economics have suggested several well-accepted mechanisms to help explain the decline in fertility (I review these in Section 1.1 of this dissertation), recent empirical evidence also points to the important role of increasing labor market risk. However, relatively little is known about the exact nature of the relationship between the rising earnings risk, declining fertility, and increasing age at first and second births. Chapter 1 offers the first quantitative exploration of such a link.

In Chapter 1, I develop a life-cycle of fertility choice in a Aiyagari-Bewley-Huggett style economy with incomplete markets and idiosyncratic earnings risk. Building on Becker (1960) and Becker and Tomes (1978), fertility decisions are modeled as sequential, irreversible choices over the number of children, accompanied by parental choices of time and market goods invested
toward improving children’s quality. I show that when earnings risk is high and persistent, young households postpone raising children, initially preferring to work and to accumulate more precautionary savings before starting a family. In presence of idiosyncratic infertility risk, this delay in turn reduces the number of births per household. I show that these findings are consistent with the observed pro-cyclicality of fertility in time-series data and the negative correlation between fertility and occupational risk in cross-sectional data. Overall, the observed increase in earnings risk can account for about one-half of the decline in U.S. completed fertility, and match well the observed increases in mean age of a mother at the first and second birth.

Chapter 2 studies the dynamics of house prices and rents over the last decade. The recent run-up (and subsequent collapse) in house prices coupled with rapidly-rising household debt and homeownership rates has raised widespread interest in studying the dynamics of the housing market. In my joint work with Paul Sullivan and Randal Verbrugge, we try to close several missing links in the existing literature by building a heterogeneous-agent equilibrium model which jointly determines house prices and rents, endogenizes the choice between buying a house or renting, and includes a number of realistic features such as equity withdrawals, lending frictions and tax advantages of home ownership. Our model is the first to generate a non-trivial relationship between house prices and rents. The key mechanism in the model generating a non-trivial relationship between house prices and rents as the macroeconomic conditions change is that the demand and supply of rental property are both endogenously determined jointly with the demand for housing. The calibrated model is used to study the impact of macroeconomic factors such as incomes, interest rates, and borrowing constraints on the equilibrium house prices and rents. We show that lower interest rates, relaxed lending standards, and higher incomes can jointly account for about one-half of the increase in house prices between 1995 and 2005. The
model correctly predicts only a small increase in rents, and broadly matches the increases in homeownership and household debt-to-income ratios over the period.
Chapter 1

Fertility Choice in a Life Cycle Model with Idiosyncratic Uninsurable Earnings Risk

1.1 Introduction

Over the last four decades, the average total fertility rate (TFR) in OECD countries has fallen dramatically: from 2.9 in the 1960s to 2.0 in 1975, and then further down to 1.6 in 2000. The decline in fertility has been accompanied by a delay in childbearing: the average age at first birth in OECD countries has increased from 24.0 in 1970 to 27.0 in 2000.\(^1\)

A number of candidate explanations have been put forward to account for declining fertility rates – I briefly survey this literature below. In this paper, I focus on a relatively unexplored mechanism: the link between delaying and reducing fertility on the one hand, and rising labor income risk on the other. In the next section I will present evidence from micro

\(^1\)The following countries were excluded from the calculation of the OECD average due to limited data availability: Australia, New Zealand, Mexico, Korea, and Turkey.
data consistent with such a link. At the same time, one can motivate exploring a potential link on theoretical grounds. First, thinking of the decision to have a child as an investment in a lumpy durable good (Becker, 1960), recent work in the literatures on irreversible investment and “consumption commitments” (i.e., big-ticket goods with sizable adjustment costs) suggests that link between risk and fertility. For example, Chetty and Szeidl (2007) or Postlewaite, Samuelson, and Silverman (2008) show that consumption commitments can amplify risk aversion with respect to earnings shocks. If earnings shocks become larger, agents may therefore be less willing to commit to children. Fisher and Gervais (2009) shows that, in the presence of large transactions costs, young households postpone homeownership when risk is high, preferring to initially rent and save more before buying a home. Using the same logic, if children are a durable good of irreversible nature that requires investment of parental resources, households could postpone (or abandon) childbearing when risk is high, initially preferring to work and save before starting a family.

Empirically, falling fertility rates have been observed during periods when labor market risk was high. One striking example is provided by the experience of the Central and Eastern European transition economies during the early 1990s, when large increases in unemployment and earnings volatility brought about by the dissolution of centralized wage- and production-setting were associated with a dramatic decline in TFRs: the average TFR in the Central and Eastern European (CEE) region kept at a stable level of about 2.2 between 1970 and 1990, but fell dramatically to about 1.2 by the year 2000.² Large changes in the TFR and mean

²The reported CEE average includes the following countries: Bulgaria, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Russian Federation, Slovak Republic, and Slovenia.
age at first birth have also been observed in Western Europe and the United States during periods of economic instability. For instance, the fertility rate fell dramatically during the Great Depression, a period characterized by record-high unemployment rates and high levels of earnings uncertainty. In contrast, the pickup of fertility in the post-war 1940s coincided with a booming U.S. economy and a compression of earnings inequality (see Kopczuk, Saez, and Song (2009) or Goldin and Margo (1992)). In 2008, fertility declined amidst global financial turmoil and rapidly increasing unemployment.

In Section 1.2 I study fertility choices of households with different earnings risk profiles. Combining the estimates of occupational risk by Saks and Shore (2005) with the Census data, I find that that the realized fertility of women with husbands in low-risk occupations (e.g., teachers or healthcare professionals) is, on average, higher than fertility of women with husbands in high-risk occupations (e.g., sales or arts and sciences). This finding is robust across different age groups. I then explore variation in household fertility over the business cycle, and confirm that the U.S. fertility rate is pro-cyclical. Since the household labor market risk is known to rise during recessions (Storesletten, Telmer, and Yaron, 2004), one interpretation of pro-cyclical fertility is that households postpone births when earnings uncertainty is high.

This paper offers the first quantitative theoretical exploration of the link between earnings risk and fertility patterns. I develop a life cycle model of fertility choice in an Aiyagari-Bewley-Huggett style economy with incomplete markets and idiosyncratic labor market risk. I study unitary households where parents make joint decisions about consumption, savings,
family size, and the allocation of resources invested into childrearing. Households face idiosyncratic wage shocks which can be partially self-insured by accumulating precautionary asset holding. Building on Becker (1960) and Becker and Tomes (1976), I model fertility decisions as sequential, irreversible choices over the number of children, accompanied by parental choices of time and money spent on improving children’s quality. The decision to have another child can only be made during the first part of the life cycle when parents are fertile. The duration of this fertile period is, however, unknown to parents, who face idiosyncratic permanent infertility shocks. Infertility risk, while low early in the life cycle, increases exponentially with the age of the household.

Using the exogenous estimates of the labor market risk for the 1990s from Meghir and Pistaferri (2004), I calibrate the model based on cross-sectional patterns of fertility, income, consumption, and saving of a U.S. cohort of households who made their fertility decisions in the 1990s – a period associated with higher levels of idiosyncratic earnings uncertainty. Next, using Meghir and Pistaferri’s (2004) risk estimates for the 1970s, the model is used to quantify the contribution of earnings uncertainty to the changes in the key U.S. fertility indicators between the two steady states.\(^3\) I show that realistic increases in the persistent labor market risk generate quantitatively large increases in the mean age at the first and second births, and are associated with a decline in the total number of births. In particular, the model predicts that households subject to the earnings risk typical in the United States

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\(^3\)Using the data from the Panel Study of Income Dynamics (PSID), a large body of literature documents sizable shifts in microeconomic earnings uncertainty since the early 1970s (see, Gottschalk (1997), Levy and Murnane (1992), or Heathcote and Violante (2004) for review). A large share of the observed increases in the earnings risk has been attributed to increases in the persistent component of household earnings (Meghir and Pistaferri, 2004) which, in turn, is commonly assumed to reflect uninsurable idiosyncratic earnings risk.
in the 1990s delay the birth and second births by 1.1 and 3.6 years, respectively, relative to households in the 1970s. At the same time, average completed fertility is lower than that in an economy with the 1970s’ levels of risk (1.9 versus 2.3 children). To put these magnitudes in context, the mean age at first and second birth increased by 3.5 and 3.6 years between 1970 and 2000, respectively. Moreover, women who made childbearing decisions in the 1960s and 1970s had on average 2.5 children compared to 1.9 births for women who made fertility choices in the 1980s and 1990s. The analysis focuses on the changes in fertility patterns since the early 1970s, as estimates of the idiosyncratic labor market risk for earlier periods are not available.\textsuperscript{4}

The key mechanism generating the postponement of births and the fertility decline in the model is that children are discrete, irreversible choices, and that childrearing requires at least a minimum amount of investment per child. When markets are incomplete and households have limited access to credit, young parents with positive wealth may respond to a fall in household wage by temporarily dis-saving, increasing labor supply (and thus reducing the hours spent on childrearing), or reducing the market expenditures devoted to childrearing. Since parents prefer to smooth consumption, households initially choose to postpone childbearing when labor market risk is high, and work and save more instead. While parents may initially consider their decisions to delay childbearing as temporary, infertility risk means that delayed fertility translates into reduced total fertility. The longer the delay of first and higher-order births, the larger the reduction in fertility. Absent fertility

\textsuperscript{4}To isolate the persistent and transitory components of the variance of the earnings residual (i.e., uninsurable and insurable labor market risk, respectively), a panel dimension of income data is needed.
risk, increased labor market risk has a smaller but still negative effect on total fertility.

A vast body of studies in microeconomics, labor economics, and macroeconomics have explored channels that likely contributed to the demographic decline. I will briefly review several recent studies of fertility choice in the quantitative theoretical tradition that are most closely related to this paper. Da Rocha and Fuster (2006) shows that high unemployment risk induces women to postpone and space births, which in turn reduces the fertility rate. Other papers try to connect three trends: increasing female education, increasing female labor market participation, and declining fertility. Conesa (2000) suggests that changes in timing of fertility decisions resulting from increasing female access to higher education can partially account for the recent fertility decline in advanced economies. In a related study, Caucutt, Guner, and Knowles (2002) argue that better education can explain less than one-third of the increase in mean age at birth, and that the delayed fertility is driven by changes in the marriage markets and increasing returns to female labor market participation. Education and female participation are beyond the scope of the present paper, but it would be interesting to explore the feedback from fertility to these trends. In particular, if women choose to delay fertility in response to labor market risk, they have more time available for education and work.

In terms of the longer-term demographic trend, Greenwood and Seshadri (2002a), Greenwood and Seshadri (2002b), or Doepke (2004) explain the decline as a result of a production shift from low-skill, labor-intensive agriculture to high-skill manufacturing. Greenwood, Seshadri, and Vandenbroucke (2005) argues that the secular decline is due to the relentless
rise in real wages that increased the opportunity cost of children, while the baby boom of the 1950s and 1960s can be explained by an technological progress that lowered the cost of having children. Knowles (2007) argues that improved opportunities for contraception and abortion can partly account for increasing female labor participation and lower fertility rates, a theory consistent with the empirical evidence in Goldin and Katz (2002). Boldrin, De Nardi, Jones, and Madrid (2005) argue an increase in government provided old-age pension can help explain the both the long-run demographic decline as well as account for the differences in fertility rates across countries.

The paper is organized as follows. Section 1.2 presents empirical evidence on the relationship of idiosyncratic labor risk and household fertility. In Section 1.3, I develop a life cycle model with heterogeneous households, fertility choice, and idiosyncratic earnings risk. Section 2.4 describes the model’s calibration. In Section 1.5, I discuss the predictions of the benchmark model. Section 1.6 describes the extent to which increases in earnings risk affect household fertility decisions, and reconciles the model’s predictions with the actual changes in household fertility behavior in the U.S. data. Section 2.6 concludes with a discussion of possible extensions and directions of the future research.

1.2 Empirical Evidence on Earnings Risk and Fertility

In this section, I show that households with high earnings risk have lower fertility than households with low earnings risk, and that the U.S. fertility rate is pro-cyclical. I also

\[5\] These papers focus only on explaining the secular decline in the fertility rates, but ignore the corresponding changes in timing of births.
support these finding with additional international evidence.

1.2.1 Occupational Risk and Fertility in Cross-sectional Data

In this section, I examine whether women in high earnings-risk households have different fertility patterns than women in households subject to low levels of earnings risk. In order to answer this question, I use the riskiness of husband’s occupation as a proxy for the earnings risk faced by households. Using the PSID income data for male household heads, Saks and Shore (2005) find that teachers, health-care professionals, and engineers face the lowest levels of earnings uncertainty, while men with occupations in math and sciences, sales, and arts and entertainment typically experience high levels of earnings risk.

To construct the data on fertility of households from these occupational groups, I use the 5 percent sample of the 1990 Decennial Census, concentrating on married couples where the husband is not self-employed. Following the estimation strategy of Saks and Shore (2005), I control for the educational attainment of husbands. After the selection criteria are applied, the sample consists of roughly 100,000 married couples with with wives between ages 20 and

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6Occupation is considered a career choice that is connected with a significant accumulation of human capital. Since changes in occupation typically involve large losses of the accumulated human capital, the perceived riskiness of the occupation represents a good proxy for the perceived riskiness of lifetime income.

7Husbands are assigned the following occupations: teachers, health-care professionals (e.g., medical and dental technicians, nurses, optometrists, and pharmacists), engineers, managers, math and sciences (e.g., mathematicians, physicists, and other natural scientists), sales workers, and arts and entertainments (e.g., artists and arts teachers, actors, dancers and dance teachers, musicians and music teachers). Occupations not studied in Saks and Shore (2005) have been omitted from the analysis.

8Since self-employed individuals have been shown to face higher earnings risk than individuals working for wage or salary across occupations (Carroll and Samwick (1997), Carroll and Samwick (1998), or Saks and Shore (2005)), I focus on individuals who are not self-employed. The fertility patterns of the self-employed exhibit similar patterns.

9Saks and Shore (2005) estimate the occupational risk for male heads with at least a college degree. For details on sample selection in this paper, see Appendix A.
Figure 1.1: Fertility and earnings risk of husband’s occupation


43 years.

Figure 1.1 plots the average number of births from the Census data against the estimates of the occupational earnings risk by Saks and Shore (2005) for various age-groups. I find that the realized fertility of women with husbands in low-risk occupations is, on average, higher than fertility of women with husbands in high-risk occupations. This finding is robust across different age groups.

To examine the negative correlation between fertility and earnings risk in a more formal framework, I estimate a simple OLS model of completed fertility using the cross-sectional data set from Census. The dependent variable is the number of births for any given couple. The regressors include occupational dummies and other basic household characteristics such
as wife’s age, and wife’s and husband’s income. In Table 1.1, the occupational dummies are ordered by the earnings risk: the higher the riskiness of the occupation, the lower the rank in the table. The dummy for teachers (i.e., the lowest-risk occupation) is omitted from the regression, and the estimated dummies therefore correspond to the difference in fertility between a given occupational group and fertility of families where husband (teacher) has a low earnings risk.

As can be seen in the table, the correlation between the number of births and the riskiness of husband’s occupation remains, on average, negative, since the estimated dummies tend to be lower (more negative) for high-risk occupations. Interestingly, the effect of husband’s income level on the number of births is positive, indicating that the demand for children rises with household income. On the other hand, the negative relationship between wife’s income and the number of births is consistent with the “price of time” theory which posits that higher-earning women have smaller families due to the higher opportunity cost of raising children. The coefficients on all variables are statistically significant at the one percent level, except for the occupational dummy for healthcare professionals which is significant at the 10 percent level.

1.2.2 Pro-cyclicality of the U.S. Fertility

In order to study the co-movement between fertility and the business cycle, I analyze evolution of the U.S. general fertility rate over the past century.\textsuperscript{10} There were three major swings

\textsuperscript{10}The evolution of fertility over the business cycle can be measured using (i) the crude birth rate, and (ii) the general fertility rate. The crude birth rate is calculated as the number of live births per 1,000 people in a given year, and represents a traditional measure of household fertility. However, its usefulness as a
Table 1.1: Regression analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>(Std. Err.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>age of wife</td>
<td>0.390</td>
<td>(0.006)</td>
</tr>
<tr>
<td>age of wife squared</td>
<td>-0.005</td>
<td>(0.000)</td>
</tr>
<tr>
<td>healthcare</td>
<td>-0.036</td>
<td>(0.022)</td>
</tr>
<tr>
<td>engineer</td>
<td>-0.148</td>
<td>(0.015)</td>
</tr>
<tr>
<td>manager</td>
<td>-0.129</td>
<td>(0.013)</td>
</tr>
<tr>
<td>math</td>
<td>-0.204</td>
<td>(0.061)</td>
</tr>
<tr>
<td>sales</td>
<td>-0.158</td>
<td>(0.014)</td>
</tr>
<tr>
<td>arts and entertainment</td>
<td>-0.305</td>
<td>(0.028)</td>
</tr>
<tr>
<td>husband’s total income</td>
<td>8.06e-07</td>
<td>(0.000)</td>
</tr>
<tr>
<td>wife’s total income</td>
<td>-1.77e-05</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-5.987</td>
<td>(0.107)</td>
</tr>
</tbody>
</table>

N 103271
R^2 0.254
F_{(10,103260)} 3519.162

Figure 1.2: U.S. fertility rate: 1909 - 2008

in the general fertility rate since the early 1900s: fertility fell rapidly during the 1920s and 1930s, before picking up strongly in the 1940s and 1950s, and then permanently declining during the 1960s and mid-1970s. Since the mid-1970s, the fertility rate has oscillated in a relatively narrow range (Figure 1.2). These swings have been associated with a variety of factors. The initial decline in fertility during the 1920s and 1930s in part reflected a downward trend from the previous decades, but falling incomes and greater economic uncertainty also contributed to lowering the fertility rate (U.S. Vital Statistics, Volume I, 1960). The pickup of fertility in the 1940s coincided with a booming U.S. economy and the record number of marriages following demobilization of the Armed Forces. Interestingly, the post-war recession was followed by temporarily lower fertility rates during 1947-1949. The baby bust of the 1960s was associated with a massive entry of women into the labor force and a large-scale adoption of contraceptives (the “pill” was first introduced in 1960). Macroeconomic volatility likely helped reduce fertility rates during the 1970s.

