The Role of Financial Intermediaries in Macroeconomics

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Abstract

Since its inception, the Modigliani-Miller capital structure irrelevancy principle has limited researchers’ interest in the role of financial intermediaries in macroeconomics. However, due to the spread of financial crises in emerging markets in the 1980s and 1990s, and the global financial collapse of 2008, the focus of much academic work has turned to rigorously modeling these entities. Chapter one surveys the past and current literature on all types of financial intermediaries (market makers, traditional banks, and hedge funds, among others) and discusses their role in dissemination of asymmetric information, real business cycle fluctuations, and financial crashes and contagion. In chapter two, I build a two-frequency sequential trade model which generates sharp endogenous asset price movements caused by slow dissemination of asymmetric information about economic fundamentals. The key mechanism used in the model employs a Glosten-Migrom market maker who gradually infers the value of the fundamental by trading with both uninformed and imperfectly informed agents. Information becomes "trapped" as purchases by the uninformed agent mask informative sales; a sudden price correction occurs as soon as the market maker discovers the true value of the fundamental. I also study the factors that influence the duration of the information dissemination process. In chapter three, I build a two-country DSGE model with multiple assets, incomplete markets, and an endogenous optimizing banking sector, that is capable of recreating some of the important trends and linkages observed in the financial data. International financial markets during
the past several decades have been characterized by a significant rise in gross interna-
tional equity flows, increased prominence of non-traditional financial institutions, and
globalization of the banking sector. In particular, I demonstrate that financial liber-
alization leads to an increase in a country’s gross international asset holdings and to
a positive net equity position. Finally, the model lays the groundwork for addressing
many of the global banking regulation issues (for example, capital requirements and
bankruptcy resolutions) that are now emerging in the field of international finance.

INDEX WORDS: International Finance, Market Makers, Banking, Portfolio
Choice
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Chapter 1

The Role of Financial Intermediaries in Amplifying and Transmitting Economic Fluctuations: A Survey

1.1 Introduction

Historically, macroeconomists have given relatively little thought to the role of financial markets in real economic activity. This lack of interest is at least partly attributable to the classic Modigliani-Miller (1958) theorem, known as the capital structure irrelevance principle, which states that if interest rates are constant, there are no taxes, agency costs, bankruptcy costs, or asymmetric information, and markets are efficient, then financial structure cannot impact the real economy. Because these conditions were traditionally not considered very restrictive, theoretical work has by and large taken the predictions of the Modigliani-Miller theorem as true.

The treatment of financial markets in the macroeconomic literature, therefore, falls into two distinct categories. Traditionally, financial markets have been treated as complete, the assumption being that a wide enough array of financial assets (for instance, Arrow-Debreu securities) are available to perfectly share risk between countries that are exposed to idiosyncratic shocks. More recently, due to extensive empirical evidence contradicting the complete market hypothesis, the standard has switched to modeling incomplete international financial markets with a single global risk-free bond as the only available asset.

\footnote{Notable exceptions to this are works by Irving Fisher and John Maynard Keynes.}
Neither of these two approaches is entirely satisfactory. Models with a single asset market cannot address questions related to international portfolio choice. Moreover, empirical analysis finds that the degree of international risk sharing is very small.\textsuperscript{2} The data also suggest that financial markets suffer from several frictions that are rarely addressed in macroeconomic work. The banking crisis of 2008 and the resulting global recession have brought these issues to the forefront of academic research.

In particular, the focus of many researchers has shifted to studying frictions that arise in financial markets due to the existence of financial intermediaries. The purpose of this chapter is to present an overview of the literature that analyzes the role of financial institutions in macroeconomic activity. The recent findings show that financial intermediaries come in many shapes and sizes and have a significant impact on financial markets and the real economy. Market makers are trade specialists who provide liquidity in various financial markets and facilitate the flow of information into asset prices. Traditional banks provide deposit services, coordinate lending between borrowers and depositors, and also allow for the aggregation of risk. Banks are often subject to systemic bankruptcy risk and government regulation. Finally, a rapidly growing number of non-traditional bank-like entities, such as money market funds, mutual funds, pension funds and hedge funds, has been changing the financial landscape.

The rest of the chapter is structured as follows. I begin by describing in section 1.2 the various types of financial intermediaries and their roles in the global economy. I then list the channels through which these intermediaries alter the predictions of the Modigliani-Miller theorem. In particular, financial intermediaries impact the economy through the gradual dissemination of asymmetric information (section 1.3).

\textsuperscript{2}There exists an extensive literature, both theoretical and empirical, on international consumption smoothing following the seminal work of Backus, Kehoe and Kydland (1992).
Agency costs that stem from the moral hazard associated with lender-borrow contracts contribute to the volatility of the real business cycles (section 1.4). The costs of bankruptcy and other issues associated with financial collapse are summarized in section 1.5. Finally, section 1.6 outlines several avenues of exploration of the issues associated with financial intermediation.

1.2 Financial Intermediaries

Without financial intermediaries, trillions of dollars of financial assets could not change hands as they do on a daily basis, yet much of the macroeconomic literature abstracts from modeling the key role these agents play in the economy. The papers that do study financial intermediaries form two distinct strands of literature.

The first and somewhat older strand is known as market microstructure. In this literature, the markets for assets are handled using a microeconomic approach, and the standard framework of a Walrasian auctioneer\footnote{A Walrasian auction is a simultaneous auction process in which agents submit their demand schedule to an auctioneer. The price is then set so that the total quantity demanded of all agents equals the total quantity supplied.} is replaced with micro-founded assumptions that more realistically characterize the interactions between asset traders and financial intermediaries called market makers (henceforth, MM). A number of puzzles in finance are a lot less puzzling when viewed through the market microstructure lens.

The second strand of literature, which is particularly in vogue at the moment, deals with the treatment of banks and other bank-like entities. The papers in this area primarily focus on the various roles banks that play in the economy, from providing deposit services to aggregating risk, and on the impact of government regulation on the banking sector and the subsequent spillover effects on the real economy. These
two topics are so closely intertwined that it is impossible to separate them. Indeed, it is often the assumptions about the role of banks in the economy that shape the regulation prescriptions.

1.2.1 Market Makers

In his seminal work, Walter Bagehot (1971) informally describes how the existence of market makers prevents stock managers from being able to outperform the market on a regular basis. A MM is a specialist who facilitates transactions between traders in a particular market. For example, the market for frozen concentrated orange juice has a number of competitive MMs who take buy and sell orders from market participants. The crucial role of the MM then is to guess based on the orders received (referred to as "order flow") what prices clear the market. If the price is set too low, MM ends up shorting the asset; if it is set too high, she has to hold the excess capacity. Neither of these conditions is optimal due to riskiness of the asset, so the MM constantly adjusts prices to keep her inventory as close to zero as possible.

The market maker is able to create a spread between the bid and ask prices of an asset. This is possible because the MM knows the intent of each trader (to buy or sell) before she quotes a price. Therefore, on average, the bid price (the amount that traders must pay to the MM to buy a unit of asset) is higher than ask price (the amount that the MM pays when a trader sells a unit of asset). Bagehot’s assumption is that the spread is crucial to keep MMs from being driven out of business by well-informed traders who have access to private information. Knowing that they must necessarily lose out on trades with these informed agents, MMs must make a profit on trades with uninformed agents who rely only on public information.

This theory is supported by Barnea and Logue (1975) who examine three different hypotheses as to why MMs charge a spread. In their work, Bagehot’s hypothesis of
asymmetric information is compared with the competing hypotheses that the spread represents the cost of liquidity, or that it reacts to MMs' inventory. The authors find support for the first two hypotheses (asymmetric information and liquidity cost), but fail to devise a quantitative method for testing the third hypothesis (the inventory motive).

In response to the changing nature of financial business, Garman (1976) wrote the first technical paper on market microstructure. In it, he explained why a new modeling approach was needed to study the functioning of financial markets. As trading volume increased, the traditional system of "call markets," in which transactions take place synchronously at pre-specified times, became restrictive and gave way to "continuous markets," in which trades take place asynchronously over a continuous interval. This change had already occurred in many markets well before Garman’s writing, with the NYSE switching its format to the continuous system in 1871.

The switch to continuous markets necessitated creation of the MM position. Instead of markets functioning as auction houses, in the new system markets play the role of "dealerships" with MMs as the dealers. Garman’s MM is a price-setting monopolist who is motivated by the desire to maximize profit and avoid bankruptcy. As a simplifying assumption, order flow is treated as a Poisson process; therefore, information content of trades cannot explain MM behavior. In Garman’s setup, inventory concerns are the reason for the existence of MM’s spread.\(^4\)

Glosten and Milgrom (1985), further exploring the order generating process, propose a model with informationally motivated MMs.\(^5\) In their model, two groups of

\(^4\)Further work in this area has been undertaken by Amihud and Mendelson (1980), Ho and Stoll (1981), and Ohara and Oldfield (1982).

\(^5\)Copeland and Galai (1983) had already shown that a MM with informational motivation fits the empirical facts of the time; however, in their model information about the value of the asset is immediately revealed after each transaction. Therefore, asset price fluctuations are driven exclusively by exogenous disturbances.
traders (rational informed and irrational uninformed) are randomly selected to make a trade with the MM. The MM is modeled as a competitive, price-setting, risk-neutral dealer, who counteracts uncertainty about the agent type with whom she is trading by creating a bid-ask spread. The bid price is higher than the average because, given a positive probability that the trader is informed, the transaction indicates that the asset may be undervalued. Likewise, the ask price must be lower, since a sell by an informed trader would indicate that the asset is overvalued. The MM estimates the value of the asset based on trading history by utilizing Bayesian learning to update prior information.

The Glosten-Milgrom model outlines all of the standard features now widely used in papers on market microstructure: (1) MMs set prices based on information, not inventory; (2) the group of rational traders has superior information, and (3) the irrational group (often called "liquidity traders") only has access to public information; and (4) an additional degree of uncertainty is necessary to prevent prices from fully revealing information, such as the assumption that trader type is unknown to the MM at the time of the transaction.

Under these assumptions, Glosten and Milgrom reach five main conclusions. First, the bid and ask prices straddle the price that would prevail with symmetric information. Second, transaction prices are martingale. Third, the expected value of the squared average spread times volume is bounded and related to the variance of uncertainty about trader types. Fourth, over time, private information is fully disseminated into asset prices. Fifth, spreads increase if (a) insiders have better information or (b) are more numerous, or (c) elasticity of supply and demand for liquidity trades.

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6Irrational traders are introduced to avoid the outcome of the Stokey and Milgrom (1982) No Trade Theorem that MMs do not participate in the market because they expect to lose money on all trades. Instead, the assumed risk-neutrality of MMs, coupled with the existence of irrational traders, guarantees that MMs break even in expectations.
increases. Additionally, Glosten and Milgrom show that markets shut down if the bid-ask spread becomes too large. The authors also suggest that asymmetric information may explain the "small firm effect" and the "ignored firm effect."\(^7\)

The assumption that MMs cannot identify which traders have better information may seem questionable; however, there exists evidence indicating that prominent investors (investment banks, hedge funds, etc.) employ trading tactics to mask their identity (Mercorelli, Michayluk and Hall, 2008). These tactics, aimed to counteract the adverse selection effect of MM pricing, can create market distortions and contribute to information loss. In order to become more efficient, many markets have introduced computerized MMs, often designed in a way to make trader identification impossible. Under these newer conditions, and considering the efforts of informed traders to mask their identity, the Glosten-Milgrom assumption that trader types are unknown to MMs is reasonable.

Extending the original Glosten-Milgrom framework, Das (2005) solves the MM’s problem using non-parametric estimation (as a substitute for Bayesian learning). The flexibility of the non-parametric estimation method allows Das to add many frictions (such as inventory controls, monopolistic competition, etc.) to better match many more features of stock market movements.

Bouchaud et al. (2004) also claim that the connection between traders and MMs, who they call liquidity takers and liquidity suppliers, respectively, is integral to successfully explaining asset price movements. The authors argue that the random walk nature of asset price movements comes from a balancing act between traders, who create positive price autocorrelations, and MMs, who attempt to make prices mean-

\(^7\)The "small firm effect" theory postulates that smaller firms tend to outperform larger companies. The "ignored firm effect" theorizes that firms which are not tracked by analysts outperform other firms. See Banz (1981) and Arbel and Strebel (1981) for further information about these effects.
reverting. Further, they suggest that, while in the short run prices behave *almost* as a random walk with linear dispersion, in the medium run they are mean reverting. Contrary to the original work of Glosten and Milgrom (1985) the authors find that most transactions contain no information.

Further sophisticating the nature of the MM, Weill (2007) shows that market specialists tend to "lean against the wind." Motivated by the transactions cost of time delays in trades, Weill argues that MMs have an incentive to hold excess capacity during brief periods of increased sales. When a large number of sales take place, the MM anticipates that this behavior cannot last and will soon be reversed. Rather than lowering prices to find buyers for the assets, the MM holds the excess capacity so that he will have assets on hand when the market rebounds and investors seek to rebuild their original portfolio positions. By having the assets readily available when buyers reenter the market, MMs can lower their transaction costs and earn a profit. Weill’s conclusions support the finds of Bouchaud et al. (2004), since in both papers MMs are treated as exerting mean-reverting influence on markets that would otherwise be characterized by excessively large fluctuations of asset prices.

Staveley-O’Carroll (2009), by incorporating a Glosten-Milgrom MM into a standard Lucas endowment economy, analyses the role of macroeconomic fundamentals on the timing of asset price movements. In this model, the high-frequency interaction between traders and MMs is affected by low-frequency factors such as productivity persistence or output volatility; this specification alters the dissemination of information in asset markets and therefore changes the timing of price movements. Specifically, Staveley-O’Carroll shows that countries with highly persistent shocks to productivity are more likely to suffer from large sudden asset price movements. Since the process of Bayesian learning takes longer in countries where shocks are less likely
to occur, asset prices do not adjust gradually but rather experience sharp movements when information finally becomes public.

Evans and Lyons (2002) employ the MM to explain the movement of exchange rates in currency markets. The structure of their model differs from the traditional Glosten-Milgrom archetype. Each period, trading occurs in three stages: (1) traders transact with MMs (called dealers); (2) interdealer trade generates order flow; and (3) MMs note the overall demand and supply of the asset based on order flow, adjust prices accordingly, and transact with traders for the second time. Using this framework, the authors show that order flow can explain medium-run exchange rate movements an order of magnitude better than traditional macroeconomic models. Their further work (Evans and Lyons, 2005), incorporates this market microstructure mechanism into a macroeconomic dynamic stochastic general equilibrium model (DSGE) to endogenize the interaction between dealers and traders.

Puzzles
Correctly modeling the behavior of market makers allows researchers to address some puzzles that exist in finance literature. Generally, traditional macroeconomic models cannot explain the observed patterns of asset price movement; fundamentals and prices show little empirical correlation (Meese and Rogoff, 1983; Roll, 1988). Furthermore, news events do a poor job of explaining sudden price changes (Romer, 1993).

Meese and Rogoff (1983) presented a serious challenge for the field of international finance when they showed that all standard approaches to modeling exchange rate behavior did no better at predicting out-of-sample exchange rate movements than a random walk process. This result, which became known as the "Exchange Rate
Disconnect" puzzle, has held up against most efforts to improve on predictive accuracy. The only area where great strides were made to address this challenge is market microstructure.\(^8\) As mentioned in the previous section, Evans and Lyons (2002) were the first to show that models which incorporate order flow perform much better than alternatives when attempting to overturn the Meese-Rogoff result.

Furthermore, Evans (2009) constructs a two-country DSGE model in which each country’s population is modeled as yeoman farmers who possess dispersed information and act as both traders and dealers. The model highlights the theoretical linkages between order flow and exchange rate movements and makes significant advances in explaining the "Exchange Rate Disconnect" puzzle.

Market microstructure models can also help address the "Forward Premium Puzzle." Empirical work studying international bond returns notes an odd feature of the data: currencies whose investment opportunities have high returns tend to appreciate relative to currencies with low returns. This goes against the predictions of standard international macroeconomic models; in particular, uncovered interest parity is at odds with this observation, since local investors who purchase foreign currency to invest abroad enjoy higher returns in addition to capital gains from currency appreciation.\(^9\)

Burnside, Eichenbaum and Rebelo (2009) attempt to explain this puzzle by employing a Glosten-Milgrom MM framework. They imagine a world with two types of investors, one informed and the other uniformed, who trade in currency. If one currency pays a higher interest rate on its bonds, then it is expected to depreciate, and uninformed traders are assumed to sell it. Consequently, the only traders who

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\(^8\)For a survey of the recent work on market microstructure and exchange rates see Osler and Mizrach (2008).

\(^9\)A general survey of theoretical and quantitative work on the forward premium puzzle can be found in Engel (1996).
wish to buy this currency are the informed investors who believe it is about to appreciate. To avoid losing money on currency transactions due to this adverse selection problem, MMs must charge a high price to anyone who buys a currency that is publicly expected to depreciate. Thus, the price of the currency will rise, generating the correlation captured in the forward premium puzzle.

1.2.2 Banks

While market makers facilitate financial transactions on a continuous basis, there exists another distinct group of financial intermediaries – banks – who offer a much broader spectrum of financial services. The essential role of banks is to provide a conduit to funnel capital from those agents who wish to save to those who wish to borrow (generally, to invest). In this chapter, I do not restrict the definition of banks to only include deposit holding companies. Instead, I consider all groups that borrow short term and lend long term, including money market funds, hedge funds, mutual funds, and pension funds, as essentially performing the role of a bank. In this section I provide a general overview of the role of banks in macroeconomic models, including the importance of government regulation for bank behavior, and then present a brief description of the nature of bank-like entities.

Banks differ from market makers in that a MM allows traders to exchange a particular asset, whereas a bank transforms assets from savings into loanable funds. While the role of banks may seem too obvious to mention, in academic literature they have been modeled as performing four different functions. The most basic of them is provision of deposit services, such as check cashing and automatic teller machines. Banks also act as insurers of idiosyncratic risk through aggregation of capital and provide the economy with liquid assets in the form of deposit receipts. Banks act as financial intermediaries between borrowers and lenders, thus reducing loan monitoring
costs. Finally, banks have recently begun securitizing assets to reduce idiosyncratic risk.

Banks in Kareken and Wallace (1978) are assigned the simple role of providing "deposit services." The bank is allowed to issue both debt and deposit receipts; the latter grant some non-specific benefits which make demand for deposits less than perfectly elastic. In this setup, the bank acquires funds and then chooses where to invest them, thus performing the role of financial intermediary. Returns from investments are used to pay interest on deposits and loans and cover service costs, with the remainder adding to bank’s profits. The bank’s only role is to provide deposit services; however, the authors assume that agents can invest directly in the same range of assets that is available to the bank. In this sense, banks in this model are not true financial intermediary.

Diamond and Dibvig (1983) do not treat the banking sector as a true financial intermediary either when they model the impact of bank runs on the real economy. In their paper, banks provide liquidity that acts as a form of insurance against idiosyncratic shocks. Agents are subject to consumption shocks and so use banks to shield their investments from individual risk. However, due to inherent risk of illiquidity, banks are subject to runs in bad equilibria. These bank runs are costly to the economy, providing motivation for mandating deposit insurance.