Since the war presented a major shock that influenced demography and fertility for a number of decades, I study the relationship between business cycle and fertility using detrended data. Panel A in Figure 1.3 suggests that output movements influenced fertility through much of the 20th century – a hypothesis partly confirmed by an impulse response function from a simple bivariate VAR with output and fertility detrended using the Hodrick-Prescott (HP) filter ($\lambda = 100$). These illustrative results are even stronger in the period after mid-1970s. Panel B plots a measure of economic cycle against the fertility rate detrended measure of fertility is somewhat limited by the fact that it does not take into account changes in the age and sex composition of the population over time. The general fertility rate, defined as the number of live births per 1,000 women aged 15-44 in a given year, corrects for the short-comings of the crude birth rate and is therefore used to study the business cycle properties of fertility in this paper.
Figure 1.3: Fertility over the U.S. business cycle (alternative samples)

Notes: Left: co-movement of the U.S. fertility rate and GDP, with and without HP-filter. Right: response of the fertility rate to the output changes from a simple bivariate VAR.
using the HP filter as in Panel A. The corresponding impulse response function points to a statistically significant relationship between output shocks and fertility changes during 1975-2005, with a lead time of about 1-3 years. The effect of an output shock on the fertility rate dies off after about 5-6 years.

These findings suggest that households postpone births at times of economic downturns when idiosyncratic earnings risk is high.\textsuperscript{11} In particular, increases in earnings uncertainty, which typically follow a negative output shock, reduce the household willingness to conceive a child in a near future. Over time, as the economic activity returns back to potential and the earnings risk declines, the willingness of households to have more children rises again.

\subsection*{1.2.3 International Evidence}

Cross-country studies also support the hypothesis that fertility responds negatively to the labor market risk. Using a panel of 23 OECD nations, Adsera (2004) finds that high unemployment and unstable contracts, common in Southern Europe, depress fertility, particularly of young women. Adsera (2005) uses the 1994-2000 waves of the European Household Panel Survey (EHPS) to reconstruct fertility histories of around 48,000 women in 13 West European countries, and finds that high long-term unemployment rates depress completed fertility and decrease the probability of a transition to higher-order births.\textsuperscript{12} Adsera (2006) uses the

\textsuperscript{11}Using the data from the Panel Study of Income Dynamics (PSID), Storesletten, Telmer, and Yaron (2004) find that household idiosyncratic earnings risk is highly persistent and strongly countercyclical, with a conditional standard deviation that increases by roughly 75 percent as the macroeconomy moves from peak to trough.

\textsuperscript{12}Adsera (2005) uses the Cox hazard transitions to the first three births with a Cox proportional-hazard model. The model predicts that in countries where male and female unemployment rates are similar and joblessness is short-lived, fertility rates are around 1.81 (a level close to the replacement rate). However, when unemployment rates are particularly high for women and unemployment is highly persistent, the estimated
1985 and 1999 Spanish Fertility Surveys to study differences between desired and actual fertility and finds that women in that mid-twenties facing high unemployment rates tend to reduce their fertility below desired levels. Using the 1992-2002 data from the German Socio-Economic Panel (GSOEP), Bhaumik and Nugent (2006) estimate a strong negative relationship between unemployment uncertainty and childbearing decisions for East German women. Mira and Ahn (2002) show that the fertility rate responds negatively to unemployment over the business cycle in 15 out of 21 OECD countries. Finally, using the data from the 1991 Spanish Socio-demographic Survey, Mira and Ahn (2001) confirm that spells of non-employment have a strong negative effect on the hazard of marriage, although the estimated direct effects of joblessness and part-time work on birth hazards conditional on marriage are small and/or not significant.

1.3 The Benchmark Model

To study the impact of changes in the labor market risk on household fertility decisions in a formal framework, I build a model based on an Aiyagari-Bewley-Huggett style economy with incomplete markets and uninsurable earnings risk. Building on Becker and Tomes’ (1976) idea of children as a durable good whose “quality” can be improved by parental investment of time and money, I next add a dynamic sequential fertility choice, and endogenize the allocation of resources devoted toward childrearing. I consider unitary households where parents make joint decisions about consumption, saving, their family size, and the inputs fertility is only around 1.28.
invested toward improving their children’s quality.\textsuperscript{13}

The model mirrors the following assumptions. Young households, which start their life cycle childless and with zero asset holdings, have limited access to credit and face idiosyncratic earnings shocks which can be partially self-insured by accumulating precautionary asset holdings. Parents enjoy having children and care for their children’s quality which is secured through parental inputs of time and market goods. Children are discrete and irreversible choices that are born in increments of one (e.g., no twins are allowed). The decision to have another child can be made only during the first part of the life cycle when parents are fertile. The exact timing of the last fertile period is, however, unknown to parents who face infertility shocks which render them permanently infertile.

1.3.1 The Demography and Endowments

The model economy is inhabited by a continuum of the same-age husband-wife households with identical preferences. The model period is one year. Households start their life together at age 16, and live until age 80 with certainty. During the first 50 periods of life, the household wage process is determined according to an idiosyncratic stochastic process:

\[
\ln w_t = \ln w_0 + h(t) + \epsilon_t + \nu_t, \quad (1.1)
\]

where \(h(t)\) governs the average age-profile of wages, \(\nu_t \sim N(0, \sigma^2_\nu)\) is a transitory shock to income received every period, and \(\epsilon_t\) is a persistent shock, also received each period, which

\textsuperscript{13}In future research, I plan to relax the assumption of unitary households and model male and female wage processes separately.
follows a first-order autoregressive process:

\[ \epsilon_t = \rho \epsilon_{t-1} + \psi_t \text{ with } \psi_t \sim IID(0, \sigma^2_t) \text{ and } \epsilon_1 = 0. \]  

(1.2)

At age 65, households retire and receive a lump-sum pension:

\[ w_t = \bar{w} \text{ for } t > 65, \]  

(1.3)

in the form of a transfer from the government.

1.3.2 Preferences

In the spirit of Becker and Tomes (1976), I assume that each household has a utility function of the form:

\[ U = U(c_t, n_t, q_t), \]  

(1.4)

where \( c_t \) stands for the parental consumption of a nondurable market good, \( n_t \) is the number of children at home, and \( q_t \) is the quality of each child. In this model, households are not altruistic toward their offspring, leaving no bequest to their children. The expected discounted lifetime utility can then be written as:

\[ E_0 \sum_{t=16}^{80} \beta^{t-16} U(c_t, n_t, q_t), \]
with the discount factor \( \beta \in (0, 1) \).

The quality of children is determined by parents through their inputs of time, \( l_t \), and goods, \( x_t \), spent on childrearing. Similarly as in Becker and Tomes (1976), I assume that the quality of each child within a family is the same and takes on the form:

\[
q_t = f(x_t, l_t, n_t),
\]

where \( x_t \) and \( l_t \) are the total amounts of goods and time invested toward childrearing. Although households do not value leisure, their labor supply is determined endogenously within the model as a fraction of the total time that is not spent on childrearing. Lastly, while household spending on children is discretionary, a minimum level of investment in children’s quality is required for families with children so that \( q_t \geq q \) if \( n_t > 0 \).

### 1.3.3 Process for Dependent Children

In order to build a model that can match the life cycle household patterns of parental spending on children, I assume that parents enjoy and make expenditures only on children which are young and live at home. Ideally, one might like to think of such children as children younger than a certain age. However, the recursive structure of this model makes keeping track of children’s ages difficult, as it requires integrating a history of the past fertility decisions into the state space of the problem (for details, see Hotz and Miller (1988) or Conesa (2000)).\(^{14}\) For the purposes of this paper, I assume that parents have two types

\(^{14}\)The history of the past fertility decisions would be summarized by a vector of zeros and ones, depending on whether the household had a child or not at each period of the fertile horizon. When the number of fertile
of children: children which are young and still live at home ($n_t$), and children which have become financially independent and have left home. The law of motion of the children ever born to the household ($n^b_t$) is deterministic and follows the process:

$$n^b_{t+1} = n^b_t + K_t$$

where $K_t = \{0, 1\}$, (1.7)

with $K_t = 1$ when a household has an additional child next period and $K_t = 0$ otherwise.

The number of dependent children which still live at home is assumed to be distributed binomially:

$$n_{t+1} \sim Bi(n_t + K_t, p)$$

with $n^b_1 = n_1 = 0$, (1.8)

where $p$ is a time-invariant probability that a child becomes independent and leaves home.

### 1.3.4 Infertility Risk

In order to build a further element of realism into the model, households face a binary idiosyncratic infertility shock $f_t = \{I, F\}$ which arrives at the beginning of every period. Only parents that are fertile in a given period (i.e., $f_t = F$) can choose to have another child, while parents once hit by the infertility shock remain infertile forever (i.e., if $f_t = I$, then $f_{t+j} = I \forall j \geq 0$). The time-variant probability $p^I_t$ that a household becomes permanently infertile is assumed to rise exponentially with the age of parents, $t$, and to become one at periods is large, keeping track of children’s ages becomes computationally intractable.
age 45 after which no households are able to conceive a child.\footnote{The fertile horizon of the household is based on the fertility cycle of the wife and reflects the fact that most women do not give birth after the age of 45.}

\[
p_t^f = \begin{cases} 
\omega_1 \exp^{\omega_2 t} & \text{if } t < 45 \\
1 & \text{if } t \geq 45.
\end{cases} \tag{1.9}
\]

### 1.3.5 Dynamic Program of Fertile Parents

Parents who have not lost their ability to bear children (i.e., \( f_t = F \)) solve the recursive problem:

\[
V_t(a_t, n_t, w_t, f_t = F) = \max_{c_t, a_{t+1}, x_t, l_t, K_t = \{0,1\}} u(c_t, n_t, q_t) + \beta E_t V_{t+1}(a_{t+1}, n_{t+1}, w_{t+1}, f_{t+1} = \{I, F\})
\]

subject to

\[
A_{t+1} = \begin{cases} 
(1 + r)(A_t + (1 - l_t)w_t - c_t - x_t) & \text{if } t \leq R ; \\
(1 + r)(A_t - c_t + \bar{w}) & \text{if } R < t \leq T ,
\end{cases} \tag{1.10}
\]

\[
q_t = f(x_t, l_t, n_t) \text{ with } q_t \geq \bar{q} \text{ if } n_t > 0, \tag{1.11}
\]

\[
n_{t+1}^b = n_t^b + K_t \text{ with } n_1^b = 0, \tag{1.12}
\]

\[
n_{t+1} \sim Bi(n_t + K_t, p) \text{ with } n_1 = 0, \tag{1.13}
\]

\[
A_{t+1} \geq 0, \tag{1.14}
\]

by choosing the parental consumption of the nondurable market good, \( c_t \), savings, \( A_{t+1} \), and the time, \( l_t \), and market goods, \( x_t \), inputs into the production function for the children’s
quality, $f(x_t, t, n_t)$, with $q$ imposing a lower bound on children’s quality. In addition to these continuous choices, households also make a discrete decision whether to have a child next period ($K_t = 1$) or not ($K_t = 0$). Households also face uncertainty about their ability to bear children next period: $f_{t+1} = \{I, F\}$ follows the process from Section 1.3.4. Equation (1.12) determines the law of motion for the stock of children ever born, $n_t^b$, to the household, while equation (1.13) summarizes the law of motion for children at home, $n_t$. In the baseline model, no borrowing is allowed (equation 1.14). $(1 + r)$ is the gross rate of return on a single asset in the economy; and $w_t$ represents household wages and follows the process described in the equation (1.1).

### 1.3.6 Dynamic Program of Infertile Parents

Parents who have lost their ability to bear children (i.e., $f_t = I$) can no longer increase their family size and, therefore, solve the problem:

$$V_t(a_t, n_t, w_t, f_t = I) = \max_{c_t, a_{t+1}, x_t, f_t, q_t} u(c_t, n_t, q_t) + \beta E_t V_{t+1}(a_{t+1}, n_{t+1}, w_{t+1}, f_{t+1} = I),$$

by choosing the optimal allocations of consumption, savings, and resources devoted to child-rearing, subject to the set of constraints and transition equations (1.10), (1.11), and (1.14), and subject to the law of motion:

$$n_{t+1} \sim Bi(n_t, p) \text{ with } n_1 = 0. \quad (1.15)$$
1.4 Calibration

The calibration strategy involves fixing some parameter values exogenously, and estimating
the remaining parameters using the method of simulated moments based on cross-sectional
patterns of fertility, income, consumption, and saving. Table 2.1 summarizes parameters
which were drawn from other studies or were calculated directly from the data. Table 1.3
contains eight estimated parameters based on moments described in Table 2.3 that are
constructed using the data from the 1979 National Longitudinal Survey of Youth (NLSY79),
the 2004 American Time Use Survey (ATUS), and the 2000 waves of the Consumption
Expenditure Survey (CEX). Appendix 1.8.1 provides details on the sample selection and the
calculation of moment conditions from these data sets.

1.4.1 Infertility Risk and and Earnings Process

Trussell and Wilson (1985) provide point estimates for the fraction of couples who are perma-
nently infertile by the woman’s age. The authors’ point estimates, fitted by an exponential
function in \( t \), represent the benchmark cumulative distribution function (c.d.f.) of the per-
manent infertility risk (Figure 1.4). The c.d.f. of the permanent infertility risk is in turn
used to calculate the sequence of the time-variant probabilities, \( p_t^I \), in equation (1.9) which
are derived so that the fraction of permanently infertile households of any given age in the
model matches exactly the corresponding fraction in the data.\(^{16}\)

Three parameters are needed to parametrize the stochastic components of the idiosyn-

\(^{16}\)In the data, about 97 percent of all couples are infertile at age 45. In the model, the cumulative
probability that a household is permanently infertile at age 45 is set to 1.
cratic earnings process in equation (1.1): the serial correlation coefficient, $\rho$, the standard deviation of the innovation term, $\sigma_\epsilon$, for the persistent shock, and a standard deviation of the innovation, $\sigma_\nu$, for the transitory shock.

Various authors have estimated the stochastic process for logged labor earnings using the PSID data. Controlling for the household observable characteristics (such as education and age), Card (1991a), Hubbard, Skinner, and Zeldes (1995b), and Storesletten, Telmer, and Yaron (1998) estimate a $\rho$ in the range from 0.88 to 0.96, and a $\sigma_\epsilon$ in the range between 0.12 and 0.25. Assuming the presence of unit root, Meghir and Pistaferri (2004) find that $\sigma_\epsilon$ increased from about 0.15 in the 1970s to 0.21 in the 1980s.\footnote{Using PSID data, Meghir and Pistaferri (2004) provide historical estimates for the variance of persistent shock for the period 1969-1991. Since the variance is known to fluctuate year by year, I compute the mean variance for the 1980s and 1990s by taking an arithmetic average across the authors' variances for the periods 1970-1979 and 1980-1989.} Meanwhile, the estimates for $\sigma_\nu$ range between 0.15 and 0.24.

For the purposes of this paper, $\rho$ and $\sigma_\nu$ are set to the middle of the spectrum of the available estimates, i.e., 0.95 and 0.17, respectively. Since the model is calibrated to match fertility choices of the NLSY79 cohort of agents who mostly made their fertility decisions in the 1980s and 1990s, my choice for $\sigma_\epsilon$ of 0.21 lies at the upper end of the available estimates, as work by Meghir and Pistaferri (2004) suggests that households in the 1980s and the 1990s faced on average a higher level of persistent labor earnings uncertainty than the earlier cohorts.

To avoid numerical integration, earnings process (1.1) is implemented as a discrete approximation to the otherwise continuous earnings process. The autoregressive process is approximated with a seven-state Markov chain with innovations being i.i.d. and transition
probabilities chosen following Tauchen (1986a). For the transitory shocks, I use an i.i.d. two-state Markov chain.

The average age-profile for wages, \( h(t) \), is calculated from the 2004 CPS by dividing the family labor income, defined as a sum of yearly earnings of both spouses in husband-wife families, by the sum of total hours worked by the couple (Figure 1.5). The average age of the couple is taken to represent the age of the household. The profile is smoothed using a cubic polynomial in age.

Retired households receive a pension transfer \( \bar{w} \) which is proportional to the household earnings in the last working period, with a replacement rate \( b \). Using the Health Retirement Survey data and the Social Security Administration records, Munnell and Soto (2006) report that the 1999-2002 median replacement rate for newly retired workers was about 42 percent of worker average indexed earnings (higher for earnings-poor individuals and lower for earnings-rich individuals, due to the progressiveness of the system). On a household basis, the Social Security benefits provide an average replacement rate of 44 percent; 58 percent for a couple with a non-working spouse and 41 for couples where both spouses work. For the purposes of this paper, the replacement rate \( b \) is set to 0.4.