Starting with Diamond (1984), banks have been treated as true financial intermediaries. The need for the banking sector is motivated by the existence of asymmetric information between lenders and borrowers. Lenders need to monitor the behavior of agents to whom they loan money, but it is inefficient for lenders to bear the monitoring costs themselves, since some lenders may free ride on others to monitor the loans. This problem can be fixed by introducing a financial intermediary (bank). The bank accumulates interest-bearing deposits from the savers in the economy and
uses them to issue loans and cover the monitoring cost. The paper shows that the loanable funds market becomes efficient through the introduction of the bank.

Recent work has expanded the breadth of the financial sector’s involvement in the economy. Caballero (2009) points out that financial intermediaries transform asset riskiness to meet market demand. He argues that the world suffers from a shortage of "safe" debt securities, and that financial entities in the U.S. address this shortage by pooling and then tranching risky debt securities. Through the process of combining and securitizing risky mortgage debt, banks create debt assets with AAA ratings. The cost of this process is an increase in the systemic risk of the entire financial sector, which can therefore endanger the entire economy, not unlike the basic bank run model of Diamond and Dibvig (1983).

International Banking

A number of recent papers address the issues of international financial intermediation done by banks. McCauley, McGuire and von Peter (2010) describe the evolution of international banks into multinational banks, the difference being that international banks provide financial services to large companies which engage in cross-country transactions, whereas multinational banks offer their services to agents at all levels (from individual consumers to large corporations) in multiple countries. Thus, the authors show that the banking sectors of different countries are becoming more closely intertwined. This finding is supported by Stein (2009), who describes the role of the International Monetary Fund in encouraging foreign banks to buy local branches in the developing world make the local financial sectors more efficient. A number of papers have begun including a global banking sector in two-country models in order to reflect this trend.
Olivero (2010) models a monopolistically competitive global banking sector that transmits total factor productivity (TFP) shocks between countries through the lending channel. If one economy experiences a positive shock, lending rates fall in both countries as a result of increased competition between banks. Kollmann, Enders and Müller (2011) extend this work by incorporating capital requirements on competitive global banks. They show that loan losses in one country have a large negative impact on the availability of credit and therefore economic activity in the other country.

Whereas banks in the above two papers are modeled as financial intermediaries that link savers and entrepreneurs, Staveley-O’Carroll (2012) introduces a global banking sector which allows for international risk sharing through portfolio diversification. In this work, agents in each country must use the financial intermediary to exchange equity and debt assets to hedge against country-specific risk. The model illustrates the impact of asymmetric bank regulation on international portfolio imbalances by showing that more deregulated countries tend to hold larger gross portfolio holdings.

**Regulation**

No survey paper on banking can be complete without at least a cursory examination of the literature on financial regulation. Meltzer (1967) lists various arguments for and against government regulation of the financial sector. Indeed, there is little agreement in the subsequent literature as to the necessity of bank regulation.

Kareken and Wallace (1978) claim that without regulation, banks would be able to operate efficiently, and bank bankruptcy would be avoided. The crucial assumption of their paper, however, is that there is no informational advantage between different types of agents. Thus, deposit holders would not allow banks to make investments
that are too risky by threatening to pull out their money. Bank regulations, the authors argue, are only necessary in the presence of deposit insurance which removes the incentive for investors to track bank behavior themselves.

Diamond and Dibvig (1983) argue instead that without deposit insurance, banks would be subject to runs. Furthermore, once banks are insured, they must be regulated to prevent moral hazard of making risky investments the cost of which is borne by the government.

The idea that banking is more fragile, and at the same time more important to the economy, than other sectors is the underlying cause of much of the industry regulation that exists today. For an overview of the recent literature on bank regulation see Tchana Tchana (2009). An analysis of the welfare costs of capital controls on banks is conducted by Van den Heuvel (2008).

**Bank-like Entities**

Not all financial intermediaries fall under the definition of a traditional bank. There are several types of bank-like entities which are regulated differently by the government, but still provide the same fundamental service as banks – only without the ability to hold deposits. This section following provides a brief description of mutual funds, hedge funds, pension funds and money market funds.

As defined by the U.S. Securities and Exchange Commission, a mutual fund is a "company that pools money from many investors and invests the money in stocks, bonds, money-market instruments, other securities, or even cash." These entities allow investors to outsource the job of managing a balanced portfolio to a third party (mutual fund managers). Shares of the mutual fund are not traded on an exchange, but are instead traded directly by the fund at the net asset value (NAV) of the investment portfolio.
Macroeconomic interest in mutual funds comes primarily from the international finance area, since they are responsible for over 17 percent of international portfolio flows into developed economies (Gelos, 2011). The compensation mechanism for fund managers is closely tied to the performance of their portfolios against various benchmark indices, therefore affecting the investment strategies pursued. Furthermore, Kaminsky, Lyons and Schmukler (2004) show that fund managers chase returns through "momentum trading" strategies, which may explain the high volatility of portfolio flows into emerging markets.

According to Mitra (2009), while there is no accepted definition of a hedge fund, they are similar to mutual funds, except for having a more aggressive investment strategy and a more limited investor pool. While mutual funds tend to follow the buy-and-hold strategy, hedge funds shift their portfolios regularly and leverage their investments considerably. Additionally, whereas mutual funds pool money from all types of investors, hedge funds limit themselves to wealthier investors such as pension funds or rich individuals. The paper reports that the booming interest in hedge funds has increased the value of global assets under their control to over one trillion U.S. dollars.

Pension funds serve the explicit purpose of investing savings earmarked for retirement. Due to their nature, pension funds tend to be larger than other types of funds and thus more likely to impact the markets in which they operate. Investment strategies of pension funds fall into two categories: defined contribution and defined benefit. Defined contribution funds adopt strategies similar to other investment funds, whereas defined benefit plans lean towards more extreme strategies (Bodie, 1988). Furthermore, Bauer, Cremers and Frehen (2010) use CEM pension fund data to show that pension funds outperform other funds; this is particularly true for small pension funds. An additional consideration for pension fund managers is that their invest-
ments are typically not taxed, and that pension funds are often subject to investing guidelines which prevent them from taking risky positions (Pagratis, 2005).

Finally, money market funds act in much the same way as traditional banks, using their capital to supply mostly corporate loans. They are, however, less regulated and thus able to pay higher interest rates on their portfolios. Money market funds managers pay particular attention to not "breaking the buck," a phrase indicating that investors have lost a portion of their principal. Baklanova (2010) provides a survey of the literature on money market funds.

1.3 Asymmetric Information

A crucial feature in most models of financial intermediaries, whether MMs or banks, is the existence of asymmetric information. MMs use bid-ask spreads to prevent inside traders from bankrupting them, and the underlying rationale for using banks as intermediaries typically derives from the moral hazard problem connected with loan monitoring. If we are to believe that financial intermediaries play an important role in the economy, then we need quantitative proof that individual traders have access to significantly different information sets, not an easy evidence to obtain. Bouchard et al. (2008), for instance, find almost no informational content contained in the trading of French stocks, while Evans and Lyons (2002) demonstrate that order flow has significant predictive power for the behavior of exchange rates.

There are two alternative informational assumptions that are often made in this literature. The first, explicitly made in all papers where banks act as intermediaries, is that one group of investors has superior information relative to the other.¹⁰ The

¹⁰Burnside, Eichenbaum and Rebelo (2009), Das (2005), and Glosten and Milgrom (1985), among others, make this assumption explicitly. It is implicitly assumed in, for example, Bernanke, Gertler and Gilchrist (1999), Diamond (1984), and Staveley-O’Carroll (2012).
alternative assumption is that no trader has superior information, but instead that information is randomly dispersed among all agents.\textsuperscript{11} Prices serve different roles in these two setups. When some traders have superior information, prices slowly reveal it to the rest of the market; when information is dispersed, prices instead work to aggregate it.

In empirical work there are typically two ways of grouping agents: home versus foreign, and sophisticated versus non-sophisticated. In one of the first papers that addresses differences between home and foreign investors, Frankel and Schmukler (1996) identify a method for measuring the relative information sets of these two types of agents in the Mexican economy. By comparing the NAV of three Mexican closed-end country funds – the Mexico Fund (MXF), Mexico Equity and Income Fund (MXE), and Emerging Mexico Fund (MEF) – against their prices listed on the New York Stock Exchange, the authors show that local investors have an information advantage over foreign traders. NAVs are determined by aggregating local prices of the components of the funds and translating them into dollars; the prices are determined primarily by local Mexican investor behavior and thus reflect local information. Closed-end country fund prices, on the other hand, are determined on Wall Street by foreign investors, and thus reflect the foreign information set. Frankel and Schmukler show that, during the Mexican crisis of 1994, NAVs moved prior to country fund prices and, in fact, had Granger caused the movements abroad.

Choe, Kho and Stulz (2004) support this finding by studying investment returns of local and foreign money managers in South Korea. They find that local investors receive higher returns than do foreign investors, but the results of the paper are more robust for large trades carried out by professional investors. The authors explain this

\textsuperscript{11}Papers in this area include Bikhchandani and Sharma (2001), Evans and Lyons (2002, 2005), and Romer (1993).
result by conjecturing that foreign traders chase trends: foreigners tend to buy local assets when their prices have recently risen and sell following price drops. Further evidence of superior local information is provided by Hua (2001) and Dvorak (2005), although the informational advantage is found to be limited mostly to sophisticated professional traders.

These arguments are countered by Froot and Ramadorai (2008), who use portfolio data from State Street Bank to differentiate between sophisticated investing institutions and small individual investors. By examining country fund prices, NAVs, and cross-border capital flows, the authors find that investing institutions which trade directly in foreign markets have superior information to local traders. The same does not hold for small foreign investors who trade only in country funds outside of the assets’ primary markets. In a sense, Froot and Ramadorai outline the hierarchical structure of informational advantage, with large foreign institutions on top, local traders in the middle, and small foreign traders on the bottom. This finding supports the assumption present in much of the banking literature that large investment institutions, such as banks and bank-like entities, have an informational advantage over individual traders. Notice that these results do not contradict the previous findings of Frankel and Schmukler (1996), but do run counter to those of Choe, Kho and Stulz (2004), Hua (2001) and Dvorak (2005).

In a separate but related area Rothenberg and Warnock (2011) and Covrig et al. (2010) bridge these competing partitions by assuming that asset values are determined by local and global components. Foreign investment institutions are assumed to have superior information about the global value of an asset, while local investors know more about the local value.

Rothenberg and Warnock (2011) use this assumption to differentiate between sudden stops and capital flights. Sudden stop are initiated abroad by institutional
investors and are caused by a negative shock to the global component of the stock value. Capital flights, on the other hand, are initiated by local investors following a negative shock to the local component of the asset.

Covrig et al. (2010) utilize data from mutual fund holdings of 5,781 stocks from 21 developing countries to show that information about specific and common stock valuation factors can explain the patterns of international portfolio holdings. Specific factors impact only the asset values in a particular country and are thus local. Common stock valuation factors impact several countries at the same time and are thus global. Additionally, the authors find that dispersion across assets of the relative importance of common and specific factors can explain the observed differences in equity portfolio holdings among mutual funds.

Finally, a unique approach to the asymmetric information problem is presented in Pagratis (2005). In his paper, ratings agencies have an informational advantage over investors, and announcements made by the former may increase the amplitude of price fluctuations.

1.4 Business Cycle

Given the vastness of the real business cycle (RBC) literature, it comes as no surprise that the interaction between financial intermediaries and business cycle properties has been analyzed in great detail. While it is clear from the data that banks and other financial entities can affect economic fluctuations, up until recently the Modigliani-Miller theorem (stating that financial structure has no impact on the real economy) provided a serious roadblock for the development of the literature. As Bernanke, Gertler and Gilchrist (1999) point out, however, the existence of asymmetric information between borrowers and lenders and the resulting structure of contracts aimed
at minimizing the cost of information gathering render the Modigliani-Miller theorem inapplicable.

The "cost of information gathering" that Bernanke, Gertler and Gilchrist (1999) mention goes back to Diamond’s (1984) argument for the existence of financial intermediaries, namely, that the latter reduce the cost of monitoring loans. The "structure of contracts" refers to earlier works by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), which identify and elucidate the financial accelerator effect.

The financial accelerator mechanism originates in the contract structure that is necessary for banks to address the moral hazard problem of their clients. While borrowers know how likely they are to pay back a loan, this information is unavailable to the lending institutions. In response, lenders can either gather costly information about the borrower’s nature, pay to monitor the loan, or securitize the loan contract with collateral requirements. The financial accelerator effect arises in the third case because the value of collateral used to securitize loans tends to change with macroeconomic fluctuations.

Houses, stocks, bonds, land, and durable goods are frequently used as collateral for loans, imparting a dual nature to these assets. Their value is determined by market conditions. In a booming economy, asset prices increase and thus provide collateral for larger loans; in recessions, asset prices fall, thereby tightening borrowing constraints of households and businesses. As a result, real business cycle fluctuations are magnified by loan contracts.\footnote{The role of the financial accelerator in RBC models is examined in detail in, among others, Bernanke and Gertler (1989), and Bernanke, Gertler and Gilchrist (1996, 1999).}

Gertler and Kiyotaki (2010) explicitly bring the banking sector into the financial accelerator model by switching the focus of the literature from the borrowing constraints of households and firms to the borrowing limits of the financial institu-
tions. In particular, the authors assume the existence of an agency problem between banks and households, which induces an interest rate spread, thus generating a financial accelerator effect. Meh and Moran (2010) similarly focus on the bank’s balance sheet, but with special consideration given to the capital level. They show that well-capitalized banks are better insured against economic downturns and are thus less likely to exacerbate negative economic shocks.

A counter-argument to financial intermediaries amplifying real business cycles comes from Dib (2010). In his work, there exist two banking sectors with different roles: a "lender" bank and a "borrower" bank. The lender bank accepts deposits from households and lends the proceeds on the interbank market to the borrower bank, which then issues loans to entrepreneurs. Both banks have monopoly power and so can set both deposit and lending rates. Additionally, banks must choose how to allocate the riskiness of their portfolio (i.e., how much to leverage their investments), and are capable of endogenously choosing to default on some of their interbank borrowing. The paper thus focuses on the supply side of the credit channel, as opposed to the demand side studied in the financial accelerator literature. Financial intermediaries are found to actually lower the variance of real business cycles. This result stems from the active role banks take in reducing the impact of shocks on their profits.

Two papers focus on the international propagation of business cycle shocks by modeling a global financial sector. Olivero (2010) finds that real business cycles can be transmitted across borders through the lending channel, while Kollmann, Enders and Müller (2011) indicate that capital requirements have only a minimal impact on the transmission of TFP shocks between countries.
1.4.1 Monetary Policy

An analysis of the role of financial intermediaries in monetary policy, conducted by Aikman and Paustian (2006), concludes that "assigning a non-trivial role for these frictions need not materially affect optimal monetary policy." However, as a result of the financial crisis of 2008, a new branch of literature has evolved that contradicts this prescription.\(^\text{13}\)

Unable to prop the declining economic activity with traditional interest rate manipulation, the Federal Reserve initiated a number of innovative policies in the months following the collapse of the U.S. housing bubble. These policies include direct lending to non-financial entities, quantitative easing, and purchases of risky assets from the financial sector. Traditional monetary frameworks, unable to analyze these new techniques, have been updated through the explicit introduction of the private financial sector. These new model vintages can be used to better understand the complex interactions between financial intermediaries and the central bank.

Goodfriend and Bennet (2007) employ the banking sector to introduce a wedge between various interest rates in the economy: collateralized versus uncollateralized debt, treasury bills, the marginal product of capital, and the intertemporal rate. These wedges arise because of the monitoring and collateral costs associated with the bank’s production of loans. The authors suggest that in the presence of multiple interest rate spreads, the overnight rate may not be the optimal instrument of monetary policy.

Cúrdia and Woodford (2009), using a similar setup, study the variability of interest rate spreads that are generated by the financial intermediary frictions. The authors

\[^{13}\text{Papers in this area draw on the work by Bernanke and Gertler (1995), Bernanke, Gertler and Gilchrist (1999), Carlstrom and Fuerst (1997), Cooley, Marimon, and Quadrini (2004), Kiyotaki and Moore (1997), and Kocherlakota (2000), and all of which introduce agency costs to generate an external finance premium.}\]
find that while interest rates spread variations are important for the relationship between real economic activity and inflation, they do not significantly affect the optimal conduct of the central bank. This framework is extended in Cúrdia and Woodford (2011) to examine optimal monetary policy in a model where the central bank has alternative policy tools, such as quantitative easing and Federal Reserve balance sheet targeting, which are not subject to zero lower bound.

1.5 Financial Crises

Since the publication of the Modigliani-Miller theorem, most models that included financial intermediaries where aimed at explaining the Great Depression or at examining financial crises in emerging economies. The assumptions of the theorem were thought to more or less hold in developed economies, but were viewed as less applicable to emerging markets or countries experiencing periods of economic instability. However, since the financial crisis of 2008, the rapidly expanding literature on financial frictions has been used to analyze business cycles in many OECD countries. In no area of research is that shift more evident than in the field examining economic crashes and financial crises.

Below, I discuss papers analyzing formation, buildup, and subsequent crash of asset bubbles and connect them the older literature on the spread of crises between countries, a phenomenon known as "contagion."

1.5.1 Bubbles

The first issue to be addressed when discussing asset bubbles is their definition. The standard approach of identifying bubbles as periods during which asset price exceeds its fundamental value is contentious, and for good reason: it is unclear what is meant
by the term "fundamentals." If expectations and beliefs are included in the definition of fundamentals, then a price increase caused by speculative optimism is not in fact a bubble, but merely rational response of optimizing traders. True bubbles are then price increases that happen despite the rational actors’ awareness that an asset is already overvalued.

Despite this difficulty of defining a true bubble, any examination of economic crashes cannot be considered complete without a discussion of the causes of asset price buildups prior to their subsequent collapse. Therefore, below I introduce several papers that examine the rise of asset prices in the lead-up to a crash; whether these rises can be considered bubbles in the true sense of the work is irrelevant.

Pastor and Veronesi (2009) present a survey of the literature on learning in financial markets. In it, they describe the role of learning following the introduction of new technology, which brings with it a great deal of uncertainty. Much like in the market microstructure literature discussed in section 1.2, agents use Bayesian learning to determine the value and expected growth rate of the new technological industry. Overinvestment is actually optimal, once one accounts for the value of learning about the new technology. The authors provide evidence of overinvestment in technology ranging from the introduction of railroads to the internet. Although not discussed in the paper, a case can certainly be made that new financial instruments, such as collateralized debt obligations (CDO), credit default swaps (CDS), and mortgage-backed securities (MBS), represent technological advanced of the 2000s.

Greenwood and Nagel (2009) place the blame for asset bubbles on professional portfolio managers from mutual funds and other financial intermediaries. The authors find that once one controls for the age of mutual fund managers, inexperience is the root cause of asset price inflation. Using data from Morningstar, the paper illustrates that younger, and thus less experienced, traders invested more heavily in technology
stocks in the buildup of the dotcom bubble of the late 1990s than did their more experienced colleagues.

A similar buildup of technology stocks by hedge funds prior to the 2000s was observed by Brunnermeier and Nagel (2004). Unlike the previous two papers, however, they also noted that hedge funds limited their risk exposure right before the collapse by selling off a portion of their technology stocks. This pattern matches the Greater Fool bubble theory, in which agents rationally invest in overvalued assets based on the belief that they will be able to sell those assets (to even greater fools) before the bubble bursts. Such episodes are modeled in Doblas-Madrid (forthcoming) using multidimensional uncertainty based on the market microstructure literature. Agents know that an asset is overvalued, but invest in it anyway hoping to reap a profit by selling during the pre-crash frenzy, perhaps to inexperienced investment managers.