1.4.2 Preferences

Following the literature on fertility choice (see, for example, Becker, Murphy, and Tamura (1990), Ranjan (1999), de la Croix and Doepke (2003), or Jones, Schoonbroodt, and Ter-tilt (2008)), the preferences are modeled as additively separable between consumption and
fertility choices (i.e., the number of children and the children’s quality):

\[ U(c, n, q) = \frac{c^{1-\gamma}}{1-\gamma} + \frac{\zeta (nq)^{1-\kappa}}{1-\kappa}, \]  

(1.16)

with \( \gamma > 0 \) and \( \kappa > 0 \). The constant relative risk aversion preferences over consumption are standard, and are characterized by the risk aversion coefficient, \( \gamma \), which determines the household desire to smooth consumption across time and states. The existing estimates of \( \gamma \) typically lie in a range between 1 and 3. To model household preferences over the number of children and their quality, I adopt a generalized version of the preference specification in de la Croix and Doepke (2003).\(^{18}\)

\(^{18}\)In particular, de la Croix and Doepke (2003) adopt the limit case with \( \gamma \) and \( \kappa \) approaching 1.
In order to characterize the household preferences described in equation (1.16), four parameters are needed: the three which identify the utility function \((\gamma, \kappa, \zeta)\), plus the discount factor \(\beta\). In order to minimize the number of calibration targets, the annual gross interest rate \((1 + r)\) is set equal to 1.04 so that \(\beta = \frac{1}{1+r}\). I set the relative risk aversion, \(\gamma\), to 1.5. The remaining two preference parameters \(\zeta\) and \(\kappa\) are calibrated.
1.4.3 Production Function for Children’s Quality

The production function for the children’s quality takes on the constant elasticity of substitution (CES) form

\[ q_t = \left[ \mu \left( \frac{x_t}{n_{t1}} \right)^\theta + (1 - \mu) \left( \frac{l_t}{n_{t2}} \right)^\theta \right]^{1/\theta}, \]

(1.17)

where \( \mu \in [0, 1] \) is the production share, and \( \frac{1}{1-\theta} \) with \( \theta \in (-\infty, 1] \) represents the elasticity of substitution between time \((l_t)\) and goods \((x_t)\) devoted to childrearing, while parameters \(\psi_1\) and \(\psi_2\) represent the household economies of scale in the time and market expenditures spent on childrearing. The CES production function is popular in applied research due to its flexibility regarding the degree of substitution between production inputs.\(^{19}\) Since very little is known about the degree of substitutability of time and market expenditures in children’s production, no a priori assumption is made about the value of \(\theta\). Instead, both CES production parameters \(\mu\) and \(\theta\) as well as the parameters \(\psi_1\) and \(\psi_2\) are calibrated. The lower bound on children’s quality, \(q\), from section 1.3.2 is calibrated as well.

1.4.4 Process for Dependent Children

In order to identify process (1.8), a value for the time-invariant probability \(p\) that a child leaves home is needed. Since a child can separate from the household in any period, \(p\) is calibrated so that the number of children living with mature-age parents at home matches the number of children living at home in the data.\(^{20}\)

---

\(^{19}\)When \(\theta = 1\), production inputs are perfect substitutes. Conversely, when \(\theta = -\infty\), the inputs are perfect complements. \(\theta = 0\) gives a Cobb-Douglas production function.

\(^{20}\)Besides data on the number of children born to respondents, NLSY79 also collects information about the number of children living at home.
Table 1.2: Exogenous parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross interest rate ((1 + r))</td>
<td>1.04</td>
</tr>
<tr>
<td>Discount factor (\beta_t)</td>
<td>(\frac{1}{1+r})</td>
</tr>
<tr>
<td>Risk aversion coefficient (\gamma)</td>
<td>1.5</td>
</tr>
<tr>
<td>Age-profile of wages (h(t))</td>
<td>computed from 2004 CPS</td>
</tr>
<tr>
<td>Persistence coefficient (\rho)</td>
<td>0.95</td>
</tr>
<tr>
<td>Std. of persistent shock (\sigma_\epsilon)</td>
<td>0.21</td>
</tr>
<tr>
<td>Std. of transitory shock (\sigma_\nu)</td>
<td>0.17</td>
</tr>
<tr>
<td>Replacement rate (b)</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 1.3: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference curvature (\kappa)</td>
<td>0.14</td>
</tr>
<tr>
<td>Preference scale (\psi)</td>
<td>3.34</td>
</tr>
<tr>
<td>Production share (\mu)</td>
<td>0.33</td>
</tr>
<tr>
<td>Elasticity of substitution in production (\frac{1}{1-\theta})</td>
<td>(\frac{1}{1-0.73})</td>
</tr>
<tr>
<td>Lower bound on children’s consumption (q)</td>
<td>0.34</td>
</tr>
<tr>
<td>Household economies to money input to production (\psi_1)</td>
<td>0.66</td>
</tr>
<tr>
<td>Household economies to time input to production (\psi_2)</td>
<td>0.54</td>
</tr>
<tr>
<td>Probability that a child leaves (p)</td>
<td>0.98</td>
</tr>
</tbody>
</table>

1.4.5 Moment Conditions For the Simulated Method of Moments

Based on the previous discussion, eight structural parameters must be calibrated: the scale and curvature preference parameters, \(\zeta\) and \(\kappa\); the production share, \(\mu\); the elasticity of substitution between time and market goods in the production function, \(\theta\); the lower bound on the children’s quality, \(q\); the parameters of the economies of scale, \(\psi_1\) and \(\psi_2\) in the production function of children’s quality; and the probability that a child leaves home at any given period, \(p\). Let \(\Theta = (\zeta, \kappa, \mu, \theta, q, \psi_1, \psi_2, p)\) define the vector of structural parameters.
Table 1.4: Moments targeted in the estimation

<table>
<thead>
<tr>
<th>Calibration target</th>
<th>Data</th>
<th>Model</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completed fertility of households</td>
<td>1.90</td>
<td>1.90</td>
<td>NLSY79</td>
</tr>
<tr>
<td>Mean age at 1st birth</td>
<td>25.5</td>
<td>25.5</td>
<td>NLSY79</td>
</tr>
<tr>
<td>Mean number of children at home for households age 35</td>
<td>1.43</td>
<td>1.43</td>
<td>NLSY79</td>
</tr>
<tr>
<td>Expenditures on childrearing to earnings</td>
<td>0.40</td>
<td>0.40</td>
<td>Lino (1998)</td>
</tr>
<tr>
<td>Elasticity of market expenditures w.r.t. number of children</td>
<td>0.34</td>
<td>0.34</td>
<td>CEX</td>
</tr>
<tr>
<td>Elasticity of childrearing time w.r.t. number of children</td>
<td>0.25</td>
<td>0.25</td>
<td>ATUS</td>
</tr>
<tr>
<td>Correlation between earnings and fertility at age 20</td>
<td>-0.20</td>
<td>-0.20</td>
<td>NLSY79</td>
</tr>
<tr>
<td>Correlation between earnings and fertility at age 45</td>
<td>-0.02</td>
<td>-0.02</td>
<td>NLSY79</td>
</tr>
</tbody>
</table>

to calibrate. The parameter values $\Theta$ are identified so that the resulting statistics in the model economy $G_j(\Theta)$ are determined by the eight specified targets $G_j$ for $j = 1, \ldots, 8$ measured in the U.S. cross-section.

The data for the eight targets come from three different sources: NLSY79, ATUS, and CEX. The NLSY data is used to estimate the average number of children ever born to a household (1.9), the mean age of a parent at first birth (25.5), and the average number of children at home for parents age 35 (1.4), as well as the correlation coefficient between number of births and labor earnings for parents of age 20 (-0.20) and of age 45 (-0.02). The average childrearing expenditures to labor income ratio (0.40) for households with children is drawn from Lino (1998) who, using the Consumer Expenditure Survey (CEX), estimates that an average dual-earner household with two children between ages 0 and 17 spends roughly 40 percent of the household earnings on direct expenses connected with childrearing (e.g., food, housing, education, transportation, babysitting, and daycare).

The ATUS and the CEX are used to help identify the economies of scale to market
goods ($\psi_1$) and time ($\psi_2$) inputs in childrearing. Previous studies (Doepke, Hazan, and Maoz (2007)) used the slope coefficient from the regression of logged time on a constant and a logged number of children in the household to represent the economies of scale to time input. However, setting $\psi_2$ to the slope coefficient can be misleading if households trade off quality per child for bigger family sizes (e.g., $\frac{\partial q}{\partial n} < 0$) since, in such a case, the estimated slope coefficient overstates the true economies of scale. To estimate the parameters $\psi_1$ and $\psi_2$ directly, the method of indirect inference is applied (for an overview, see Smith (2008)).

First, in order to pin down the household economies to the expenditure input $\psi_1$, I run an auxiliary regression,

$$\ln x_t = \alpha_0 + \alpha_1 \ln n_t, \quad (1.18)$$

using the 2000 CEX data, with $x_t$ representing the total children-specific expenses and $n_t$ determining the number of own children in the household. The slope coefficient $\alpha_1$, estimated at 0.34, represents the elasticity of childrearing expenditures with respect to the number of children at home. Similarly, in order to estimate the economies of scale to the time input $\psi_2$, the 2004 ATUS data are used to run an OLS regression

$$\ln l_t = \gamma_0 + \gamma_1 \ln n_t, \quad (1.19)$$

with $l_t$ representing the total time spent by respondents on childrearing and $n_t$ capturing the number of own children at home. The estimated elasticity of time spend on childrearing with respect to the number of children is 0.25. Equations (1.18) and (1.19) provide the last
two moment conditions for the method of simulated moments, with the elasticities $\alpha_1$ and $\gamma_1$ yielding calibration targets.\textsuperscript{21}

1.5 Fit of the Benchmark Model

1.5.1 Savings, Consumption, and Earnings

Figure 1.6 Panel A shows the simulated life cycle profiles of earnings, savings, and consumption. The age-profiles of earnings and savings both follow the standard hump-shaped pattern, and are broadly consistent with available empirical evidence. The household earnings approximately triples between ages 25 and the peak at age 58. At the same time, the savings profile generated by the model peaks in the retirement year at about 3.7 times of the mean earnings, and the wealth of retired households slowly decays at a steady rate.\textsuperscript{22} The age-profile of parental consumption generated by the model is standard.

1.5.2 Fertility Patterns

Panels B and C in Figure 1.6 compare the age-specific cumulative births and fertility rates generated by the model with the corresponding NLSY79 estimates.\textsuperscript{23} The simulated birth probability profile matches the data well for households between ages 30 and 45, although

\textsuperscript{21}Indeed, the model predicts that the actual values for the economies of scale parameters $\psi_1$ and $\psi_2$ lie well above the corresponding elasticities $\alpha_1$ and $\gamma_1$. This implies that parents face trade-offs between the number and quality of children, and reduce the quality of each child in order to increase their family size.

\textsuperscript{22}The evidence from the Survey of Consumer Finances (SCF) 2007 suggests that the age-profile of households peaks roughly at age 55 and triples between age 22 and the peak, while the savings profile peaks at age 65 at about 3.5 times of mean earnings.

\textsuperscript{23}The age-specific birth rate captures the number of births in a given period as a fraction of total population in the model.
the average number of births by the younger agents differs slightly from the data, in part
due to the low levels of household heterogeneity early in the life cycle.\textsuperscript{24}

Table 1.5 captures the distribution of households by the number of born children by the
age 45. In both the model and the data, the median household has two offspring. Moreover,
13 percent of all households are childless compared with roughly 19 percent in the NLSY79
survey. Since in the model households enjoy having children, childless households are only

\textsuperscript{24}In the model, I assume that households are ex-ante identical at age 16 and only grow heterogeneous over
time as they start to vary in their respective earnings histories. This model abstracts from various additional
sources of households heterogeneity present in the data (such as differences in educational enrollment) that
may enter household decision to have a child.
Table 1.5: Distribution of households by the number of births at age 45

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 children</td>
<td>0.19</td>
<td>0.13</td>
</tr>
<tr>
<td>1 child</td>
<td>0.17</td>
<td>0.14</td>
</tr>
<tr>
<td>2 children</td>
<td>0.32</td>
<td>0.47</td>
</tr>
<tr>
<td>3 children</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>4 children</td>
<td>0.08</td>
<td>0.05</td>
</tr>
<tr>
<td>5+ children</td>
<td>0.05</td>
<td>0.02</td>
</tr>
</tbody>
</table>

the ones who became permanently infertile before giving the first birth. In practice, some households prefer not to have any children which could explain the higher fraction of childless households found in the data.

As discussed previously in Section 1.3.3, keeping track of children’s ages is computationally intractable. Therefore, it was assumed that the number of children at home follows a binomial distribution, identified by a time-invariant probability that a child leaves home in any given period. In Section 2.4, I estimated this constant probability $p$ by targeting the average number of children at home at age 35. Panel D Figure 1.6 shows the simulated age-profile for the number of children at home. The simulated profile does a very nice job matching the data for households younger than 40 years of age. However, the model tends to overstate the number of children at home for older households. The NLSY79 households have on average approximately 1.3 children at home at age 45, while in the model roughly 1.4 children enjoy the benefits of parental care. By parents’ retirement, approximately 0.8 children still reside at home.$^{25}$

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$^{25}$In an alternative calibration exercise, the number of children at home at the household age of 45 was targeted to pin down the parameter $p$. Such calibrated model tends to understate the number of children at home for households between ages 30 and 45, but improves the match for households aged 45+. 

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1.5.3 Expenditures on Children and Time Spent with Children

The calibration results point to a high degree of substitutability between time and market goods, with the elasticity of substitution estimated at about 3.7 (see Table 1.3). The high elasticity of substitution between time and market expenditures in childrearing has implications for the allocation of resources between the high-wage and low-wage households. In the model, low-wage households have a low opportunity cost of spending time at home and, as such, specialize in home production of children’s quality: the correlation coefficient between household wages and time spent on childrearing is -0.23. Since the opportunity cost of staying at home and caring for children increases with household wages, high-wage households prefer to substitute time at home for market expenditures in the model. The model generates a correlation of 0.71 between household wages and goods expenditures on childrearing.

In order to see how this prediction fits the data, I explore the patterns of household expenditures on children by household income in the 2000 CEX data. Figure 1.7 captures the average expenditures on a child under age 6 by dual-earner families by household income group.\(^{26}\) Not surprisingly, the CEX estimates suggest that household spending on the directly measured child-specific items increases with household income.\(^{27}\) In particular, households earning less than $20,000 per year on average spend $120 per quarter on children’s clothes, toys and equipment, while a household making over $75,000 dollars per year spends roughly

\(^{26}\)I focus on households who report positive spending on childcare services (such as baby-sitting or daycare), as reported zero spending on such services by dual-earners points to an alternative childcare arrangement (e.g., childcare is provided by an unpaid family member).

\(^{27}\)CEX collects only limited data on expenditures directly attributable to children or childrearing. In particular, CEX collect information on children’s clothing (for boys, girls and infants), toys, playground equipment, babysitting and daycare.
Figure 1.7: Expenditures on a child under age 6 by dual-earner families (CEX)

1.8 times more. The differences in household spending on babysitting and daycare by income class are, however, more pronounced. Households with yearly incomes of less than $20,000 spend roughly $300 per quarter on babysitting and daycare, while households earning over $75,000 per year spend 3.5-times more on such services. Intuitively, we know that high-income households are likely to spend more money on children's clothing and toys, because they spend more money on cars and houses as well. However, the exponential rise in spending on childcare services relative to low-income households indicates that high-income households indeed substitute some of the time devoted to childrearing with market expenditures.

The variability in the opportunity cost of childrearing over the life cycle has also implications for the predicted temporal allocations of time and market expenditures. Panel A in Figure (1.8) depicts the age-specific correlation between time and goods expenditures
and household market wage, \( w_t \), while Panel B captures age-profile of average time and goods expenditures spent on children. An interesting prediction regarding the allocation of resources over the life cycle emerges from the model. Namely, in a model with deterministic wage growth over the life cycle (as in here), young working families have a low opportunity cost of time relative to older workers. Therefore, young households choose to invest time (rather than money) into children’s production.\(^{28}\) In contrast, older workers have on average a higher opportunity cost of not working and therefore substitute time with market goods in children’s production.

\(^{28}\)Parents below age 20 do not invest any market goods into childrearing. The average expenditures on childrearing by households between ages 20 and 25 are positive but very close to zero.
1.5.4 Fertility and Income

A large number of studies has explored the relationship between income and fertility. Using the analogy with durable goods, Becker (1965) argues that the number of children and income should be positively correlated, but perhaps weakly so because childrearing is “a time-intensive activity that uses many hours which could be used to work” (page 510). While a full consensus on the relationship between income and fertility has not yet been reached (see, for example, Heckman and Walker (1990) for review), most studies agree the life cycle fertility is likely to respond to changes in household wages in non-linear fashion, in part because of the offsetting income and substitution effects. When an important cost of children is parents’ own time, higher wage families face higher price of children relative in terms of other consumption, but as long as children are a normal good, the income effect associated with the higher wage implies that the demand for children should increase. Which effect dominates then determines the sign of the total effect of the wage change on the demand for children.

Panel A in Figure 1.9 captures the correlation between household market wages and the number of births in the model. The correlation is slightly negative (around -0.1) for households between ages 18 and 21. This means that low-wage households have, on average, their first child sooner than high-wage households who face a higher opportunity cost of childrearing than their low-wage peers. However, the correlation coefficient flips in sign at age 22 when an increasing number of high-wage households have the first and second child.\footnote{While low-wage households tend to have the first birth earlier than high-wage households, high wages appear to matter for the timing of the second (and subsequent) birth. For example, the average wage of a household age 22 with one child is approximately 15 percent lower than the average wage of a same-age}
These findings suggest that, in the model, the substitution effect dominates early in the life cycle, while the income effect prevails later in life.

To fully understand this result, recall that the decision to have another child is a lumpy choice, because parents are required to satisfy the minimum requirement for each child’s quality, given by $q$. Extending the family size thus entails increasing the total amount of spending toward improving children’s quality, either in a form of time or market goods. Given that the primary input in childcare for young families is time (see the discussion in Section 1.5.3), high wage levels allow young households to reduce hours worked in the market to satisfy the minimum requirement for their children’s quality, and still be able to consume and save. The reduction in hours worked by high-wage earners with two plus children in turn lowers the household earnings and that is why the correlation between births and household earnings is lower than the correlation between births and wages (Panel B).

1.6 Results on the Relationship between Earnings Risk and Fertility

In this section, I study how changes in the idiosyncratic earnings risk affect the number of births and their timing in the benchmark model. I also discuss the impact of changes in the earnings risk on household labor supply and on the allocation of parental resources devoted toward improving children’s quality. Finally, I test whether the model can match the changes household that is childless. However, a household with two children at home has on average a 60 percent higher wage than a household with one child and roughly 45 percent higher wages than a household with no children.
Figure 1.9: Relationship between fertility, earnings, and wages

Notes: Age-specific correlation between household earnings and births for husband-wife couples (Panel A). Age-specific correlations between market wage and births (Panel B). Data source: NLSY79.

in U.S. household fertility patterns over the last 30 years.

1.6.1 Response of Fertility to Changes in Risk

Figure 1.10 summarizes the predicted relationship between the standard deviation of persistent earnings shock, $\sigma_\epsilon$, household fertility, and the timing of first and second births. As can be seen from the figure, increases in earnings uncertainty have a quantitatively large impact on the timing of births and are also associated with a decline in the number of births.

When risk rises, households postpone the first birth, the average gap between births rises, and the total number of births falls. As long as the standard deviation of earnings remains
relatively low (e.g., $\sigma$ between 0.01 and 0.12), the predicted changes in household fertility are relatively small: the number of births per family declines from roughly 2.6 to 2.4, and the mean age at first birth rises by roughly two years (from 20 to 22 years). At the same time, the average time-gap between the first and second births remains constant. However, the changes in household fertility behavior are more pronounced for medium to high levels of earnings risk. Increasing the earnings risk from 0.12 to 0.21 – an increase broadly in line with the U.S. experience since the 1970s – would reduce the number of births per household from 2.4 to 1.9. This decrease in the number of births is significant not only because of its magnitude (the number of births would fall by over 20 percent), but also because the completed fertility would drop below the replacement rate of 2.1 (i.e., the birth level needed to prevent a demographic decline). The fall in the number of births is accompanied by increases in the age at first and second births. In particular, the age at first birth rises by additional 1.5 years, while the gap between the births widens dramatically as households significantly postpone the birth of their second child: the age at second birth jumps from 25 to 31 years.

These predictions are consistent with empirical findings in Amialchuk (2008) who, using panel data constructed from 1968-1993 waves of the PSID, documents a negative effect of a persistent shock to husband’s earnings on the fertility patterns of married couples. In particular, Amialchuk (2008) finds that women with husbands displaced from a job (due to, for example, a factory closing or a lay-off) postpone births, with the postponement being particularly robust for the second and subsequent births.
Turning to the mechanism generating delayed childbearing and lower fertility response to higher risk, recall that children are discrete, irreversible choices, and that childrearing requires a minimum amount of time and money invested per each child so that the average quality per child is always above $\bar{q}$. Moreover, markets are incomplete and young households have limited access to credit. In this set-up, young households with positive wealth may respond to a fall in household wage by temporarily dis-saving, increasing the labor supply (and thus reducing the hours spent on childrearing), or reducing the market expenditures on children. Since parents prefer to smooth consumption, they initially choose to postpone...
childbearing when labor market risk is high, and work and save more instead.\textsuperscript{30} The delay in childrearing is more pronounced pronounced for higher-order births, as extending the family size requires that parents increase the total amounts of goods and parental time invested toward childrearing even further. While parents may initially consider their decision to delay childbearing as temporary, the infertility risk will tend to reduce the total number of births, and the number of households with no or only one child will rise.

Panel A in Figure 1.11 shows the age-specific birth rates under different levels of risk.\textsuperscript{30} In the model, the average savings rate with low level of risk (i.e., $\sigma = 0.01$) is roughly 10 percent lower than in the baseline.
When risk is low (i.e., $\sigma = 0.01$), an average household has over 2 children by the age of 25. In contrast, when risk rises to the U.S. level (i.e., $\sigma = 0.21$), the birth activity of households is more spread-out, and an average household has only about one child by age of 30.

Panel B in Figure 1.11 demonstrates that increases in the earnings risk have a large effect on the labor supply of young households. In particular, households between ages 16 and 35 would work 30 percent less in an economy with almost no earnings risk than in an environment with U.S. earnings volatility. This is because when risk is low households have children sooner and spend time at home caring for them when risk is low. In contrast, labor supply of households over age 35 is surprisingly inelastic with regard to changes in the earnings risk: under both low and medium levels of earnings risk, the middle-aged households spend approximately half of their time working, in part because older households invest less time and more market goods into childrearing (see the discussion in Section 1.5.3).

Finally, Panel C in Figure 1.11 depicts how the average quality per child evolves over time under different risk profiles. When risk is low, young households have children very early in the life cycle, reducing the quality of each child in exchange for large family sizes. The minimum quality requirement $q$ binds for young households mostly below age 30. Over time, as households’ earnings potential increases and some children leave home, the average quality per child rises again. In an economy with high earnings risk, the minimum quality requirement binds over a longer period of time even though on average households have one child before age 30.
1.6.2 Reconciling the Model’s Predictions with the U.S. Fertility Data

In this section, I examine whether the model can fit the main stylized facts about fertility over the past three decades. In the United States, the mean age at first birth increased by 3.5 years between 1970 and 2000 (with the steepest increase from 1975 to 1985), and women who made childrearing decisions in the 1960s and 1970s had on average 2.5 children compared to 1.9 births for women who made fertility choices in the 1980s and 1990s. The rising mean age and falling number of births coincided with large shifts in microeconomic earnings uncertainty (see, Gottschalk (1997), Levy and Murnane (1992), or Heathcote and Violante (2004) for review). A large share of the observed increases in the earnings risk has been attributed to increases in the persistent component of household earnings (Meghir and Pistaferri, 2004) which, in turn, is commonly assumed to represent the uninsurable idiosyncratic earnings risk.

Using the PSID data, Meghir and Pistaferri (2004) provide yearly estimates of the standard deviation of the persistent shock to labor earnings for the period 1969-1991, and find that the standard deviation of the persistent shock was rising throughout the 1970s and in the early 1980s. Figure 1.10 shows that as $\sigma_\epsilon$ increases from 0.15 (i.e., an arithmetic average for 1969-1979 in Meghir and Pistaferri (2004)) to a benchmark value of 0.21 (i.e., an average

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31 Rather than using the TFR which provides a misleading estimate of the actual rate of childbearing when the age at childbearing is changing (Bongaarts (1999)), I use estimates of completed cohort fertility. The average number of births for women who made childbearing choices in the 1960s and 1970s comes from Jones and Tertilt (2006) who use the Decennial Census data between 1900 and 1990 to construct the average number of births by women’s birth-cohorts. The 2000 wave of the Decennial Census no longer collects information on the number of children ever born. NLSY79 is thus used to compute births of women who made their fertility choices in the later years.
age for 1980-1991), the fertility rate falls by 0.34 (from a value of 2.24 to 1.9 children per household). This represents over a half of the actual decline in the U.S. completed fertility between the 1970s and 1990s. At the same time, the model predicts an increase in the mean age at first birth from 24.4 to 25.5 years and an increase in the mean age at second birth from 26.5 to 31.2 years. Since the actual increases were 3.1 and 3.4 years, respectively, changes in the earnings risk can, therefore, explain about a third of the actual increase in the age at first birth, while the predicted increase in the age at second birth is broadly in line with the data.

1.7 Conclusions

Since the 1970s, the childbearing patterns of U.S. households changed significantly: the completed fertility of U.S. women declined from 2.5 to 1.9, while the women’s mean age at first birth increased by 3.5 years. The rising mean age and the falling number of births coincided with large shifts in uninsurable earnings risk.

This paper explored the relationship between the labor market risk and household fertility choices from several perspectives. I started by documenting two empirical facts about the relationship between fertility patterns and earnings uncertainty in the U.S. data. First, I showed that the U.S. fertility rate is pro-cyclical. Since earnings risk is known to rise during recessions (Storesletten, Telmer, and Yaron, 2004), the pro-cyclical behavior of the U.S. fertility rate suggests that households postpone childbearing during economic downturns when earnings uncertainty is high. Second, by combining the Census micro data with estimates
of the occupational earnings risk for male heads from Saks and Shore (2005), I showed that women with husbands in high risk occupations (e.g., arts and entertainment) tend to have smaller family sizes than women with husbands in low risk occupations (e.g., teachers).

Next, I studied the relationship between household fertility choices and microeconomic uncertainty using a life cycle model of fertility in which unitary households face idiosyncratic uninsurable labor market risk, and make joint decisions about consumption, savings, the family size, and the allocation of parental time and market goods invested toward improving their children’s quality. Children were modeled as durable goods of irreversible nature that are born sequentially, and require at least a minimum amount of parental investment per child. Finally, the decision to have another child could be made only during the first part of the life cycle when parents are fertile. The duration of the fertile period is, however, unknown to parents who face idiosyncratic infertility risk.

The model was calibrated to match the cross-sectional patterns of household fertility, consumption, and earnings, and was employed to study the interaction between the idiosyncratic earnings risk and fertility choices. I showed that higher earnings uncertainty reduces the fertility rate, while increasing the mean age at first and second births. This is because when risk is high, households postpone childbearing, initially preferring to work more and to accumulate more precautionary savings. The birth postponement in turn interacts with the risk of infertility which rises exponentially with the age of the mother, reducing the total number of births per family. Moreover, higher risk was shown to reduce the amount of resources devoted to raising children’s quality. Finally, the model was used to quantify
the contribution of the increase in the idiosyncratic earnings risk to the observed changes in the U.S. fertility patterns. It was shown that the observed increase in the household risk can explain roughly one-half of the fertility decline and one-third of the increase in mean age at first birth in the United States between the 1970s and 1990s, while matching well the changes in the timing of the second birth.

This paper studied the changes in the fertility patterns in the United States, but the completed fertility and the timing of births vary widely across the OECD region. Several recent studies document sizable increases in earnings uncertainty in a cross-section of advanced economies, but also point to the role of labor market institutions, and tax and transfer systems which can offset the rise in household labor market risk (see, for example, Domeij and Floden (2009) or Jappelli and Pistaferri (2009)). In the future, I therefore plan to extend my model to study the differences in fertility across countries with varying levels of earnings uncertainty and social security institutions.

1.8 Appendix

1.8.1 Data sources

NLSY79

The National Longitudinal Survey of Young 1979 (NLSY79) is a nationally representative sample of 12,686 young men and women ages 14 to 22 when they were first interviewed in 1979. These individuals were interviewed annually through 1994 and are currently inter-
viewed on a biennial basis. This study uses the data from 1982 through 2004 to construct fertility moments used either to estimate the model, or used elsewhere in the paper. The information on the number of children ever born, the age at first birth, the number of children at home by respondents, and the wages and salaries for both the respondent and his/her spouse were used to compute the statistics reported in the paper. To compute the average number of births and the mean age at first birth, the 2004 data were used. To construct the life cycle profiles in Figures 1.6 and 1.9, the panel dimension of the data set was employed. To construct the age-profile of correlation between household earnings and births, I define the total household earnings as the sum of the nonnegative reported salaries and wages for a respondent and his/her spouse or unmarried partner. If a respondent does not have a spouse or a partner, the labor income of the spouse is replaced with zero. All earnings variables are deflated by the CPI-U. Sampling weights were employed to create a nationally representative sample of households, and respondents with the missing data for variables used to construct the data moments were dropped from the analysis.

CEX

The Consumer Expenditure Survey (CEX) is a quarterly survey of household expenditures. Household expenditures are aggregated mostly at a household level, with only few expenses being directly attributable to children (i.e., expenses on children’s clothing, toys, equipment, daycare, and babysitting are directly recorded for children). To estimate the OLS regression in equation (1.18), only the observations of child-specific expenditure components for the last quarter for husband and wife consumer units with own children younger than 18 years
of age are used is used to run the auxiliary regression for indirect inference, specified in 
equation (1.18), as well as to construct expenditure profiles in Figure 1.7.

**ATUS**

The American Time Use Survey (ATUS) is a survey on time use in the United States. ATUS respondents (i.e., one household member age 15 or over) are interviewed only one 
time about how they spent their time on the previous day, where they were, and whom 
they were with. Questions related to caring for or helping household children are asked 
in the survey. The survey defines the time spent on caring or helping household children 
as “time spent doing activities to care for or help any child (under age 18), regardless of 
relationship to the survey respondent or the physical or mental health status of the person 
being helped.” In this study, I use the 2004 data on “primary childcare activities.” These 
activities include time spent providing physical care; playing with children; reading with 
children; assistance with homework; attending children’s events; taking care of children’s 
healthcare needs; and dropping off, picking up, and waiting for children. Passive childcare 
done as a primary activity (such as “keeping an eye on my son while he swam in the pool”) 
also is included. Only respondents with (i) a spouse or unmarried partner and (ii) own 
children in the household have been included in the analysis. These reported data on minutes 
spent on primary childcare activities in the previous day have been used to estimate the 
auxiliary regression in equation (1.19), and to construct the profiles of time spent with 
children used elsewhere in the paper.

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32In the survey, a child’s presence during the activity is not enough in itself to classify the activity as childcare. For example, “watching television with my child” is coded as a leisure activity, not childcare.
Census

I use the 5 percent sample of the Decennial Census from 1990, available publicly at the IPUMS-USA website. I concentrate on married couples. Matching respondents with their spouses yields a total of about 2.5 million husband-wife observations. Since educational attainment is known to affect the timing of births, I consider only households in which the husband attained a bachelor’s degree. Similarly, since fertility patterns are known to vary between urban and rural areas, I consider only households which resided at urban, non-farm regions at the time of the interview. These subsampling criteria decrease the sample’s heterogeneity without restricting the sample size excessively. In order to explore the effect of the husband’s occupation on household fertility, I use the Census 1950 three-digit occupational codes to assign husbands to one of the seven occupational groups from Saks and Shore (2005). I assign husbands the following occupations: teachers, heath-care professionals (e.g., medical and dental technicians, nurses, optometrists, and pharmacists), engineers, managers, math and sciences (e.g., mathematicians, physicists, and other natural scientists), sales workers, and arts and entertainments (e.g., artists and arts teachers, actors, dancers and dance teachers, musicians and music teachers). Husbands whose occupation lies outside those studied in Saks and Shore (2005) have been omitted from the analysis. Since self-employed individuals have been shown to face higher earnings risk than individuals working for wage or salary across occupations (see, for example, Carroll and Samwick (1997) or Saks and Shore (2005)), I focus on individuals who are not self-employed.
1.8.2 Numerical solution and algorithm

The household problem is solved numerically by backward recursion from the terminal period. At each state, I solve for the value function and the optimal policy rules, given the current state variables and the solution to the value function in the next period. In the model, households face sterility shocks which render them permanently infertile. A household with fertile members and a state vector \((a_t, n_t, \epsilon_t, \nu_t, f_t = F, t)\) solves the problem

\[
V(a_t, n_t, \epsilon_t, \nu_t, F, t) = \max \left\{ \begin{array}{c}
\max_{a_{t+1} \times t, t} u(c_t, n_t, q_t) + \beta E_t V(a_{t+1}, n_{t+1}, \epsilon_{t+1}, \nu_{t+1}, f_{t+1}, t + 1) \\
\max_{a_{t+1} \times t, t} u(c_t, n_t, q_t) + \beta E_t V(a_{t+1}, n_{t+1}, \epsilon_{t+1}, \nu_{t+1}, f_{t+1}, t + 1)
\end{array} \right. 
\]

subject to the constraints and transition equations specified in equations (1.10) to (1.14), with \(f_{t+1} = \{I, F\}\).

A household with infertile members and state vectors \((a_t, n_t, \epsilon_t, \nu_t, f_t = I, t)\) solves the recursive problem of the form

\[
V(a_t, n_t, \epsilon_t, \nu_t, I, t) = \max_{a_{t+1} \times t, t} u(c_t, n_t, q_t) + \beta E_t V(a_{t+1}, n_{t+1}, \epsilon_{t+1}, \nu_{t+1}, I, t + 1),
\]

subject to the constraints and transition equations in equations (1.10), (1.11), (1.14), and (1.15).

The complications for the solution of the household problem arise from the presence of a discrete choice (have a child next period or not). The discrete fertility choice implies that the value function will not necessarily be concave or differentiable at any stage of the life cycle. Therefore, I employ finite dynamic programming methods and only approximate the
solution to the household problem by solving the household problem on a grid.

The algorithm used to solve the household problem is as follows. First, I guess the values for the parameters to be estimated. Given the guesses (and given the remaining parameters summarized in Table 2.1), I use finite dynamic programming to solve for optimal decision rules for savings \( a(a_t, n_t, \epsilon_t, \nu_t, f_t, t) \), number of children at home \( n(a_t, n_t, \epsilon_t, \nu_t, f_t, t) \), and time \( l(a_t, n_t, \epsilon_t, \nu_t, f_t, t) \) and market expenditures \( x(a_t, n_t, \epsilon_t, \nu_t, f_t, t) \) devoted to childrearing. Next, I simulate the shock histories for 10,000 households. Using the simulated histories and the optimal decision rules, I compute the targeted moments for the model economy. Last, I use the method of simulated moments to pin down the values for estimated parameters which produce moments summarized in Table (1.3). Since the differentiability of the objective function in the estimated parameters is not guaranteed, I use a minimization procedure that does not rely on the existence of the gradient (simplex). Once the estimated parameters are identified, I resolve the household problem and save the optimal decision rules.
Chapter 2

Run-up in the House Price-Rent Ratio: How Much Can Be Explained by Fundamentals?

2.1 Introduction

The sharp increase and subsequent collapse in U.S. house prices over the past decade has been well documented. Real house prices rose by only 3.7 percent between 1985 and 1995, but increased by 46 percent between 1995 and 2005. In sharp contrast, real rents remained virtually unchanged during the recent increase in house prices, so that in 2006 the house price-rent ratio peaked at approximately forty percent above its level in the year 2000 (Figure 2.1). The house price-rent ratio is widely used as an indicator of over and undervaluation of the housing market. Yet, despite the widespread use of the price-rent ratio as a key housing market statistic, surprisingly little is known about the theoretical relationship between the price-rent ratio and market fundamentals such as interest rates, income, down payment
requirements, and features of the U.S. tax code which favor homeownership over renting and provide sizable tax subsidies to landlords.