1.5.2 Crashes

Financial intermediaries were always believed to influence real activity during periods of economic and financial collapse. Under normal conditions, the assumptions of the Modigliani-Miller theorem (no bankruptcy or agency cost, symmetric information, and efficient financial markets) do not seem implausible; however, during financial panic, when bankruptcy rates rise, and frozen asset markets stall the information dissemination process, the importance of the financial structure of the economy cannot be understated.

One of the common explanations of economic collapse is the presence of asymmetric information. Traditionally, sudden declines in asset prices were blamed on herding behavior of investors. Indeed, many studies referenced in this paper find evi-
dence of momentum trading by professional investors. The theory of herding aims to explain sudden changes in portfolio positions of agents through a loss-of-information mechanism. In market microstructure models with continuously operating markets, traders must use a combination of their own information while still learning from price signals. Thus, a series of asset sales and the subsequent price decline might cause investors to ignore their own information and sell based on the observed market outcomes. In this setup, information becomes trapped as prices are simply reflecting the first several trades and are no longer aggregating market sentiment (Bikhchandani and Sharma, 2001).

Yuan (2005) offers a different take on the traditional herding mechanism by introducing borrowing constraints into her model. She assumes the existence of sophisticated professional traders who possess superior private information. Other traders wish to learn from these informed agents, but may become "confused" when prices fall, unsure if sells are occurring for informational reasons or because sophisticated investors have reached their borrowing limits. Borrowing constraints are assumed to be binding, as portfolio managers often have to sell off positions in response to investor capital withdrawals from their funds.

Staveley-O’Carroll (2009) shows that the speed of price changes may depend on the persistence of macroeconomic variables. If agents fundamentals to be fairly stable, the information dissemination process is slowed, and price adjustments become much more abrupt.

Separate from asymmetric information, regulation issues often receive part of the blame for financial collapses. Berg and Eitrheim (2009) illustrate using Norwegian experience that regulators behave in a cyclical manner. In good times, regulators

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14See, for example, Bouchaud, Gefen, Potters, and Wyart (2004), Choe, Kho, and Stulz (2004), Froot and Ramadorai (2008), Greenwood and Nagel (2009), and Tille and van Wincoop (2009).
relax restrictions and allow financial entities a great deal of leeway. During crises, regulators instead tighten restrictions and introduce new legislation. Such behavior corroborates Caballero’s (2009) explanation of the underlying reason for financial collapse as an imperfect attempt by the financial sector to supply the world with "safe" assets. In doing so, he argues, risky debt is tranched and repackaged to create AAA-rated bonds that can only fail in the event of a systemic collapse. Thus, the assets are safe during normal times, but increase the risk to the global economy of a systemic financial failure. Financial intermediaries engaged in tranching precisely because regulations were weakened during the decades leading to the 2008 financial crisis.

In the initial stages of a financial crash, it may be unclear whether an economy is experiencing a liquidity shortage, or if the financial sector is truly insolvent. Liquidity, while readily available under normal economic conditions, can evaporate quickly when asset prices abruptly decline. This complicates operations of financial firms who value their portfolios based on market prices, and leaves investors, intermediaries and regulators asking the question "If the market for an asset is not operating, then how much is the asset worth?"

As discussed previously, Weill (2007) argues that during downturns liquidity is often provided by MMs who prefer to "lean against the wind." This finding, however, is questioned by Lagos, Rocheteau and Weill (2011), who find that under a variety of market parameterizations, MMs may optimally choose not to provide liquidity. Papers discussed below show that capital constraints, credit crunches, and consumer default may lead to lower liquidity provision and therefore further deepen the financial distress.

In a series of papers, Gertler and Kiyotaki (2010) and Gertler, Kiyotaki and Queraltó (2011) explicitly model a banking sector that must optimally choose its capital
structure in order to provide loans to entrepreneurs. Banks prefer to raise capital from equity sales (since dividends are never guaranteed), but because of investors’ concerns about embezzlement, a portion of the capital must be borrowed (embezzling borrowed funds is assumed to be more difficult because of the regularity with which debts must be maintained). As a result of risky bank portfolios, banks must reduce the number of loans issued during a downturn which increases the external finance premium and thus lowers economic activity. This economic stress can be relieved through direct lending to the production sector of the economy by the central bank.

De Walque, Pierrard and Rouabah (2010) present a model with heterogeneous banks and exogenous default probabilities of firms and banks. The banking sector plays an important role in the propagation of financial distress, but the authors show that Basel II regulations help to shield the economy from the effects of shocks. In another paper with exogenous default by firms, von Peter (2009) illustrates the nonlinear dynamics between loan default and asset price movements when banks act as a propagation mechanism though their balance sheet effects (an equivalent of the financial accelerator mechanism, described in section 1.4). This model allows von Peter to identify the causes of credit crunches, financial instability, and banking crises using case studies of the U.S. Great Depression, Japan’s Lost Decade, and Norway’s banking crisis of the late 1980s. Further work by Gerali et al. (2009) uses Bayesian estimation to demonstrate that shocks to bank capital were the leading cause of output decline in Europe in 2008.

Iacoviello (2011) models two distinct borrowing groups, impatient households who borrow to buy a house, and entrepreneurs who borrow to invest in productive capital. When impatient households default on a portion of their debt, banks, which must satisfy certain capital requirements, react by deleveraging their balance sheets, leading to reduction in capital investment and output. Thus, capital requirements
are found to be the underlying cause of financial crises. In a similar study, Angeloni and Faia (2009) suggest that capital requirements should be mildly counter-cyclical, a recommendation which, in the framework used by Iacoviello (2011), would reduce the output cost of household loan defaults.

1.5.3 Contagion

In the 1990s, a number of financial crises spread throughout the developed world and gave rise to the theory of contagion. Kaminsky, Reinhart, and Vegh (2003) define contagion as “an episode in which there are significant immediate effects in a number of countries following an event.” The "event" generally refers to a sudden stop of capital inflow into a single country, which then affects foreign investment in other countries and gives rise to a chain of financial crises.

The blame for these crises is often placed on financial intermediaries; thus, Kaminsky, Reinhart, and Vegh (2003) outline each outbreak and attribute it to a particular group of investors (among them, hedge funds, mutual funds, and U.S., European and Japanese banks). The connection, as identified by the authors, between countries that experience contagion is a leveraged common creditor. Once this entity experiences a loss from the initial shock that occurs in any one country, it is forced to rebalance its portfolio and reduce its risk exposure in all other countries (Kaminsky and Reinhart, 2000).

On the empirical side, Broner, Gelos and Reinhart (2006) employ a broad data set covering hundreds of equity funds with a focus on emerging markets for the 1996-2000 period to illustrate the tendency of funds to rebalance towards a benchmark portfolio when returns are below average. Such behavior on the part of many mutual and hedge funds may lead to contagion. Further evidence is provided by Boyer, Kumagai and Yuan (2006), who compare the co-movement of stock prices in emerging markets
across two categories: assets that may be purchased by a foreign investor, and those that may not. The authors show that stocks which can are available to foreigners co-move to a much higher degree, suggesting that mutual funds and their investors may be contributing to financial contagion.

Jotikasthira, Lundblad and Ramadorai (forthcoming) find evidence that financial intermediaries are a conduit, rather than a cause, of contagion. Using hedge fund and mutual fund data, the authors show that fund managers react to binding borrowing constraints in the face of investor withdrawals (due to an adverse shock) by selling assets at fire-sale prices and thus increasing asset price co-movement between developing markets. This hypothesis is supported by Kaminsky, Lyons and Schmukler (2004) who analyze the behavior of fund manager and investors to show that contemporaneous momentum trading during crises is dominated by the behavior of the latter group. Pavlova and Rigonbon (2008) present a theoretical three-country model in which borrowing constraints cause excessive co-movement between asset prices in two countries as a result of the third country reaching its credit limit.

Yuan (2005) and Calvo (2005) argue that binding constraints can explain only a portion of the contagion story. By combining borrowing constraints with the informational advantage fund managers are assumed to possess, the authors demonstrate that information can become lost and traders "confused." The confusion arises because traders cannot tell if fund managers are forced to sell because of investor funds withdrawal, or if they are acting on superior information. Traders may incorrectly infer that managers have received a negative signal about the future state of the economy, and so can initiate a bout of contagion.

Finally, Kollmann, Enders, and Müller (2011) present a two-country model with a single global banking sector and show that in their framework a large loan default in
one country has an immediate negative impact on both economies, providing further theoretical support for the theory of contagion.

1.6 Conclusions

It is clear that the field of macroeconomics has come to recognize the important role financial intermediaries play not only in emerging markets but in the developed world as well. The literature is rife with papers that diminish the applicability of the Modigliani-Miller theorem and rigorously model the structure and behavior of modern financial entities. As empirical papers collect stylized facts about the workings of financial markets, theoretical research supplies new models to explain the mechanisms involved.

As the researchers’ ability to model these increasingly complex financial interactions improves, the literature on financial regulation must be updated as well. Up until the 2008 crisis, most economists called for removal of barriers to financial competition. This liberalization increased consumer prosperity, but at the same time raised the degree of systemic global risk. Stiglitz (2010) actually argues that due to the existence of bankruptcy risk in the financial sector, it may not be optimal for some economies to fully integrate into the global financial system.

As a result of the recent (and in some places still continuing) crisis and ongoing concerns about the stability of the world’s financial system, global regulators are reacting by advocating financial re-regulation. Now that financial intermediaries can be properly modeled in macroeconomic papers, researchers in the field should turn their attention to regulation recommendations on both national and international scale.
2.1 Introduction

Seminal Meese and Rogoff (1983) and Roll (1988) papers have pointed out that macroeconomic fundamentals do a very poor job of predicting future exchange rates and asset prices, and that a random walk outperforms the predictive power of existing models. Following a period of gloomy acceptance of the fact that our models cannot explain future asset prices, a new promising strand of literature emerged, which uses order flows to address this problem. Evans and Lyons (2002, 2005) and Evans’ (2009) series of microstructure papers develop the theory and quantitative analysis that demonstrate why order flow is a better predictor of exchange rates than are lagged fundamentals. Work in this field has succeeded in explaining many features of currency price behavior, perhaps most importantly why exchange rates are disconnected from macroeconomic fundamentals (Evans (2009)).