This paper bridges the gap in the existing literature by studying the joint dynamics of endogenously determined house prices and rents in a dynamic equilibrium model of housing tenure choice with fully specified markets for homeownership and rental properties. Our framework is an Aiyagari-Bewley-Huggett style economy with a stochastic life cycle and heterogeneous households who are subject to idiosyncratic earnings shocks. Households derive utility from nondurable consumption and shelter services which are obtained either via renting or through homeownership. Markets are incomplete. Households can partially self-insure earnings risk by accumulating precautionary financial assets: deposits. In addition to deposits, households can hold a non-financial asset: houses. Houses are modeled as durable, indivisible, discrete-sized items which provide housing services, grant access to collateralized borrowing, and can serve as a source of rental income for homeowners who choose to become landlords. The supply of rental housing is thus determined endogenously within the model, as homeowners weigh their utility from shelter space against rental income, taking into account the tax implications of their decisions.\footnote{Using the data from the Property Owners and Managers Survey, Chambers, Garriga, and Schlagenhauf (2008) use micro data evidence to document that a vast majority of U.S. rental property is owned by households, rather than firms. Namely, 86 percent of the U.S. rental property is owned by individual investors (or husband and wife), and fully 94 percent of all rental property is owned by non-institutional investors. The remainder is controlled by real estate corporations, other corporations, non-profit organizations, or church.} Mortgages are available to finance purchases of housing, but home-buyers must satisfy a minimum down payment requirement. Moreover, home purchases and sales are subject to lumpy transaction costs and the housing stock is subject to depreciation. Households who do not own houses rent housing services in the rental
market and do not have access to borrowing or to the preferential tax treatment of owner-occupied housing and rental properties embedded in the U.S. tax code. Both house prices and rents are determined in equilibrium through clearing of housing and rental markets.

The calibrated model is used to study the impact of macroeconomic factors such as incomes, interest rates, and borrowing constraints on the equilibrium price-rent ratio. Our rational expectations model of the housing market demonstrates that the rising incomes, historically low interest rates, and easing of down payment requirements observed in the

Figure 2.1: FHFA House Price Index and BLS Rent of Primary Residence Index
data can explain about one-half of the increase in U.S. house prices between 1995 and 2005.\(^2\)

In addition, the model predicts that changes in these factors will have only a small positive effect on equilibrium rents, a result that is consistent with the U.S. data.\(^3\) The price and rent dynamics generated by the model coincide with increases in the homeownership rate and household debt-to-income ratio that are also similar to the actual developments in the U.S. housing market between 1995 and 2005.\(^4\)

The key mechanism in the model generating the run-up in the equilibrium price-rent ratio as macroeconomic conditions change is that the supply and demand of rental property are endogenously determined jointly with the demand for housing. When the mortgage interest rate and required down payment fall, the demand for rental property falls because households switch from renting to owning as homeownership becomes more affordable. At the same time, the supply of rental property increases because investment in rental property becomes more attractive relative to the alternative of holding bank deposits as the interest rate falls.\(^5\) As a result, the equilibrium rent falls. At the same time, the demand for housing increases because more households can afford to purchase homes, and existing homeowners

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\(^2\)A large body of empirical literature has investigated the relationship between house prices and macroeconomics aggregates. For example, regression analysis by by Englund and Ioannides (1997), Malpezzi (1999), Muellbauer and Murphy (1997), Muellbauer and Murphy (2008), Otrok and Terrones (2008) show that real interest rates, income, income growth, and financial liberalization have a statistically significant effect on the dynamics of real house prices.

\(^3\)Poterba (1984), Topel and Rosen (1988) and Muellbauer and Murphy (1997) model the relationship between house prices and rents using asset pricing models which predict that the expected return on housing equals (up to a constant) the rate of return on alternative investments. In general, this type of model cannot explain the coexistence of rising house prices and relatively constant or declining rents.

\(^4\)The total household debt to disposable income ratio has increased from 80 percent in 1985 to 93 percent in 1995 and to a whopping 141 percent in 2007. At the same time, the U.S. homeownership rate, initially flat at 64 percent between 1983 and 1995, rose to 69 percent by 2005.

\(^5\)In the United States, the buy-to-let markets have grown substantially since the mid-1990s (OECD, 2006). The portion of sales attributable to such investors has risen sharply since the late 1990s, reaching around 15 percent of all home purchases in 2004, much higher than the normal 5 percent (Morgan Stanley, 2005).
can afford larger homes. Given that the supply of housing is fixed, the equilibrium house price rises. An increase in income that is symmetric across all wage groups leads to a roughly proportional increase in house prices and rents, leaving the price-rent ratio unchanged, as it roughly offsets the initial decline in rents while further boosting house prices.

The model provides a number of additional insights about the mechanisms that jointly determine house prices and rents. Both the house price and rent are relatively inelastic with respect to the down payment requirement, so a lessening of credit constraints cannot by itself account for the run-up in the house prices observed in recent years. The key to understanding the small effect of decreases in the required down payment on equilibrium house prices is to realize that changes in equilibrium house prices are primarily driven by shifts in the housing demand by households who find the minimum down payment a binding constraint and, therefore, increase their demand for housing when the lending standards are relaxed. However, relative to the entire market demand for housing, this increase in demand is relatively small, so the resulting house price increase is small. The corresponding increase in household borrowing as credit constraints are relaxed is skewed toward low-income households, as poorer households gain access to mortgage markets and borrow large amounts relative to their labor income to finance their home purchases.

Furthermore, we find that falling interest rates create large increase in house prices, since cheap credit and a low opportunity cost of borrowing boost household willingness and ability to purchase big properties and to finance them using large mortgages. In our economy with a fixed supply of housing, a falling interest rate thus pushes up house prices.
As expected, falling interest rates lead to a large increase in household borrowing, since the low interest rate decreases the cost of mortgage financing and, at the same time, lowers the return on household savings. Somewhat surprisingly, a decline in the interest rate reduces the homeownership rate. This happens because as the interest rate falls and equilibrium house prices rise, some low income households are no longer able to afford the minimum down payment on a house.

This paper builds on the growing body of literature which studies housing using quantitative macroeconomics models with heterogeneous households. See, for example, Díaz and Luengo-Prado (2008), Chambers, Garriga, and Schlagenhauf (2008), Chambers, Garriga, and Schlagenhauf (2009a), Chambers, Garriga, and Schlagenhauf (2009b), Favilukis, Ludvigson, and Van Nieuwerburgh (2009), Kiyotaki, Michaelides, and Nikolov (2008), Nakajima (2008), Ríos-Rull and Sánchez-Marcos (2008), Ortalo-Magné and Rady (2006), and Iacoviello and Neri (2007). The studies most closely related to ours are Chambers, Garriga and Schlagenhauf (2008, 2009a, 2009b) and Díaz and Luengo-Prado (2008) in terms of the model, and Favilukis, Ludvigson, and Van Nieuwerburgh (2009) and Kiyotaki, Michaelides, and Nikolov (2008) in terms of the theme. Díaz and Luengo-Prado (2008) build a partial equilibrium economy with a number of realistic features such as collateral borrowing, non-convex adjustment costs, taxes, and idiosyncratic earnings risk. However, in their model, housing and rental markets exist only insofar as both house prices and rents follow exogenous processes. Chambers, Garriga and Schlagenhauf (2008, 2009a, 2009b) use the American Housing Survey to document that the vast majority of U.S. rental property is owned by households
instead of firms, and develop a model where rental property is supplied by households who choose to become landlords as a result of optimal investment strategies. However, the authors allow rents but not house prices to be determined endogenously within their model.

This paper adopts the structure of rental markets from Chambers, Garriga, and Schlagenhauf (2009a), but also explicitly models a housing market so that both house prices and rents are determined in an equilibrium. Turning to the dynamics of the price-rent ratio, Kiyotaki, Michaelides, and Nikolov (2008) briefly explore the equilibrium relationship between house prices and rents in a more stylized model where production capital (i.e., factories) can be costlessly transformed into housing structures, and where rent is determined as a factor price of this production capital. The authors, however, focus primarily on the response of welfare to changes in fundamentals. Lastly, Favilukis, Ludvigson, and Van Nieuwerburgh (2009) study the evolution of the price-rent ratio, but their model does not include a rental market. Instead, they impute rent for homeowners using the marginal rate of substitution between consumption and housing. Moreover, the supply of housing in their economy is highly elastic, as the authors abstract from features such as a fixed supply of land or fixed

6Alternative models that allow for renting typically adopt the representative zero-profit rental firm framework as in Gervais (2002) or Nakajima (2008) in which the supply of rental property is perfectly elastic and, by construction, rents are positively correlated with house prices through a simple arbitrage condition. However, this positive correlation does not always hold in the data. For example, Panel B of Figure 2.1 shows that there have been protracted periods during which U.S. house prices grew while rents declined. We therefore follow Chambers, Garriga, and Schlagenhauf (2009a) and assume that rental property is supplied by households who choose to become landlords as a result of optimal investment strategies. This approach to modeling the rental market allows the supply of rental property to respond to changes in fundamentals in a non-trivial fashion so that the positive correlation between house prices and rents need not hold. In addition, this framework accounts for the effects of moral hazard in rental markets and the preferential tax treatment of landlords on the supply of rental property.

Chambers, Garriga and Schlagenhauf (2008, 2009a, 2009b) have, however, other equilibrium objects, such as interest rates.

In a model such as ours with discrete choices, lumpy adjustment costs, and borrowing constraints, the relationship between the MRS, market rent, and the cost of housing is theoretically ambiguous.
supply of housing.

This paper is organized as follows. In Section 2.2, we develop a quantitatively rich stochastic life cycle model of the housing market with fully specified household choices with respect to consumption, saving, and homeownership, and provide rationale for our modeling assumptions. Section 2.3 defines the equilibrium of the economy, while Section 2.4 describes the model’s calibration and discusses the fit of the benchmark model. In Section 2.5, we discuss predictions of the benchmark model, and reconcile these with the actual dynamics of house prices and rents in the U.S. data. Section 2.6 concludes with a discussion of possible extensions and directions for the future research.

2.2 The Model Economy

The baseline is a small open economy in steady state with inflexible supply of housing and endogenously determined supply of rental properties. The time-invariant house price and rent are determined endogenously within the model through clearing of housing and rental markets.

2.2.1 Demography and Endowments

Our framework is an overlapping generations heterogenous-agent economy with incomplete markets and uninsurable idiosyncratic income risk.\(^9\) We follow Heathcote (2005) in modeling the life cycle as a stochastic transition between various labor productivity states that, in a

\(^9\)As discussed in Castaneda, Díaz-Gimenez, and Ríos-Rull (2003), when insurance markets are allowed, the model economy collapses to a representative agent model, as long as the right initial condition holds.
stylized way, also allows households to age. Namely, we use the one-dimensional stochastic state variable, $w$, to denote the household’s labor endowment. We assume that the process for $w$ is independently and identically distributed across households, that it takes on values in the $J$-dimensional set \{\(w_1, \ldots, w_J\)\} = \mathcal{W}, and that it follows a finite-state Markov chain $\pi_w (w'|w)$ which is intended to parsimoniously estimate a richer stochastic process. A detailed description of the endowment income process is presented in Section 2.4.1. In this model, we do not allow for inter-generational transfers of wealth (financial or non-financial) or human capital. Instead, we assume that, upon death, estates are taxed at a 100 percent rate by the government and immediately resold, and young households are born as renters and can accumulate assets only gradually through saving or housing investment.\(^{10}\)

### 2.2.2 Preferences

Each household derives utility from consumption of a nondurable good, $c$, (which is the numeraire) and shelter services, $s$, provided by residential capital, $h'$.\(^{11}\) The expected lifetime utility of a household who does not value leisure is

$$E_t \sum_{t=0}^{\infty} \beta^t \chi (s_t, h_{t+1}) u (c_t, s_t), \quad (2.1)$$

and $\beta \in (0, 1)$ is the time-discount factor.

---

\(^{10}\)This removes the bequest motive from the saving decision. To ensure that such assumption does not lead households to excessively borrow during their lives, we carefully calibrate the model (see Section 2.4 ) to ensure that the household borrowing patterns align with the data.

\(^{11}\)We suppress the index of household $i$ when we describe a typical household. Furthermore, the notation $x'$ denotes the value of generic variable $x$ at the end of the period (or equivalently, the instant a new period begins). For example, $h'$ is the level of housing chosen by an agent after within-period shocks have been realized.)
Shelter services may be obtained either via a rental market at a constant price $\rho$ per unit of housing, or through ownership of housing at a constant price $q$ per unit of housing.\footnote{The prices $(q, \rho)$ are time-invariant due to the fact that we solve for the steady-state of the economy. For details, see Huggett (1993) or others.} A linear technology transforms the housing investment, $h'$, into housing services, $s$, so that one unit of housing provides one unit of shelter services. Households cannot rent and be homeowners at the same time, i.e. $s \leq h'$. Homeowners can, however, become landlords. Namely, as in Chambers, Garriga and Schlagenhauf (2007), homeowners may choose to set $s < h'$, in which case $(h' - s) =: l$ is leased to renters at rental rate $\rho$. Being a landlord, however, implies a constant utility loss $\chi$ caused by the burden of managing and maintaining a rental property. The landlord utility loss is

$$\chi(s, h') = \begin{cases} 1 - \chi & \text{if } s < h' \\ 0 & \text{otherwise.} \end{cases} \quad (2.2)$$

### 2.2.3 Assets and Market Arrangements

There are three types of assets in the economy: residential capital, $h \geq 0$, deposits, $d \geq 0$, and collateral debt, $m \geq 0$, taking on values in sets $\mathcal{H}$, $\mathcal{D}$ and $\mathcal{M}$, respectively. Deposits offer an exogenous return $r$, while collateral debt (mortgage debt and equity loans) carries an exogenous interest payment $r^m$. There is no uncertainty about interest rates. Households may alter their individual holdings of the assets $h, d$, and $m$ to the new levels $h', d'$, and $m'$ at the beginning of period. Homeownership is lumpy in that houses have a minimum size (i.e., $h_t \geq h$), and come in discrete sizes (i.e., $h_t \in \{0, h(1), ..., h(m)\}$). Agents also make
a discrete choice about shelter consumption. Households can rent a small unit of shelter, $\underline{s}$, which is smaller that than the minimum house size available for purchase, $\underline{s} < h(1)$. To maintain symmetry between shelter sizes available to homeowners and renters, we assume that all other levels of shelter consumption must match a point on the housing grid, so $s_t \in \{\underline{s}, h(1), ..., h(m)\}$.

Only households with residential capital (i.e., homeowners) can access to collateralized borrowing. In particular, we assume that, in any given period, a homeowner faces the borrowing constraint

$$m' \leq (1 - \theta)q h'$$  \hspace{1cm} (2.3)$$

with a minimum equity requirement, $\theta > 0$. The equity requirement effectively disposes of free-entry to the housing market, since households interested in buying a house with a market value $q h'$ must put down at least a fraction $\theta$ of the value of the house. By the same token, households who wish to sell their house and move to a different size house or become renters must repay all the outstanding debt, since the option of a mortgage default is not available. The accumulated housing equity above the down payment can, however, be used as collateral for home equity loans.\footnote{Similarly to Díaz and Luengo-Prado (2008), we abstract from income requirements when purchasing houses. See their paper for further discussion.} Moreover, households can access the additional housing equity through costless refinancing. In general, the collateral borrowing is modeled in a spirit of home lines of credit: households with collateral debt are subject to only the per-period interest payments, but do not need to make payments toward the principle.\footnote{Chambers, Garriga and Schlagenhauf (2006) and Campbell and Cocco (2003) offer a more complete analysis of mortgage choice. See Li and Yao (2005) for an alternative model with refinancing costs.}
There are no other limits to credit availability: regardless of age or income, if a household can pay the down payment, they receive a mortgage.\textsuperscript{15}

The total housing stock, $H$, is fully owned by households and its size does not change over time.\textsuperscript{16} Our set-up with endogenous house prices and inflexible housing supply thus represents an alternative to a production economy where land – the input factor into the housing production – is in fixed supply.

Buying and selling a house is costly: a fraction of the house value is lost when bought or sold. A household which buys a house pays a transaction cost, $\tau^b$, proportional to the value of the new house (the total buying cost thus equals $\tau^b qh'$). Similarly, a household which sells a house pays a transaction cost, $\tau^s$, proportional to the value of the old house, so selling costs equal $\tau^s qh$. Since there are no realtors in this model, we model the transaction costs as taxes, but interpret them as brokerage fees and other costs related to moving. Importantly, the presence of transactions costs makes housing a relatively illiquid asset, and can generate sizeable inaction regions with regard to the household decision to buy or sell.

Homeowners incur maintenance expenses, which for convenience we take to be immediate. The actual expense depends both upon the value of housing and upon the level of $s$ in relation to $h'$ (e.g., the amount of the property that is rented to other households). Housing which

\textsuperscript{15}As discussed in Section 2.2.1, if the household dies, the government receives the housing asset and resells it right away.

\textsuperscript{16}Indeed, the available empirical evidence suggests that the housing supply grew in the U.S. metropolitan regions grew only modestly since 1995. Namely, according to the Census data, the median square footage per housing unit increased by 4 percent between 1997 and 2007 in the United States, but most of these increases were observed outside the metropolitan statistical areas. For example, outside MSAs, the median square footage increased by 13 percent between 1997 and 2007. In a sharp contrast, the median square footage per housing unit in MSA cities decreased at -0.2 percent between 1997 and 2007, while in MSA suburbs the square footage per house grew by 1.5 percent over the period. Moreover, the increases in the aggregate housing supply coincided with population growth which increased the U.S. population increased by 12.5 percent between 1997 and 2007 (4.7 percent between 2000 and 2005).
is consumed by the owner depreciates at rate $\delta_o$. We assume that a moral hazard problem exists in the rental market for housing services, namely that housing occupied by a renter depreciates more rapidly than owner occupied housing. This problem arises because renters decide how intensely to utilize a house but may not actually pay the resulting cost, which creates an incentive to overutilize the property. The depreciation rate for rented property is $\delta_r$, and $\delta_r > \delta_o$. Thus, current total maintenance costs facing an agent who has just chosen housing equal to $h'$ are given by

$$M(h', s) = I_{h'\neq 0} [\delta_0 s + I_{h'>s} \delta_r (h' - s)],$$

(2.4)

with the binary indicator $I_{h\neq 0}$ denoting that a household is a homeowner, and $I_{h'>s}$ indicating that a household is also a landlord.

### 2.2.4 The Government

We follow Díaz and Luengo-Prado (2008) in modeling a tax system with a preferential tax treatment of owner-occupied housing that mimics the U.S. system in a stylized way. Namely, in addition to the taxation of household labor and asset income, the government imposes a proportional property tax on housing which is fully deductible from income taxes, and allows deductions for interest payments on collateral debt (mortgages and home equity). As in the U.S. tax code, the imputed rental value of owner-occupied housing is excluded from taxable income. We expand on the tax treatment of rental property in existing models of the housing market by allowing landlords to deduct depreciation of the rental property from
their taxable income. For simplicity, we assume proportional income taxation at the rate $\tau^y$. We do not require a balanced budget every period.

The total taxable income is thus defined as

$$\tilde{y} = w + rd + I^{h' \neq 0}[-\tau^m r^m_m - \tau^h q h'] + I^{h' > s}[\rho (h' - s) - \tau^{LL} q (h' - s) - \delta_r q (h' - s)],$$

(2.5)

where $w + rd$ represents household labor income plus earned interest. The first term in brackets represents the tax deduction received by homeowners, where $\tau^m r^m_m$ is the mortgage interest deduction, and $\tau^h q h'$ is the fully deductible property tax payment made by the household. The next term in brackets represents the taxable rental income of landlords, which equals total rents received, $\rho (h' - s)$, minus the tax deductions available to landlords. The term $\tau^{LL} q (h' - s)$ represents the tax deduction for depreciation of the rental property, where $\tau^{LL}$ represents the fraction of the total value of the rental property that is tax deductible in each year. The final term that determines taxable rental income, $\delta_r q (h' - s)$, represents tax deductible maintenance expenses. If the tax deductions for the rental property exceed rental income, so $\rho (h' - s) < \tau^{LL} q (h' - s) + \delta_r q (h' - s)$, then rental losses will reduce the households’ tax liability by offsetting income from wages and interest, $w + rd$.

At this point it is useful to discuss the current U.S. tax treatment of landlords and explain how the key features of the tax code are incorporated into our model. Landlords must pay income taxes on rental income. However, landlords are permitted to deduct many different expenses associated with operating a rental property from their gross rental income when determining the amount of rental income that is subject to income taxes. Among
the major tax deductible rental expenditures incorporated into our model are mortgage interest payments, property taxes paid on the rental property, depreciation of the rental structure, and maintenance expenditures.\textsuperscript{17} The amount of the depreciation deduction is specified in the U.S. tax code, and we discuss the exact depreciation rate used in our model in Section 2.4. In addition, landlords who meet a minimum standard of involvement with their rental property may use rental losses to offset income earned from sources other than real estate.\textsuperscript{18}

2.2.5 Households’ Problem

Each period the economy-wide state is a measure of households, $\lambda$, defined over $B$, an appropriate family of subsets of $\{D \times M \times H \times W\}$. As far as each individual household is concerned, the state variables are the realization of the household-specific shock, $w$, the current asset position, $(d, m, h)$, and the aggregate state, $\lambda$. Let $x = (w, d, m, h)$. In a steady state, the measure of households, $\lambda$, remains time-invariant, implying that household’s state variable is simply the vector $x$.

\textsuperscript{17}Other expenses that are tax deductible but not incorporated in our model are expenses related to advertising, travel to the rental property, commissions, insurance, legal and professional fees, management fees, supplies, and utilities. See IRS publication 527 for details on the tax treatment of residential rental property.

\textsuperscript{18}A maximum of $25,000 in rental property losses can be used to offset income from other sources, and this deduction is phased out between $100,000 and $150,000 of income. In our stylized model we abstract away from the $25,000 limit and we do not incorporate the phasing out of this deduction for high income households into our model of the tax system.
Timing of events

A household starts any given period $t$ with a stock of residential capital, $h \geq 0$, deposits, $d \geq 0$, and collateral debt (mortgage debt and equity loans), $m \geq 0$. Households observe the idiosyncratic earnings shocks, $w$, and – given the current prices $(q, \rho)$ – choose new levels of nondurable consumption, $c$, shelter, $s$, as well as their new asset position $(h', d', m')$. Namely, homeowners $(h > 0)$ choose whether to adjust the size of their house (so that $h' \neq h$), and whether or not to become a landlord ($h' > s$). Households currently renting $(h = 0)$ choose whether to continue to rent ($h' = 0$), or enter the housing market ($h' > 0$). If a household enters the housing market, they can become a landlord. Households receive interest on deposits, $r$, and pay interest on collateral debt, $r^m$. There is no uncertainty about interest rates. Landlords receive rent payments from their tenants, $\rho(h' - s)$. Households pay taxes and homeowners cover maintenance cost, $qM(h', s)$. Households which are buying or selling a house $(h' \neq h)$ incur transaction cost $\tau^b qh'$ and $\tau^s qh$, respectively. In particular, homeowners who increase or decrease the size of their homes pay both the buying and selling fees. Renters who newly become homeowners incur buying fees only. Similarly, former homeowners who sell their property and become homeowners incur selling fees only.

The Dynamic Programming Problem

Each period, a household whose state is $x = (w, d, m, h)$ solves the dynamic program:

$$v(w, d, m, h) = \max_{c, s, h', d', m'} \chi(s, h) u(c, s) + \beta \sum_{w' \in W} \pi(w' | w) v(w', d', m', h')$$  \hspace{1cm} (2.6)
subject to the constraints

\[
c + \rho (s - h') + d' - m' + q(h' - h) + I^s\tau^s qh + I^b\tau^b qh' \\
\leq w + (1 + r)d - (1 + r^m) m - \tau^y\tilde{y} - \tau^b qh' - qM (h', s)
\]

\[
m' \leq (1 - \theta) qh'
\]

\[
m' \geq 0
\]

\[
d' \geq 0
\]

\[
h' \geq s
\]

by choosing consumption, \(c\), and shelter, \(s\), as well as current levels of housing investment, \(h'\), deposits, \(d'\), and collateral debt, \(m'\). \(\rho (s - h')\) represents either a rental payment by renters (i.e., households with \(h' = 0\)), or the rental income received by landlords (i.e., households with \(h' > s\)). \(q(h' - h)\) captures the cost of new housing investment over its current value. \(\tau^s qh\) represents the transaction fees incurred when a property is sold (i.e., \(I^s = 1\) if \(h_t \neq h_{t-1} > 0\); zero otherwise), while \(\tau^b qh'\) captures the fees incurred when a new property is purchased (i.e., \(I^b = 1\) if \(0 < h_t \neq h_{t-1}; zero otherwise). \(w\) represents the household income and follows a process \(\pi_w(w_t|w_{t-1})\) described in Section 2.2.1. \(rd\) and \(r^m m\) capture the interest income on deposits and the mortgage payment, respectively. \(\tau^y\tilde{y}\) is the total income tax paid of the taxable income \(\tilde{y}\) in Equation 2.5. \(\tau^b qh'\) describes the property tax paid by homeowners. Finally, \(qM (h', s)\) represents the maintenance expenses for homeowners in Equation 2.4.
2.3 Definition of a Stationary Equilibrium

A steady state equilibrium for the baseline economy is a household value function, \( v(x) \), a household policy \( \{c(x), s(x), d'(x), m'(x), h'(x)\} \), a probability measure of agents over the individual states, \( \lambda \), and price vector \( (q, \rho) \) satisfying:

1. \( c(x), s(x), d'(x), m'(x), \) and \( h'(x) \) are optimal decision rules to the households’ decision problem, given prices \( q \) and \( \rho \)

2. Markets clear:
   
   (a) Housing market clearing: \( \int h'(x)d\lambda = H \), where \( H \) is fixed
   
   (b) Rental market clearing: \( \int (h'(x) - s(x))d\lambda = 0 \),

   where integrals are defined over the state space \( \{D \times M \times H \times W\} \).

3. \( \lambda \) is a stationary probability measure.

2.4 Calibration

The method of simulated moments is used to calibrate the model based on cross-sectional patterns of income, wealth, homeownership, and landlord characteristics. Table 2.1 summarizes parameters which were drawn from other studies or were calculated directly from the data. Table 2.2 contains four estimated parameters based on the moments described in Table 2.3.
Table 2.1: Exogenous Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation $\rho_w$</td>
<td>0.90</td>
</tr>
<tr>
<td>Standard Deviation $\sigma_w$</td>
<td>0.20</td>
</tr>
<tr>
<td>Risk Aversion $\sigma$</td>
<td>2.00</td>
</tr>
<tr>
<td>Down Payment Requirement $\theta$</td>
<td>0.20</td>
</tr>
<tr>
<td>Selling Cost $\tau^s$</td>
<td>0.07</td>
</tr>
<tr>
<td>Buying Cost $\tau^b$</td>
<td>0.025</td>
</tr>
<tr>
<td>Risk-free Interest Rate $r$</td>
<td>0.04</td>
</tr>
<tr>
<td>Spread $\kappa$</td>
<td>0.015</td>
</tr>
<tr>
<td>Depreciation Rate for Homeowner-Occupiers $\delta_0$</td>
<td>0.025</td>
</tr>
<tr>
<td>Property Tax Rate $\tau^h$</td>
<td>0.01</td>
</tr>
<tr>
<td>Mortgage Deductibility Rate $\tau^m$</td>
<td>1.00</td>
</tr>
<tr>
<td>Deductibility Rate for Depreciation of Rental Property $\tau^{LL}$</td>
<td>0.023</td>
</tr>
<tr>
<td>Income Tax $\tau^y$</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 2.2: Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor $\beta$</td>
<td>0.959</td>
</tr>
<tr>
<td>Consumption Share $\alpha$</td>
<td>0.720</td>
</tr>
<tr>
<td>Depreciation of Rental Property $\delta_r$</td>
<td>0.037</td>
</tr>
<tr>
<td>Landlord Utility Loss $\chi$</td>
<td>0.024</td>
</tr>
</tbody>
</table>

2.4.1 The Endowment Process

A time period in the model is one year. As discussed previously, we consider a version of the stochastic-aging economy that is designed to capture the idea that liquidity constraints may be most important for younger individuals who are at the bottom of an upward-sloping lifetime earnings profile. We follow Heathcote (2005) and allow households to transit from state $w$ via two mechanisms: (i) aging and (ii) productivity shocks, where the events of aging and receiving productivity shocks are assumed to be mutually exclusive. The probability of
Table 2.3: Calibration Targets

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home-ownership rate</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>Landlord rate</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Imputed rent-to-wage ratio</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Fraction of homeowners with collateral debt</td>
<td>0.65</td>
<td>0.64</td>
</tr>
</tbody>
</table>

transiting from a state $w_j$ via aging is equal to $\phi_j = 1/(p_jL)$, where $p_j$ is the fraction of population with productivity $w_j$ in the ergodic distribution over the support $W$, and $L$ is a constant equal to the expected lifetime. Similarly, the conditional probability of transiting from a working-age state $w_j$ to a working-age state $w_i$ due to a productivity shock is defined as $P(w_i|w_j)$. The overall probability of moving from state $j$ to state $i$, denoted by $\pi_{ji}$, is therefore equal to the probability of transition from $j$ to $i$ via aging, plus the probability of transition from $j$ to $i$ via a productivity shock, conditional on not aging, so that

\[
\Pi = \begin{bmatrix}
0 & \phi_1 & 0 & 0 \\
0 & 0 & \ldots & 0 \\
0 & 0 & 0 & \phi_{J-1} \\
\phi_J & 0 & 0 & 0
\end{bmatrix} + \begin{bmatrix}
(1 - \phi_1) & 0 & 0 & 0 \\
0 & \ldots & 0 & 0 \\
0 & 0 & (1 - \phi_{J-1}) & 0 \\
0 & 0 & 0 & (1 - \phi_J)
\end{bmatrix} P. \quad (2.11)
\]

The fractions $p_j$ are the solutions to the system of equations $p = p\Pi$.

To calibrate the stochastic aging economy, we assume that households live, on average, 50 periods (e.g., $L = 50$). In terms of the process for household productivity, many papers in the quantitative macroeconomics literature adopt simple AR(1) specification to capture the earnings dynamics for working-age households that is characterized by the serial correlation
coefficient, $\rho_w$, and the standard deviation of the innovation term, $\sigma_w$.\footnote{Heathcote (2005) discusses alternatives to the AR(1) specification in a technical appendix which is available on the Review of Economic Studies web site.} Using the data from the Panel Study of Income Dynamics (PSID), work by Card (1991b), Hubbard, Skinner, and Zeldes (1995a) and Heathcote, Storesletten and Violante (2003) indicates a $\rho_w$ in the range 0.88 to 0.96, and a $\sigma_w$ in the range 0.12 to 0.25. For the purposes of this paper, we set $\rho_w$ and $\sigma_w$ to 0.90 and 0.20, respectively, and follow Tauchen (1986b) to approximate an otherwise continuous process with a discrete number (7) states.

2.4.2 Preferences

We assume that preferences over the consumption of goods and housing services can be represented by the following utility function,

$$u(c, s) = \left(\frac{c^{\alpha} s^{1-\alpha}}{1-\sigma}\right)^{1-\sigma}.$$  \hfill (2.12)

To characterize household preferences, we must choose values for four parameters. The risk aversion parameter, $\sigma$, is set to 2. The discount factor ($\beta$), Cobb-Douglas share parameter ($\alpha$), and landlord utility loss parameter ($\chi$) are calibrated. The share parameter $\alpha$ affects the allocation of income between the two expenditure components. Using the data from 1980, 1990, and 2000 Decennial Census of Housing, Davis and Ortalo-Magné (2008) estimate the share of expenditures on housing services by renters to be roughly 0.25, and find that the share has been constant across time and MSA regions. We thus calibrate $\alpha$ to match this share. Moreover, the discount factor $\beta$ is calibrated to match the fraction of owner-occupiers
with collateral debt. According to data from the 1994-1998 American Housing Survey (ASH), approximately 65 percent of homeowners report collateral debt balances.\textsuperscript{20} The parameter $\chi$ that characterizes the utility loss for landlords in equation (2.2) is set to match the average fraction of homeowners (0.66) in the United States between 1995 and 2005.

### 2.4.3 Market Arrangements

In the benchmark model, a minimum down payment of 20 percent is required to purchase a home.\textsuperscript{21} With regard to the transaction costs, Gruber and Martin (2003), using the data from the Consumption Expenditure Survey (CE), document that selling cost for housing can be up to 7 percent, while buying costs are around 2.5 percent. We use the authors’ estimates and set $\tau^s = 0.025$ and $\tau^b = 0.07$.

To calibrate the interest rates on deposits and collateral debt, we follow Díaz and Luengo-Prado (2008) and assume that the collateral debt is associated with an interest rate $r^m = r + \kappa$, where $\kappa > 0$ represents the spread between the two rates. Based on data from the Federal Reserve Statistical Release, the average spread between the nominal interest rate on a 30-year fixed-rate conventional home mortgage and the interest rate on nominal 30-year constant maturity Treasury (or T-bond) between 1977 and 2008 is 1.5 percent, so that $\kappa$ is set to 0.015.\textsuperscript{22} For consistency, we use the interest rate on the same 30-year constant

\textsuperscript{20}The discount pattern $\beta$ governs household borrowing behavior in our model. Since deceased agents in our model are replaced by newborn descendants who do not, however, inherit the asset positions of the dead, we calibrate $\beta$ to ensure that households do not borrow excessively and to generate a realistic borrowing behavior of households in our model economy.

\textsuperscript{21}Using the American Housing Survey 1993, Chambers, Garriga and Schlagenhauf document that the average down payment is approximately 20 percent.

\textsuperscript{22}The spread has fluctuated between 0.73 and 3.32 percent between years 1977 and 2008. The average spread for the period is 1.59 percentage points while the median spread is 1.5. For the data used to construct
maturity T-bonds to represent the interest rate on deposits, $r$. The average rate for the period between 1977 and 2008 fluctuated between -2.23 and 8.04, with an average for the period of 3.76.\footnote{The median interest rate for the period is 3.83.} We thus set the real interest rate to 4 percent so that $r = 0.04$.

To parametrize the maintenance cost function $M(h', s)$ in equation (2.4), we follow Harding, Rosenthal, and Sirmans (2007) who estimate the depreciation rate for housing units used as shelter between 2.5 and 3 percent. We thus set $\delta_0 = 0.025$ and estimate the depreciation rate of rental property, $\delta_r$, so that the model delivers a landlord rate and homeownership rate comparable to that in the U.S. economy. Chambers, Garriga and Schlagenhauf (2008) use the American Housing Survey data to compute the fraction of homeowners who claim to receive rental income. The authors find that approximately 10 percent of the sampled homeowners receive rental income. We use the authors’ estimate of the “landlord rate” to help identify $\delta_r$.

### 2.4.4 Taxes

Using data from the 2007 American Community Survey, Díaz and Luengo-Prado (2009) compute the median property tax rate for the median house value and report a housing property tax rate of 0.95 percent. Moreover, the authors, using information from TAXSIM, the deduction percentage for interest payments of 0.9. We thus set $\tau^h = 0.01$, and allow mortgages to be fully deductible so that $\tau^m = 1$. The U.S. tax code assumes that a rental structure depreciates over a 27.5 year horizon, which implies an annual depreciation rate of

\[\text{the spread, see Federal Reserve Statistical Release, H15, Selected Interest Rates.}\]
3.63 percent. However, only structures are depreciable for tax purposes, and the value of a house in our model includes both the value of the structure and the land that the house is situated on. Davis and Heathcote (2007) find that on average, land accounts for 36 percent of the value of a house in the U.S. between 1975 and 2006. Based on their findings, we set the depreciation rate of rental property for tax purposes to $\tau^{LL} = (1 - .36) \times .0363 = .023$.