The above papers go a long way in connecting high frequency trading behavior with low-frequency movements of fundamentals. However, such an approach has not been applied to other markets, in part because data sets on representative order flow do not exist outside of the foreign exchange market. However, even in the presence

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1. Macroeconomic fundamentals are generally considered to be GDP, current account balance, short term interest rates, money supply, etc.
2. Order flow, as the name suggests, is the net flow of transaction orders that are filled in a given time period, signed to indicate whether an order is initiated as a sell or a buy request.
of data limitations, extending the work of Evans and Lyons into other markets seems a fertile ground for research because it can shed light on the mechanisms governing the behavior of asset prices in the short and medium run.

The contribution of this chapter lies in connecting a typical low-frequency DSGE framework with a standard microstructure model to explain how stock prices may become decoupled from the fundamental value of the stock.\(^3\) Using this setup, I can take a step towards explaining the timing of stock market crashes, which in my model depends on the speed with which asymmetric information disseminates into the market.

The model consists of two trader types (one local, one foreign), with the local agent being rational and welfare-maximizing with an informational advantage, and the foreign agent being an irrational noise trader. The traders have two assets available for exchange, a riskless bond and a risky asset, with the latter representing the state of the local economy. Both agent types trade indirectly through competitive market makers, who cannot tell whether any given transaction is initiated by a local or a foreign agent, and therefore are forced to infer the true value of the risky asset (imperfectly known by the local trader) through the history of observed trades.\(^4\) This causes market makers to sometimes misinterpret the trade history, which leads in turn to an information lag between informed (local) traders and the market. The fact that market makers do not know the identity of their clients in a common feature in sequential trade models which focus on insider trading. Mercorelli, Michayluk and Hall (2008) describe various strategies traders utilize to avoid being identified; there is an adverse selection problem since dealers may condition the ask and bid prices on the

\(^3\)Examples of the standard microstructure model used in this paper include, among others, Burnside, Eichenbaum, and Rebelo (2009), Das (2005) and Evans (2011).

\(^4\)The market maker setup is drawn directly from Glosten and Milgrom (1985).
identity of their clients. In fact, as Mercorelli, Michayluk and Hall point out, many markets are switching to electronic order takers that maintain trader anonymity.

Even though local agents trade based on superior information about the risky asset’s future dividend growth, this should not be interpreted as insider trading. Evans and Lyons (2005) note that most macroeconomic data start as disaggregated micro-level information about individual consumers’ preferences, companies’ output, etc., which means that locals necessarily have an informational advantage. Choe, Kho and Stulz (2004) demonstrate that local investors in Korea buy (sell) stocks prior to higher (lower) returns.5

The model successfully creates price movements that are tied to the fundamental movements in the economy. Thus, I am able to examine the timing of asset price movements, something that could not be achieved in standard DSGE models.

Beyond the successful integration of a low-frequency model with a microstructure model, the model generates two important results. First, since locals are assumed to be more knowledgeable about the risky asset, the opening up of asset markets to foreign investors naturally increases the chance that there will be a lag in private information entering the market (especially if these foreigner investors have non-informational reasons to buy). This lag can lead to a sudden price correction when market makers discover their error. Second, the persistence of the shock to the dividend process can affect the length of time it takes for the extra information available to local traders to influence stock market prices. According to the model,

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5Further support for this interpretation is given by Frankel and Schmukler (1996) who, in an investigation of the 1994 Mexican financial crisis, show using daily price data that information about the start of the crisis originated in local Mexican markets; they dub this "closeness to information". Rothenberg and Warnock (2011) find quarterly evidence that many sudden stops are actually sudden flights triggered by informed local investors sending capital to a foreign safe haven before an oncoming financial crisis.
a country with a history of low GDP growth volatility will learn about an upcoming recession later than a country with high growth volatility.

Future work in this field should focus on two specific areas. Primarily, the non-informational reasons for trade by the foreign trader should be endogenized to gain greater insight into the microstructure learning process. My first pass attempt to do this employing the incomplete market structure from Wang (1994) indicates that non-informational reasons for trade are not strong enough to allow markets to function. Additionally, this model is conceived in the simplest manifestation of a DSGE structure possible. Adding layers of complexity (such as extra risky assets, exchange rates, and production and capital accumulation) can shed light on other issues associated with the timing of a recession.

The rest of the chapter is organized as follows. Section 2.2 introduces the model and discusses quarterly and trading period problems of the agents. Section 2.3 discusses the relevant parameters and their chosen values. Section 2.4 presents the main results of the model and performs robustness checks. Finally, section 2.5 summarizes the main findings and lists several extensions for future research.

2.2 Model

I generalize the framework outlined in "Exchange-Rate Dynamics" by Martin Evans by incorporating a high-frequency sequential trade model into an endowment economy framework. This framework endogenously models low-frequency behavior of asset prices and fundamentals, while sequential trading incorporates the market maker and asymmetric information. Because both frameworks are employed together, it is

\[\text{Without non-informational reasons for trade, a market with asymmetric information meets the conditions of the Stokey-Milgrom "No Trade" Theorem. In essence, if non-informational reasons are not strong enough, MM will always set price spreads that are too high for either trader to act, and the risky-asset market will become illiquid.}\]
convenient to measure time in two separate metrics: low-frequency periods and high-frequency (trading) periods. Each low-frequency interval contains $T$ trading periods. Because there is a continuum of agents, each agent has zero probability of trading during the high-frequency periods, so the low-frequency problem can be solved first without concern for the high-frequency problem. Thereafter, the sequential trading problem is addressed inside the framework of the low-frequency model. In the rest of the chapter, low-frequency variables are denoted with the subscript $\tau$, while high-frequency variables are denoted with the subscript $t$.

For simplicity, all goods in the economy are assumed to be perishable and perfectly substitutable.

2.2.1 Agents

There exist three types of agents, all acting in an endowment economy setting: local consumers, foreign investors and market makers. Local consumers (LC) make up the entire emerging market economy and receive private signals about future dividend payments of the risky asset during the trading periods. Foreign investors (FI) trade for non-informational reasons, and market makers (MM) set prices for the local asset and a risk free bond to maximize profit.

Notationally, variables related to the LC are denoted with tildas, variables pertaining to the FI are denoted with hats and variables describing MM have no decoration.

Traders

Traders in the emerging market economy consist of a continuum of LC of measure $\theta$ and a continuum of FI of measure $1 - \theta$, each with log utility. During low-frequency periods, each LC attempts to maximize her discounted lifetime utility $\tilde{V}_\tau$ by choosing
her holdings of the bonds $\tilde{B}_\tau$ and the risky asset $\tilde{A}_\tau$, as well as her real consumption $\tilde{C}_\tau$. The bond pays one unit of consumption in period $\tau + 1$, while the local asset pays a stochastic dividend $D_{\tau+1}$. A representative LC’s maximized lifetime utility can be expressed as follows:

$$V_\tau = \max_{\tilde{A}_\tau, \tilde{B}_\tau, \tilde{C}_\tau} E \left[ \sum_{i=0}^{\infty} \beta^i \ln \left( \tilde{C}_{\tau+i} \right) | \tilde{\Omega}_\tau \right]$$

subject to the budget constraint

$$Q_\tau \tilde{B}_\tau + P_\tau \tilde{A}_\tau + \tilde{C}_\tau = \tilde{W}_\tau$$

where

$$\tilde{W}_\tau = \tilde{B}_{\tau-1} + (P_\tau + D_\tau) \tilde{A}_{\tau-1},$$

$Q_\tau$ and $P_\tau$ are the price of the bond and risky asset, respectively, and $\tilde{\Omega}_\tau$ and $\tilde{W}_\tau$ represent LC’s information set and real wealth.

This maximization results in the following Euler equations:

$$1 = E \left[ \tilde{M}_{\tau+1} \frac{1}{Q_\tau} | \tilde{\Omega}_\tau \right]$$  \hspace{1cm} (2.1a)

$$1 = E \left[ \tilde{M}_{\tau+1} \frac{P_{\tau+1} + D_{\tau+1}}{P_\tau} | \tilde{\Omega}_\tau \right],$$  \hspace{1cm} (2.1b)

where

$$\tilde{M}_{\tau+1} \equiv \beta \left( \frac{\tilde{C}_{\tau+1}}{\tilde{C}_\tau} \right)^{-1}.$$  

The term $\tilde{M}_{\tau+1}$ represents LC’s stochastic discount factor. FI’s low-frequency problem is analogous.

During trading periods, LC continues to maximize her future utility stream given a private signal about the risky asset. The FI instead reacts to random idiosyncratic liquidity shocks.
Market Makers

LC and FI are not allowed to trade directly with each other. Instead, market liquidity is provided by a countable number of perfectly competitive, risk-neutral market makers who set ask and bid prices for the bond and the risky asset and agree to hold excess risky assets (or meet excess demand for them) at those prices.

In the model, MM is of the Glosten-Milgrom type: he maximizes expected profits assuming that the fundamental value of the risky asset is given by LC’s discounted value of tomorrow’s dividend and price. MM resets prices only in response to informational changes instead of changes to his own asset position. This setup results in two convenient features: asset prices are martingale (Glosten and Milgrom, 1985), and the focus of the problem remains on the information dissemination process.

Additionally, it is assumed that all MMs have perfect knowledge of the actions of all other MMs, so that no rents can be gained from informational advantage.