Lastly, we follow Díaz and Luengo-Prado (2008) and Prescott (2004) and set the income tax rate, $\tau^y$, to 0.20.

### 2.4.5 Calibration Results

#### Moment Conditions

As discussed previously, our calibration is designed to match the U.S. homeownership rate (0.66), the fraction of households who receive income from rental property (0.10), the fraction of homeowners with collateral debt (0.65), and the ratio of housing services expenditures to wages (0.25). Targeting the homeownership and landlord moments implies that we are also implicitly targeting the fraction of households who are renters (0.34) and owner-occupiers (0.56) because the landlord, renter, and owner-occupier categories are mutually exclusive and collectively exhaustive. As can be seen in Table 2.3, we match these moments well.

Table 2.4 reports several other important statistics generated by the model and compares these with the estimates that are either drawn from other studies or the official AHS tables, or are computed from the 2007 Survey of Consumer Finances. Appendix A describes how we compute the SCF statistics in the data. As can be seen in the table, the average net
worth-to-income ratio for homeowners, where net worth is defined as the sum of deposits and housing wealth net of collateral debt, generated by the model is 2.9, which is close to the 2007 SCF estimate of 3.2. The house value-to-income ratio for homeowners of 3.64 lies between the comparable estimates in the AHS and SCF: 3.1 and 4.0, respectively. The loan-to-value ratio for homeowners of 1.19 aligns nicely with the 2007 SCF estimate of 1.16. At the same time, the loan-to-value ratio for homeowners of 0.31 matches closely the 2007 SCF estimate of 0.28, but both the model and the 2007 SCF estimate understate the 2005 AHS statistics of 0.55. The model also predicts a ratio of rental income to total income for landlords at 0.28, which is close to the ratio of 0.31 estimated in the 2005 AHS. Finally, the model generates a house price-rent ratio of roughly 11.6. The U.S. Department of Housing and Urban Development and the U.S. Census Bureau report a price-rent ratio of 10 in the 2001 Residential Finance Survey (chapter 4, Table 4-2). Garner and Verbrugge (2009), using Consumer Expenditure Survey (CES) data drawn from five cities over the years 1982–2002, report that the house price to rent ratio ranges from 8 to 15.5 with a mean of approximately 12. The house price rent ratio of 11.6 generated by the model therefore falls well within the range of recent estimates based on U.S. data. Overall, the ability of the model to fit a number of key moments that were not targeted during the calibration is encouraging.

Cross-sectional Implications of the Model

There are twelve discrete shelter sizes in our model economy: eleven self-standing discrete-size housing structures that can be purchased in the housing market, and a very small living

\[ \text{The cities included in this analysis are Chicago, Houston, Los Angeles, New York, and Philadelphia.} \]

80
Table 2.4: Other Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net worth to total income ratio for homeowners</td>
<td>2.94</td>
<td>3.17</td>
<td>SCF 2007</td>
</tr>
<tr>
<td>Housing value to total income ratio for homeowners</td>
<td>3.64</td>
<td>4.02 / 3.1</td>
<td>SCF 2007 / AHS 2005</td>
</tr>
<tr>
<td>Loan to total income ratio for homeowners</td>
<td>1.19</td>
<td>1.16</td>
<td>SCF 2007</td>
</tr>
<tr>
<td>Loan to value ratio for homeowners</td>
<td>0.31</td>
<td>0.28 / 0.55</td>
<td>SCF 2007 / AHS 2005</td>
</tr>
<tr>
<td>Rental income receipts to income ratio for landlords</td>
<td>0.28</td>
<td>0.31</td>
<td>AHS 2005</td>
</tr>
<tr>
<td>House price-rent ratio</td>
<td>11.4</td>
<td>8 - 15.5</td>
<td>Various studies</td>
</tr>
</tbody>
</table>

Table 2.5: Distribution of Households Across House Sizes

<table>
<thead>
<tr>
<th>Housing Owned (h)</th>
<th>Shelter Services Consumed (s)</th>
<th>% HHs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Room</td>
<td>Small shelter-size</td>
</tr>
<tr>
<td>Renter (h = 0)</td>
<td>67.90</td>
<td>32.10</td>
</tr>
<tr>
<td>Small-size property</td>
<td>0.63</td>
<td>99.37</td>
</tr>
<tr>
<td>Medium-size property</td>
<td>1.57</td>
<td>6.52</td>
</tr>
<tr>
<td>Large-size property</td>
<td>0.00</td>
<td>0.58</td>
</tr>
<tr>
<td>% HHs</td>
<td>23.74</td>
<td>27.77</td>
</tr>
</tbody>
</table>
Table 2.6: Distribution of Landlords by Labor Income

<table>
<thead>
<tr>
<th>Income group</th>
<th>% Landlords</th>
<th>% Total Rental Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>3.32</td>
<td>1.7</td>
</tr>
<tr>
<td>Group 2</td>
<td>15.02</td>
<td>10.2</td>
</tr>
<tr>
<td>Group 3</td>
<td>33.85</td>
<td>20.7</td>
</tr>
<tr>
<td>Group 4</td>
<td>15.44</td>
<td>20.8</td>
</tr>
<tr>
<td>Group 5</td>
<td>14.47</td>
<td>20.8</td>
</tr>
<tr>
<td>Group 6</td>
<td>12.32</td>
<td>17.7</td>
</tr>
<tr>
<td>Group 7</td>
<td>5.58</td>
<td>7.8</td>
</tr>
</tbody>
</table>

space that can be rented out but is not available for sale. Discreteness in housing captures the idea that housing units typically come in discrete sizes, such as one bedroom, two bedroom, or four bedroom. At the same time, the smallest-size shelter unit, which we call a “room,” captures the idea that agents can also rent a very small living space that is not, however, available for sale so that, for example, a person can share a room with a roommate or can rent a room while sharing the kitchen. For clarity of exposition, we divide the properties owned by households into three groups called small, medium, and large size properties. The small properties represent starter homes, while medium sized properties are owned by agents who represent the average households in terms of wealth and income. Finally, large properties are in general used for investment, as these often serve as rental units.

Table 2.5 shows the relationship between units of housing owned and units of shelter consumed. As can be seen in the table, 68 percent of renters live in a room, while the remaining 32 percent of renters inhabit the small size house. The renters are typically hand-to-mouth agents who are at the bottom of the wealth distribution and have savings that are below the minimum down payment requirement for the smallest house.
The renters lease housing services from homeowners who choose to become landlords by consuming less shelter than they currently own. The landlords are typically highly leveraged and often low earnings households who partially lease out their homes to boost their income level. Table 2.5 shows that 8.1 percent of the owners of medium sized properties are landlords, and supply 39 percent of the total amount of shelter that is rented. Virtually all owners of large properties are landlords (99.9 percent). Although these households comprise only 5.6 percent of the population, they supply 61 percent of the shelter services that are obtained through the rental market. Table 2.6 shows that low and middle income agents account for a large fraction of the landlords in the model economy. This prediction is consistent with the findings of Chambers, Garriga and Schlagenhauf (2007) who, using the 1996 Property Owners and Managers Survey, find that 25 percent of households receiving rental income are low-income households with annual earnings below $30,000, compared to 30 percent of high-income households with annual earnings over $100,000 (see their Table 4).

Owner-occupiers consume all of the housing services provided by their property. The vast majority of owner-occupiers are divided between the small and medium house sizes and represent the average household in terms of earnings and financial wealth. The remaining owner occupiers live in large properties, represent only 0.1 percent of the population, and are very rich people with medium to high wages.

In general, homeownership is preferred to renting. Households who can afford a down payment on a house typically enter the housing market and become homeowners. Inter-
estingly, the option to become a landlord plays an important role in our model economy, as rental income helps low and medium income households who are typically highly lever-aged to keep up with homeownership expenses and payments. For example, the average owner-occupier of a medium-size house has a large amount of financial wealth and receives a wage endowment that is roughly 30 percent higher than the economy’s average, while an average landlord who owns the same size house earns a wage that is 8 percent lower than average, and is in debt. The option to become a landlord is, however, also popular among rich homeowners who purchases sizeable properties as an investment.

2.5 What Explains the Changes in the Price-Rent Ratio?

The estimated model is employed to analyze the observed changes in the house prices, rents, and the price-to-rent ratio since mid-1990s. We first study the model’s predictions about the responsiveness of house prices and rents, and the price-rent ratio, to changes in interest rates, borrowing constraints, and household incomes. Then we consider the combined effects of these macroeconomic factors on the housing market equilibrium. As a cross-check, we also study the model’s implications for the homeownership rate, loan-to-income, and loan-to-value ratios.
2.5.1 Relaxation of Down Payment Requirements

Since the early 1990s, a number of developments have occurred with respect to the financing of housing investment. Financial innovations such as interest-only loans and combo mortgages provided households with greater choices in mortgage debt financing and significantly reduced down payment requirements. Moreover, policies enacted by the Clinton and Bush Administrations targeted lowering of the down payment requirement to increase households’ access to mortgage financing and to generate additional first time home buyers. As a result, the average down payment declined from about 20 percent in the mid-1990s to 15 percent in the 2000s.

Figure 2.2 illustrates the impact of variation in the minimum down payment requirement, $\theta$, on equilibrium housing market outcomes. As the down payment requirement falls from 40 percent to 15 percent, both the equilibrium house price and rent increase by roughly 8 percent, so the price-rent ratio remains virtually unchanged. A reduction in the average down payment requirement in line with the recent U.S. experience from 0.20 to 0.15, leads to a 5.8 percent increase in the house price and a 5.3 percent increase in rent. Since both the house price and rent are relatively inelastic with respect to the down payment requirement,

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25 The Clinton Administration enacted policies through the Federal Home Administration (FHA) to lower the downpayment requirements with mortgage insured loans, while the Bush Administration developed the Zero-Downpayment Initiative for FHA to generate additional first-time buyers.

26 Chambers, Garriga and Schlagenhauf (2008), using the data from the American Housing Survey (AHS), document that between 1995 and 2003 the average downpayment for FHA loans declined from 21.6 percent in 1995 to 13.8 percent in 1999 before rising again to 16.3 percent in 2003. At the same time, the average downpayment on a non-FHA loan has decreased from 29.8 percent in 1995 to 24.1 percent by 2003. Chomsisengphet and Pennington-Cross (2006) document similar trends in the subprime lending markets. In addition, the fraction of households with a loan to value ratio greater than 90 percent rose from 10 percent in 1990 to 25 percent by 1995 before retracting slightly to 18 percent in 2005, according to the Federal Finance Board. More generally, the down payment requirements were significantly relaxed during the periods 1995-1998 and 2001-2004, although the financial markets tightened slightly temporarily in the wake of the 1998 Asian crisis.
a lessening of credit constraints cannot by itself explain the run-up in house prices observed in recent years.\textsuperscript{27}

That said, lower down payment requirements lead to large increases in the homeownership rate as poorer households gain access to mortgage markets and borrow larger amounts to finance home purchases. When $\theta$ falls from 0.40 to 0.15, the homeownership rate increases from 66 percent to 80 percent. At the same time, the loan-to-wage ratio jumps up from

\textsuperscript{27}The U.S. real house prices rose by 11 percent while the real rents grew by 3 percent between 1995 and 2000. During this period, the real deposit rate on 10 year constant maturity T-bonds oscillated in a relatively narrow range between 3 and 4 percent.
Table 2.7: The Distribution of Owned Housing Under Different Downpayment Requirements

<table>
<thead>
<tr>
<th>House Size</th>
<th>20% Downpayment</th>
<th>15% Downpayment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% Households</td>
<td>% Housing Stock</td>
</tr>
<tr>
<td>Renter</td>
<td>33.8</td>
<td>0.0</td>
</tr>
<tr>
<td>Small-size property</td>
<td>13.9</td>
<td>11.4</td>
</tr>
<tr>
<td>Medium-size property</td>
<td>46.7</td>
<td>63.7</td>
</tr>
<tr>
<td>Large-size property</td>
<td>5.5</td>
<td>24.9</td>
</tr>
</tbody>
</table>

0.7 to 1.4, while the fraction of homeowners in debt rises from 53 percent to 64 percent. The increase in household borrowing is skewed toward low-earnings households, as relatively more low-wage households enter the housing market.

The key to understanding the small effect of decreases in the required down payment on house prices is to realize that the housing market responses are primarily driven by households who find the minimum down payment to be a binding constraint. Decreasing the down payment requirement thus primarily affects low-income, low-savings households who wish to become homeowners but are unable to because of the high equity requirement. As a result, when the down payment requirement falls from 20 percent to 15 percent, the proportion of renters decreases from 34 to 20 percent as previously credit constrained households switch from renting to owning a small-sized house (Table 2.7).

Table 2.8 provides further details on how changes in the down payment requirements affect the housing market. Column (2) shows the impact of a decline in down payment requirement from 20 to 15 percent under the restriction that house prices and rents are not allowed to change (i.e., both house prices and rents are held fixed at their equilibrium values from the baseline version of the model). Column (3) reports the impact of a decrease on the
Table 2.8: The Partial and Equilibrium Effects of a Reduction in the Equity Requirement to 15%

<table>
<thead>
<tr>
<th></th>
<th>Baseline (1)</th>
<th>15% Equity Requirement Fixed Prices (2)</th>
<th>Equilibrium Prices (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>House Price</td>
<td>2.55</td>
<td>2.55</td>
<td>2.70</td>
</tr>
<tr>
<td>Rent</td>
<td>0.22</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>Share of Homeowners</td>
<td>0.66</td>
<td>0.81</td>
<td>0.80</td>
</tr>
<tr>
<td>Share of Renters</td>
<td>0.34</td>
<td>0.19</td>
<td>0.20</td>
</tr>
<tr>
<td>Share of Landlords</td>
<td>0.10</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>Share of Owner-Occupiers</td>
<td>0.56</td>
<td>0.70</td>
<td>0.69</td>
</tr>
<tr>
<td>Share of Homeowners in Debt</td>
<td>0.64</td>
<td>0.69</td>
<td>0.70</td>
</tr>
</tbody>
</table>

down payment requirement when both house prices and rents are allowed to adjust to clear the housing and rental markets at the lower down payment requirement. As can be seen in the table, since house prices changed relatively little, the share of landlords among homeowners remains broadly unchanged (Columns (2) and (3)) relative to the baseline case. In addition, while a number of households switch from renting to homeownership, only a small increase in the house price is needed to achieve the redistribution of properties to accommodate the entrants to the housing market. For example, while the percentage of households who own small-size houses jumps up because of the influx of new homeowners into the housing market as the down payment requirement falls from 4 to 2 percent, the fraction of households owning the medium size-property declines (Table 2.7). In addition, fewer households purchase the large houses which are exclusively occupied by landlords.

Our results are consistent with several recent studies which document the positive correlation between the size of the down payment requirement and homeownership (e.g., Chambers, Garriga, and Schlagenhauf (2008), Díaz and Luengo-Prado (2008), and Ortalo-Magné and
Rady (2006)). These studies suggest that, while the financial sector innovations have minimal impact for existing homeowners, lower down payment requirements do affect households who are excluded from the housing market due to a high down payment constraint. The authors suggest that when down payment requirements are relaxed, the initially excluded households enter the housing market and the homeownership rate rises. This mechanism is supported by the empirical findings in Ortalo-Magné and Rady (1999) who document that decreases in the down payment requirements in England and Wales after the financial liberalization of the early 1980s were associated with unprecedented increases in the homeownership for young households. Using regression analysis, Muellbauer and Murphy (1990), Muellbauer and Murphy (1997) show that while the current income and short-term demographic changes were the most important factors behind the U.K. house price boom during the 1980s, but that the increase in homeownership was mostly due to the credit market liberalization and the extrapolative price expectations.

In summary, the model clearly indicates that in the absence of changes in other factors, a relaxation of borrowing constraints cannot by itself account for the magnitude of the recent increase in the price-rent ratio. With this result in mind, the next sections of the paper examine the impact of changes in the interest rate and income on the equilibrium price-rent ratio.
2.5.2 Changes in the Interest Rate

Figure 2.3 shows the evolution of the real contract and effective mortgage rates on conventional single-family mortgages in the United States between 1985 and 2005.\textsuperscript{28} As can be seen in the figure, the real mortgage rate for residential property oscillated around the 5 percent mark between 1990 and 1997, but started to fall following the 1998 Asian Financial Crisis, before 2.5 percent in 2005.\textsuperscript{29}

Figure 2.4 captures the impact of changes in the real risk-free rate, $r$, on the steady state

\textsuperscript{28}The effective rate represents the sum of the contract rate and the discounted initial fees and charges. The estimates provided by the Federal Housing Financing Board.

\textsuperscript{29}The mortgage spread, defined as the difference between the real mortgage rate on a 30-year conventional fixed-rate mortgage and the interest rate on a 30-year constant maturity Treasury, fluctuated in a relatively narrow range between 1 and 2 percent since 1995, although the mark-up fell temporarily below one percent between 1991 and 1993.
equilibrium of the housing market. Changes in the risk-free rate interest rate in our model directly translate into changes in the mortgage interest rate because the mortgage interest rate is determined by a constant markup, \( \kappa \), over the risk-free rate. Therefore, changes in the risk-free interest rate affect both the cost of borrowing and the rate of return on saving. As can be seen in the figure, when the real interest rate falls from 6 percent to 1 percent, the equilibrium house price increases by 32 percent, the equilibrium rent decreases by 15 percent, and the price-rent ratio increases by 54 percent from 9.9 to 15.2. When the interest rate declines from 4 percent to 2 percent – a decrease broadly consistent with the actual decline between 1995 and 2005 – the house price level rises by 17 percent, the rent falls by 1.8 percent, and the price-rent ratio rises by 19 percent from 11.4 to 13.6.

As expected, a lower interest rate leads to a large increase in household borrowing, since the low interest rate decreases the cost of mortgage financing and, at the same time, lowers the rate of return on household savings. The average loan-to-wage ratio increases from 0.7 to 3.3 when the interest rate permanently declines from 6 percent to 1 percent, while the fraction of homeowners with mortgage debt rises from 41 percent to 85 percent. For a decline in the interest rate from 4 percent to 2 percent, the loan-to-wage ratio roughly doubles, rising from 1.4 to 2.7.