Because he reacts to information transmitted by transactions (a buy, $B$, or a sell, $S$), MM can set "no regret" prices. Since, in general, some of his customers have an informational advantage, "no regret" prices already take into account the information that the next transaction will convey. Thus, an ask price will always be higher than a bid price, because the act of MM selling a unit of the local asset to an informed agent implies that the asset is more valuable than previously thought.

MM maximizes his expected profit:

$$E \left[ \Pi_r | \Omega_r \right] = \left( P_r^a - E \left[ \bar{M}_{r+1} \left( P_{r+1} + D_{r+1} \right) | \bar{B}_r, \Omega_r \right] \right) \Pr \left( \bar{B}_r | \Omega_r \right)$$
$$+ \left( E \left[ \bar{M}_{r+1} \left( P_{r+1} + D_{r+1} \right) | \bar{S}_r, \Omega_r \right] - P_r^b \right) \Pr \left( \bar{S}_r | \Omega_r \right)$$
$$+ \left( E \left[ \bar{M}_{r+1} | \bar{B}_r, \Omega_r \right] - Q_r^b \right) \Pr \left( \bar{B}_r | \Omega_r \right)$$
$$+ \left( Q_r^a - E \left[ \bar{M}_{r+1} | \bar{S}_r, \Omega_r \right] \right) \Pr \left( \bar{S}_r | \Omega_r \right)$$
where $\Omega_\tau$ is MM’s information set. Notationally throughout the chapter, $B_t$ signifies a purchase of the asset from MM ($S_t$ denotes a purchase of the asset by MM).\textsuperscript{7} MM maximizes his expected profit each period by choosing an ask and bid price for the local asset. He then accept all trades at these prices. The budget constraint indicates MM’s bond holdings:

$$Q_\tau B_\tau + P_\tau A_\tau = B_{\tau-1} + (P_\tau + D_\tau) A_{\tau-1}.$$

Since all MMs are competitive and act on the same information, they must all quote the same prices. Specifically, MMs must set

$$P^a_\tau = E \left[ \tilde{M}_{\tau+1} \left( P_{\tau+1} + D_{\tau+1} \right) \mid B_\tau, \Omega_\tau \right] $$

$$P^b_\tau = E \left[ \tilde{M}_{\tau+1} \left( P_{\tau+1} + D_{\tau+1} \right) \mid S_\tau, \Omega_\tau \right] $$

$$Q^b_\tau = E \left[ \tilde{M}_{\tau+1} \mid B_\tau, \Omega_\tau \right] $$

$$Q^a_\tau = E \left[ \tilde{M}_{\tau+1} \mid S_\tau, \Omega_\tau \right].$$

These are the MM’s first order conditions.

MM cannot identify the trader type of his client. This prevents him from perfectly inferring each agent’s private signal, and reflects the findings of Mercorelli, Michayluk, and Hall (2008) that traders go to some lengths to keep their identities hidden, especially if they are perceived to have valuable information.

Finally, if information is symmetric between all agents, MM does not need to learn from the transaction process and thus allows an unlimited number of trades for unlimited amounts of the local asset; however, if information is asymmetric, MM limits trades to a sequential process for a fixed amount of the asset in order to learn

\textsuperscript{7}More specifically, these purchases $B_t$ can be split into two categories: $\tilde{B}_t$ denotes a purchase by the local consumer and $\hat{B}_t$ denotes a purchase by the foreign investor. We use the notation $B_t$ to indicate a purchase when the market maker does not know who initiated it.
from traders. This behavior sets the stage for the trading period problem discussed below.

2.2.2 Low-Frequency Equilibrium

The total number of outstanding local and foreign shares is normalized to unity:

\[ \theta \hat{A}_r + (1 - \theta) \hat{A}_r = 1. \tag{2.3} \]

In equilibrium, bonds must be in zero net supply:

\[ \theta \hat{B}_r + (1 - \theta) \hat{B}_r = 0. \tag{2.4} \]

Because goods are perishable, total consumption must equal total dividend payments:

\[ \theta \hat{C}_r + (1 - \theta) \hat{C}_r = D_r. \tag{2.5} \]

Notice that, since there is a countable number of MM, their actions do not impact the market clearing conditions.

Information regarding future dividends is transmitted through current dividend payments and is therefore common and symmetric in the low-frequency problem. It follows that the information set of each agent is the same

\[ \Omega_r = \hat{\Omega}_r = \hat{\Omega}_r = \{D_r, \Delta D_r\}. \]

Transactions taking place at low frequency convey no extra information (since there are no private signals); thus, each MM will accept all trades for all amounts. Therefore, MM’s first order conditions (2.2) are completely described by the traders’ Euler equations (2.1).

The growth process for the risky asset’s dividend is modeled as a simple two-state symmetric Markov chain. The dividend’s growth can be low or high and is denoted
Generally, the conditional probability of a particular future realization of the dividend growth is written as \( \Lambda_{ij} = \Pr (\Delta D_{t+1} = \Delta D_i | \Delta D = \Delta D_j) \), with \( \Pr (\Delta D_{t+1} = \Delta D_i | \Delta D = \Delta D_i) = \lambda \) for \( i, j \in \{1, 2\} \), where 1 indicates low dividend growth and 2 high growth. This process can be represented in matrix notation as follows:

\[
\Delta D = \begin{bmatrix} l \\ h \end{bmatrix},
\]

\[
\Lambda = \begin{bmatrix} \lambda & 1 - \lambda \\ 1 - \lambda & \lambda \end{bmatrix}.
\]

The current dividend level is then written as a history of growth rate realizations

\[
D_t = \Delta D_{t} \times \Delta D_{t-1} \times \ldots \times \Delta D_1 \times D_0
\]

where \( D_0 \) is the starting dividend level. Because the model contains only one shock and one risky asset, markets are complete. Therefore, the wealth ratio of the two traders, LC and FI, remains constant:

\[
\omega_0 = \frac{\hat{W}_t}{\hat{W}_\tau} \quad \forall \tau.
\]

Thus, the entire state \( X_t \) of the economy is described by the current dividend level and the most recent dividend growth rate:

\[
X_t = \{D_t, i\}
\]

where \( i = 1 \) if \( \Delta D_t = l \) and \( i = 2 \) if \( \Delta D_t = h \).

An equilibrium for the low-frequency problem is defined as the set of decision rules

\[
\left\{ \tilde{A}(X_t), \tilde{A}(X_t), \tilde{B}(X_t), \tilde{B}(X_t), \tilde{C}(X_t), \tilde{C}(X_t) \right\}
\]

and equilibrium prices

\[
\{Q(X_t), P(X_t)\}
\]
that satisfy for all states \( \{X_\tau\} \) (i) all agents' Euler equations (2.1) and (ii) asset market clearing conditions (2.3)-(2.5).

The equilibrium can be solved analytically (See Appendix A.1). Agents’ value functions, can be described using their wealth and the last dividend growth rate:

\[
\tilde{V}(\tilde{W}_\tau, i) = \frac{\ln[(1 - \beta)\tilde{W}_\tau] + \beta \Lambda_i (I - \beta \Lambda)^{-1} \ln(\Delta D)}{1 - \beta} \\
\hat{V}(\hat{W}_\tau, i) = \frac{\ln[(1 - \beta)\hat{W}_\tau] + \beta \Lambda_i (I - \beta \Lambda)^{-1} \ln(\Delta D)}{1 - \beta}.
\]

### 2.2.3 High-Frequency Equilibrium

The purpose of the low-frequency problem is to set the stage for the trading model. It is in this sequential trading model that information about asset values may become trapped and not enter the market. Due to the setup of the low-frequency problem, information is necessarily revealed at the beginning of each new low-frequency period. This differs from Evans and Lyons (2005) and Tille and van Wincoop (2010), who add an extra layer of uncertainty to the dividend process that inhibits the fundamental from ever being revealed. My model restricts the analysis of price and information flows to the trading periods, a simplifying assumption that allows for the low-frequency equilibrium to be found independently of the high-frequency equilibrium. Thus, the low-frequency periods exist purely as a framework to endogenize asset prices and enrich the traders daily problem.

The local asset is divided into infinitesimally small blocks, and in equilibrium only one block can be traded each period. The block size is \(1/Z\) for an individual trader. In other words, if all LC were to buy one block of the local asset, they would increase the aggregate holdings by \(1/Z\). This assumption creates a liquidity constraint in the model and is imposed to restrict the amount of information transmitted via

---

*The notation \( \Lambda_i \) indicates a \((1 \times 2)\) matrix from the \(i^{th}\) row of \( \Lambda \).*
each trade. The size of the trading blocks $Z$ does not matter as much, however, as
the restriction that LC and FI cannot choose the amount they would like to trade.
Otherwise, MM would be able to determine the trading partner based on the size of
the trade, and the information transmission would become trivial. It has been shown
that traders often make many small sales rather than one large one in order to mask
their actions (Bouchaud, Gefen, Potters, and Wyart, 2004).

Since only one trade is made per period, the aggregate asset holdings of each agent
type do not change during trading periods. This is a strong assumption that causes
aggregate asset positions to change only during low-frequency periods.