Turning to the rental market, lower interest rates increase the supply of rental property because, holding the rent fixed, a lower interest rate increases the rate of return to investing in rental property for landlords with mortgages. In addition, investing in rental properties also becomes more attractive relative to the alternative of holding bank deposits. The increase
Figure 2.4: The Housing Market Equilibrium Under Different Interest Rates

in rental supply decreases the equilibrium rent, so rents are falling even though house prices are rising. For example, when the interest rate decreases from 4 to 2 percent, the aggregate supply of rental property rises increases by 4 percent while the rent falls from 0.22 to 0.21. At the same time, the relative importance of owners of small-sized homes in the rental market increases significantly because with higher house prices and greater leverage, homeowners use rental income to partially cover the interest rate payments on their outstanding mortgage loans. For example, as can be seen in Table 2.9, when interest rate falls from 4 to 2 percent, both the owners of the large properties as well as the owners of the medium-sized properties
Table 2.9: The Distribution of Owned Housing and Landlords Under Different Interest Rates

<table>
<thead>
<tr>
<th>House Size</th>
<th>6% Interest Rate</th>
<th>4% Interest Rate</th>
<th>2% Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% HHs</td>
<td>% Landlords</td>
<td>% HHs</td>
</tr>
<tr>
<td>Renter</td>
<td>18.9</td>
<td>0.0</td>
<td>33.7</td>
</tr>
<tr>
<td>Small property</td>
<td>46.2</td>
<td>0.9</td>
<td>13.9</td>
</tr>
<tr>
<td>Medium property</td>
<td>31.4</td>
<td>45.4</td>
<td>46.9</td>
</tr>
<tr>
<td>Large property</td>
<td>3.5</td>
<td>53.7</td>
<td>5.5</td>
</tr>
</tbody>
</table>

account for a smaller percentage of all landlords (54.4 percent to 46.8 percent for large-property owners, and 45.0 percent to 27.9 percent for medium-sized property owners). The opposite trend occurs for owners of small houses, who account for a much larger share of all landlords when the interest rate reaches 2 percent (0.9 to 25.3 percent).

Figure 2.4 shows that the steady state homeownership rate is constant for interest rates between one and 4 percent. One might expect the homeownership rate to rise as the interest rate decreases from 4 to one percent because a falling risk-free interest rate both decreases the cost of mortgage financing and reduces the attractiveness of saving relative to housing investment. However, this is a situation where accounting for equilibrium price effects is critical, as Table 2.10 illustrates. As can be seen in Column (2), a reduction in \( r \) from 4 percent to 2 percent under the restriction that the house price and rent are not allowed to adjust would result in a 15 percent increase in the homeownership rate from 0.66 to 0.81, and an increase in household borrowing because of the lower cost of mortgage financing. At the same time, the fraction of landlords in the economy would rise by 39 percent from 0.10 to 0.49, because households would purchase bigger properties and use rental income to keep up with the mortgage payments. However, in the general equilibrium where the house
Table 2.10: The Partial and Equilibrium Effects of the Interest Rate Reduction to 2%

<table>
<thead>
<tr>
<th></th>
<th>Baseline ($r = 0.04$)</th>
<th>Reduction of Interest Rate to 2% Fixed Prices</th>
<th>Equilibrium Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>House Price</td>
<td>2.55</td>
<td>2.55</td>
<td>2.97</td>
</tr>
<tr>
<td>Rent</td>
<td>0.22</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>Share of Homeowners</td>
<td>0.66</td>
<td>0.81</td>
<td>0.66</td>
</tr>
<tr>
<td>Share of Renters</td>
<td>0.34</td>
<td>0.19</td>
<td>0.34</td>
</tr>
<tr>
<td>Share of Landlords</td>
<td>0.10</td>
<td>0.49</td>
<td>0.10</td>
</tr>
<tr>
<td>Share of Owner-Occupiers</td>
<td>0.56</td>
<td>0.32</td>
<td>0.55</td>
</tr>
<tr>
<td>Share of Homeowners in Debt</td>
<td>0.64</td>
<td>0.94</td>
<td>0.84</td>
</tr>
</tbody>
</table>

In equilibrium, the decline in the interest rate has no effect on the homeownership rate because the higher house price increases the minimum down payment requirement while a lower $r$ decreases the speed at which the aspiring first-time buyers are able to save up for it. Also, the equilibrium rent decreases slightly from 0.22 to 0.21 even though house prices are rising, which further discourages households from becoming homeowners. All of these effects offset the fact that mortgage interest payments decrease when the interest rate falls.

Interestingly, when the interest rate is well above 4 percent, households do not in general borrow to purchase housing properties, preferring to use the accumulated saving to pay for the house upfront. Although households see their savings grow quickly when interest rate is high, they live, on average, in smaller-sized properties, and move up the house size ladder only when sufficient amount of savings was accumulated to pay for the house in cash. At the same time, the supply of rental property contracts significantly, since homeowners substitute rental income for the interest rate income on their savings, and the rent rises. As a result, the price-rent ratio falls below 9.0, increasing the affordability of owning relative to renting. The higher affordability of owning vs. renting in turn further increases the homeownership rate.
2.5.3 Changes in Income

A large body of empirical literature identifies the level and growth rate of income as an important determinant of house price dynamics (see, for example, Poterba (1991), Englund and Ioannides (1997), Muellbauer and Murphy (1997), Malpezzi (1999), and Sutton (2002)). In the United States, real hourly wages increased by 9.4 percent between 1995 and 2005.\footnote{This calculation is based on the BLS Current Employment Statistics (CES) real wage data, series ID CES0500000032.}

Figure 2.5 summarizes the impact of changes in income on the housing market equilibrium. In our experiment, we assume that household wages rise at the same rate across all wage groups. The model suggests that both house prices and rents increase linearly at about the same rate as wages.\footnote{The actual changes in the income levels were not, however, symmetric. Heathcote, Perri, and Violante (2009) document the changes in the U.S. earnings inequality between 1967 and 2006. Using the CPS data, the authors find that the real earnings of the bottom decile of the earnings distribution did not, on average, grow between 1985 and 2000, although the earnings of the top earnings distribution grew steadily over the sample period (see their Figure 7). The authors also find that the wage dynamics of the bottom decile of the earnings distribution is very similar to those for the median workers (e.g., workers in the 45-55 percentile of the earnings distribution).} For example, when the wage level increases by 10 percent relative to the benchmark economy, the equilibrium house price and the rent rise by approximately 11 percent. As a result, the house price-rent ratio stays approximately flat. Since the relative price of obtaining housing services through the rental market compared to the market for owned housing remains unchanged, symmetric changes in income of the sort examined here have no effect on the homeownership and landlord rates.

Table 2.11 helps to explain why the homeownership rate does not rise with income. Again, Column (2) shows the impact of a 10 percent increase in income under the restriction
Figure 2.5: The Housing Market Equilibrium Under Different Income Levels

that house prices and rents are not allowed to change (i.e., both house prices and rents are fixed at their equilibrium values from the baseline version of the model). Column (3) reports the impact of a 10 percent increase in income when both house prices and rents are allowed to adjust to clear the housing and rental markets at the higher income level. When house prices and rents are not allowed to adjust, rising income has a substantial impact on the housing market, with the homeownership rate increasing from 66 to 92 percent as more households are able to afford the down payment and mortgage payments required to purchase a house. In addition, many households stop renting out their units as they can more easily
Table 2.11: The Partial and Equilibrium Effects of a 10% Increase in Income

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>10% Increase in Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Fixed Prices</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>House Price</td>
<td>2.55</td>
<td>2.55</td>
</tr>
<tr>
<td>Rent</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>Share of Homeowners</td>
<td>0.66</td>
<td>0.92</td>
</tr>
<tr>
<td>Share of Renters</td>
<td>0.34</td>
<td>0.21</td>
</tr>
<tr>
<td>Share of Landlords</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>Share of Owner-Occupiers</td>
<td>0.56</td>
<td>0.71</td>
</tr>
<tr>
<td>Share of Homeowners in Debt</td>
<td>0.64</td>
<td>0.70</td>
</tr>
</tbody>
</table>

cover their mortgage payments: the share of owner-occupied housing increases from 0.56 to 0.71. However, once the house prices and rents are allowed to adjust to higher incomes, homeownership returns approximately to its baseline level (Column (3)). When income increases by 10 percent, the equilibrium house price increases by about the same rate from 2.55 to 2.85, while the equilibrium rent increases from 0.22 to 0.25. The relative cost of renting and owning remains unchanged, keeping the proportions of renters, homeowners, landlords, and owner-occupiers in the economy essentially the same as in the benchmark specification.

2.5.4 Combined Effects of Changes in the Market Fundamentals

As discussed in the preceding sections, neither declines in the real interest rate, relaxation of borrowing constraints, nor rising incomes can on their own account for the increase in the price-rent ratio, homeownership rate, and household debt between 1995 and 2005. This section examines the combined effects of changes in these fundamentals on equilibrium housing
market outcomes. Figure 2.6 depicts the percentage deviation of the steady state price-rent ratio from the baseline economy for a range of interest rates and required down payments. Point A represents the calibrated baseline economy with an interest rate on deposits, \( r \), of 4 percent and a required down payment, \( \theta \), of 20 percent. As the interest rate and the required down payment decrease, the price-rent ratio steadily rises. The price-rent ratio increases by 20 percent over its baseline value when the interest rate is 2 percent and the required down payment is 15 percent. These changes in the interest rate and down payment seem to be a reasonable representation of the recent U.S. experience. For comparison, the U.S. price-rent ratio increased by 36 percent from 1995 to 2005, and by 26 percent between 2000 and 2005.

Table 2.12 provides a more comprehensive analysis of the simulated effects by showing the percentage deviations in house prices, rents, and the price-rent ratio from their baseline values (Column (1)). To facilitate a comparison of the model’s predictions to the data, Columns (8) and (9) show recent changes in the U.S. data. Columns (2) through (4) show that when income is held constant, lowering \( \theta \) and \( r \) raises house prices, lowers rents, and consequently increases the price-rent ratio. Columns (5) through (7) of Table 2.12 show that increasing wages by 10 percent while decreasing \( \theta \) and \( r \) does not change the price-rent ratio compared to the scenarios where income is held constant.\(^{33}\) However, the model also predicts that higher income will cause a small increase in rents that is quite close to the growth in rents observed in the United States. As noted above, the actual increase in the house price-rent ratio from 1995 to 2005 was about 36 percent, so a plausible calibration of

\[^{33}\text{A 10 percent increase in real wages is approximately what was observed in the U.S. between 1995 and 2005.}\]
Table 2.12: The Combined Effects of Interest Rate, Required Downpayment, and Income Changes

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Changes in r and θ (%Δ from Baseline )</th>
<th>U.S. Data (%Δ)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>r=0.02</td>
<td>r=0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>θ=0.17</td>
<td>θ=0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>house price</td>
<td>2.55</td>
<td>15.7%</td>
<td>16.0%</td>
</tr>
<tr>
<td>rental price</td>
<td>0.225</td>
<td>-3.5%</td>
<td>-3.6%</td>
</tr>
<tr>
<td>price-rent ratio</td>
<td>10.8</td>
<td>19.6%</td>
<td>20.0%</td>
</tr>
</tbody>
</table>

Notes: Columns (2)-(7) show percent changes in the equilibrium value of each variable from the baseline model shown in column (1). Columns (8) and (9) show the actual percent changes observed in the U.S. over two different time periods.
the model can account for over one-half of the observed increase. Of course, this estimate must be viewed in the context of our admittedly stylized steady state model of the housing market. However, our results suggest that the changes in the interest rate and required down payment observed in the United States had a substantial impact on the price-rent ratio. In addition, the ability of our model to simultaneously predict large increases in house prices and sluggish rents is consistent with recent developments in the U.S. housing market and stands in marked contrast with predictions of simpler models of the housing market which
imply that equilibrium house prices and rents must change in the same direction and at the same rate.

In our model, holding house prices and rents constant, when the mortgage interest rate and required down payment fall, the demand curve for rental property shifts inward because households switch from renting to owning as homeownership becomes more affordable.\textsuperscript{34} At the same time, the supply curve for rental property shifts to the right because when \( \theta \) and the interest rate decrease, more households are able to afford down payments and mortgage payments on rental properties. In addition, since both the mortgage rate and rate of return on deposits fall when interest rates decrease, investing in rental property becomes more attractive relative to the alternative of holding bank deposits. The net result of the declining demand and increasing supply in the rental market is a decrease in the equilibrium rent. At the same time, the demand for housing (or homeownership) increases when the interest rate and the required down payment decrease because more households can afford to purchase homes, and existing homeowners can afford larger homes. Given that the supply of housing is fixed, the equilibrium house price rises. It follows that the price-rent ratio increases as the house price increases and rent falls in response to the change in fundamentals.

\textbf{2.6 Conclusion}

This paper develops a dynamic equilibrium model of the housing market in which both house prices and rents are determined endogenously. We use the model to study the relationship

\textsuperscript{34}When the downpayment requirement declines from 20 to 10 percent and the interest rate falls from 4 to 2 percent, the homeownership rate increases by 1.4 percent from 0.66 to 0.674.
between the steady state house price-rent ratio and fundamentals such as the interest rate, required down payment, and income. This analysis is motivated by the fact that although the price-rent ratio is a widely used economic indicator, its determinants are not well understood. Without a theoretical understanding of how the price-rent ratio is determined, it is not possible to determine whether observed changes in the relationship between house prices and rents reflect changing fundamentals or an asset price bubble.

The model predicts that the combination of low interest rates, reduced down payment requirements, and rising wages observed in the United States leads to a large increase in the steady state, rational expectations equilibrium price-rent ratio. However, changes in these fundamentals are capable of explaining only about one-half of the 36 percent increase in the price-rent ratio observed between 1995 and 2005. At the same time, changes in fundamentals generate increases in the homeownership rate and household debt that are consistent with the recent U.S. experience.

2.7 Appendix

2.7.1 Finding Equilibrium in the Housing and Rental Markets

Equilibrium in the housing and rental markets is formally defined by the conditions presented in Section 2.3. In practice, the market clearing rent \((\rho^*)\) and house price \((q^*)\) are found by finding the \((q^*, \rho^*)\) pair that simultaneously clear both the housing and shelter markets in a simulated economy. The market clearing conditions for a simulated cross section of \(N\) agents
The optimal housing and shelter demands for each agent are functions of the market clearing steady state prices and the agents other state variables \((x)\). Solving for the equilibrium of the housing market is a time consuming process because it involves repeatedly re-solving the optimization problem at potential equilibrium prices and simulating data to check for market clearing until the equilibrium prices are found. The algorithm outlined in the following section exploits theoretical properties of the model such as downward sloping demand when searching for market clearing prices. Taking advantage of these properties decreases the amount of time required to find the equilibrium far below that of a more naive search algorithm.

**2.7.2 The Algorithm**

Let \(q_k\) represent the \(k\)th guess of the market clearing house price, let \(\rho_k\) represent a guess of the equilibrium rent, and let \(\rho_k(q_k)\) represent the rent that clears the market for housing conditional on house price \(q_k\). The algorithm that searches for equilibrium is based on the

\[
\sum_{i=1}^{N} h'_i(q^*, \rho^* | x) = H \tag{2.13}
\]

\[
\sum_{i=1}^{N} s'_i(q^*, \rho^* | x) = H. \tag{2.14}
\]
The following excess demand functions

\[ ED_h^k(q_k, \rho_k) = \sum_{i=1}^{N} h_i'(q_k, \rho_k|x) - H \]  

(2.15)

\[ ED_s^k(q_k, \rho_k) = \sum_{i=1}^{N} s_i'(q_k, \rho_k|x) - H. \]  

(2.16)

The equilibrium prices \( q^* \) and \( \rho^* \) simultaneously clear the markets for housing and shelter, so

\[ ED_h^k(q^*, \rho^*) = 0 \]  

(2.17)

\[ ED_s^k(q^*, \rho^*) = 0. \]  

(2.18)

The following algorithm is used to find the market clearing house price and rent.

1. Make an initial guess of the market clearing house price \( q_k \).

2. Search for the rent \( \rho_k(q_k) \) which clears the market for owned housing conditional on the current guess of the equilibrium house price, \( q_k \). The problem is to find the value of \( \rho_k(q_k) \) such that \( ED_h^k(q_k, \rho_k(q_k)) = 0 \). This step of the algorithm requires re-solving the agents’ optimization problem at each trial value of \( \rho_k(q_k) \), simulating data using the policy functions, and checking for market clearing in the simulated data. One useful property of the excess demand function \( ED_h^k(q_k, \rho_k(q_k)) \) is that conditional on \( q_k \), it is a strictly decreasing function of \( \rho_k \). Based on this property, \( \rho_k(q_k) \) can be found efficiently using bisection.
3. Given that the housing market clears at prices \((q_k, \rho_k(q_k))\), check if this pair of prices also clears the market for shelter by evaluating \(ED^s_k(q_k, \rho_k(q_k))\).

(a) If \(ED^s_k(q_k, \rho_k(q_k)) < 0\) and \(k = 1\), the initial guess \(q_1\) is too high, so set \(q_{k+1} = q_k - \varepsilon\) and go to step (2). This initial house price guess \(q_1\) is too high if \(ED^s_k(q_k, \rho_k(q_k)) < 0\) because \(ED^s_k(q_k, \rho_k(q_k))\) is decreasing in \(q_k\).

(b) If \(ED^s_k(q_k, \rho_k(q_k)) > 0\) set \(k = k + 1\) and \(q_{k+1} = q_k + \varepsilon\) and go to step (2).

(c) If \(ED^s_k(q_k, \rho_k(q_k)) = 0\), the equilibrium prices are \(q^* = q_k, \rho^* = \rho_k(q_k)\), so stop.
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