At the beginning of every period, all LCs receive a common private signal about the
risky asset’s next dividend payout, and all FIs receive an independent idiosyncratic
shock which determines their liquidity requirements for the period. Concurrently,
MM sets prices based on his beliefs about LC signal history up through the current
period. Once the prices are set, only one agent, chosen randomly, is allowed to trade
with one MM for a unit of the local asset; LC trades each period with exogenous
probability $\theta$, and FI trade with probability $1 - \theta$. Let $\Theta_t \in \{0,1\}$ represent this
choice, with 0 indicating a trade by a FI and 1 indicating trade by a LC. Once
the trading agent is chosen, the trade ($T_t$) may be a buy ($B_t$), sell ($S_t$) or pass ($P_t$).
Numerically these transactions are written in units of the asset as:

\[
B_t = \frac{1}{Z}, \quad S_t = -\frac{1}{Z}, \quad P_t = 0.
\]

After the trade occurs, MM updates his beliefs about LC and FI’s signal histories to
include the current period.
Local Consumer’s High-Frequency Problem

The imperfect signal about the future dividend growth of the risky asset arrives at the beginning of each trading period. The signal, $S_t = \{l, h\}$, follows a two-state discrete process with a publicly known probability $\nu$ of correctly predicting next quarter’s dividend. This can be expressed as

$$\nu \equiv \text{Pr}(\Delta D_{t+1} = \Delta D_i | S_t = S_i) \quad \text{for} \quad i \in \{1, 2\}$$

where $\nu$ is the strength of the signal. The probability of the signal being correct can be expressed as $\Psi_{ij} = \text{Pr}(\Delta D_{t+1} = \Delta D_i | S_t = S_j)$ or in matrix notation as

$$\Psi = \begin{bmatrix} \nu & 1 - \nu \\ 1 - \nu & \nu \end{bmatrix}.$$  

Aggregation of the signal history from trading period 1 to the current period $t$ is done by setting $S^t \equiv j$, where $j$ is the number of high signals LC has received. Thus, LC’s information set $\bar{\Omega}_t$ contains last period’s dividend growth rate and the signal history and can easily be used to find the probability of next period’s dividend growth realization. For example, to find the probability that next period’s growth is high, combine the Markov probability matrix with the probability that all of the high signals where correct and the probability that all of the low signals were incorrect:

$$\text{Pr} \left( \Delta D_{t+1} | \bar{\Omega}_t \right) = \text{Pr} \left( \Delta D_{t+1} = \Delta D_k | S^t = j, \Delta D_r = \Delta D_i \right)$$  

$$= \frac{\Lambda_{ki} (\Psi_{k1})^{t-j} (\Psi_{k2})^j}{\sum_k \Lambda_{ki} (\Psi_{k1})^{t-j} (\Psi_{k2})^j}.$$  

Using her information set updated with the signal, LC would like to maximize her lifetime utility, described in the low-frequency equilibrium, by rebalancing her portfolio. Since there are no dividend or bond payouts during a trading period, there cannot be any consumption choice either. This assumption is motivated by the
observation that most households allocate resources to consumption on a pay-period frequency (or less), while their portfolios can be rebalanced much more frequently in response to daily news. Additionally, many consumption purchases (mortgages, cars payments, etc.) are lumpy.

LC’s trading problem can be summarized as

\[ \tilde{T}_{i,t} = \arg \max E \left[ \tilde{V} \left( \tilde{W}_{i,\tau+1}, j \right) \mid \Omega_{t} \right] \tag{2.8} \]

where \( \tilde{V} \left( \tilde{W}_{i,\tau+1}, j \right) \) is the maximized lifetime utility given in (2.6) and \( \tilde{W}_{i,\tau+1} \) is the wealth of trader \( i \) given her action \( \tilde{T}_{i,t} \). In other words, LC chooses asset holdings while taking her next quarter maximizing decision rules as given.

Since each LC is infinitesimal, the probability of one agent being randomly chosen to trade twice is zero. This simplifies the model in two ways. First, agents make their trading period decision knowing that their next chance to trade is in period \( \tau + 1 \). Second, any agent chosen to trade in period \( t \) has wealth \( \tilde{W}_{\tau} \). The solution to LC’s trading period problem (2.8) is the trading period decision rule \( \tilde{T}_{t} \in \{ \tilde{B}_{t}, \tilde{S}_{t}, \tilde{P}_{t} \} \):

\[ \tilde{T} \left( S', P_{t} \right) = \arg \max \sum_{\Delta D_{\tau+1}} \Pr \left( \Delta D_{\tau+1} \mid \Omega_{t} \right) \tilde{V} \left( \tilde{W}_{i,\tau+1}, j \right) \tag{2.9} \]

\[ \text{s.t.} \quad \tilde{W}_{i,\tau+1} = \tilde{W}_{\tau+1} + (P_{\tau+1} + D_{\tau+1} - P_{t}) \tilde{T}_{t} \]

\[ P_{t} = \begin{cases} P_{t}^{a} & \text{if } \tilde{T}_{t} = \tilde{B}_{t} \\ P_{t}^{b} & \text{if } \tilde{T}_{t} = \tilde{S}_{t} \end{cases} \]

where \( \Pr \left( \Delta D_{\tau+1} \mid \Omega_{t} \right) \) is given by (2.7).

**Foreign Investor’s High-Frequency Problem**

During the trading periods, FI does not get a private signal, but is instead hit with an idiosyncratic liquidity shock. This shock will cause FI to either buy or sell a unit
of the risky asset with equal probability. Thus,

\[
\Pr\left(\hat{T}_t = j\right) = \frac{1}{2} \quad \text{where} \quad j = \{\hat{B}_t, \hat{S}_t\}. \tag{2.10}
\]

Theoretically, FI must have a non-informational reason for trading. As shown in Stokey and Milgrom’s No Trade Theorem, MM would never be willing to engage in asset trade with only the (more informed) LC. Therefore, we must add FI (who is just as uninformed about the state of the local asset as MM) and impose the restriction that MM does not know whether he trades with LC or FI. This way, MM can expect to regain his losses from trades with LC by also trading with the uninformed FI.

**Market Makers’ High-Frequency Problem**

Because of asymmetric information, MM reduces the liquidity available in the market. Prices are set for a specific trade amount, \(1/Z\), and only one trade is allowed to occur at the set price before prices are adjusted again. Additionally, because consumption decisions are not made during the trading periods, the price of the assets no longer have to be discounted. Thus, equations (2.2) must be rewritten with \(\hat{M}_{t+1} = 1\). In this case, \(Q^a_t = Q^b_t = 1\) for all \(t\) and

\[
P^a_t = E[P_{t+1} + D_{t+1}|\hat{B}_t, \Omega_t] \tag{2.11a}
\]

\[
P^b_t = E[P_{t+1} + D_{t+1}|\hat{S}_t, \Omega_t]. \tag{2.11b}
\]

To set these prices, MM employs Bayesian learning to determine the probability of each signal history occurring given the trade history he has witnessed:

\[
\Pr\left(S^t = j|\hat{T}_t, \Omega_t\right) \quad \text{for} \quad j \in \{0, t\}. \tag{2.12}
\]

These probabilities are found by updating prior beliefs that MM has at the beginning of the period. The probabilities (2.12) are combined with LC’s beliefs given the signal
history\(^9\) (2.7) to solve for MM’s beliefs about the future dividend growth rate:

\[ \text{Pr} (\Delta D_{t+1} = \Delta D_k | T_t, \Omega_t) = \sum_{j=0}^{t} \text{Pr} (\Delta D_{t+1} = \Delta D_k | S^t = j) \text{Pr} (S^t = j | T_t, \Omega_t). \]

To employ Bayesian learning, MM must find cutoff points using LC’s decision rule. Let \(g_o (h_o)\) denote the highest possible signal history \(S^t\) that LC could have and still want to sell a unit of the local asset (the lowest possible signal history LC could have and still want to buy a unit of the local asset). These bounds give MM a simple way to calculate the probability of each possible signal history given all possible trader actions. For a given set of bounds, MM calculates \(P^a (h_o)\) and \(P^b (g_o)\) (see Appendix A.3 for details).

MM first iterates over an initial guess for \(g^0\) to find the correct value \(g_o\). Since \(g_o\) is an upper bound, the first guess is \(g^0 = t\). If LC’s trade is consistent with the current guess \(g^j\), i.e. \(\tilde{T} (g^j, P^b (g^j)) = \tilde{S}_t\) and \(\tilde{T} (g^j + 1, P^b (g^j)) \neq \tilde{S}_t\), then \(g^j = g_o\), and the iteration process is complete. Otherwise, update \(g^{j+1} = g^j - 1\) and continue iterating. The bound \(h_o\) is found using an initial guess \(h^0 = 0\), which is incrementally increased until \(\tilde{T} (h^j, P^a (h^j)) = \tilde{B}_t\) and \(\tilde{T} (h^j - 1, P^a (h^j)) \neq \tilde{B}_t\).

MM comes into each period with prior beliefs about the signal history and the future dividend growth of the risky asset, which he uses to set optimal prices:

\[ \pi_{s_{j,t}} \equiv \text{Pr} (S^{t-1} = j | \Omega_t) \]
\[ \pi_{d_{k,t}} \equiv \text{Pr} (\Delta D_{t+1} = \Delta D_k | \Omega_t). \]

Once the trade is completed, MM must update these beliefs in order to be able to set prices next period. This is done by finding MM’s beliefs given a trade with a certain agent (either LC or FI) and weighting these beliefs by the conditional probability

\(^9\)LC’s beliefs may be used in this case, since the signal history \(S^t\) contains strictly more information than the trade history.
that he traded with each of these agents:

\[
\pi_{j,t+1}^s = \Pr(S^t = j|\hat{T}_t, \Omega_t) \Pr(\hat{T}_t|T_t, \Omega_t) \\
+ \Pr(S^t = j|\hat{T}_t, \Omega_t) \Pr(\hat{T}_t|T_t, \Omega_t) \\
\pi_{k,t+1}^d = \Pr(\Delta D_{t+1} = \Delta D_k|\hat{T}_t, \Omega_t) \Pr(\hat{T}_t|T_t, \Omega_t) \\
+ \Pr(\Delta D_{t+1} = \Delta D_k|\hat{T}_t, \Omega_t) \Pr(\hat{T}_t|T_t, \Omega_t).
\]

(2.13a)

(2.13b)

**High-Frequency Equilibrium**

The definition of the high-frequency equilibrium is (i) a set of prices \( \{P_t^a, P_t^b\} \) that satisfy (2.11), (ii) trading decisions \( \{\hat{T}_t, \hat{T}_t, \hat{T}_t\} \) that satisfy (2.9), (2.10) and

\[ T_t = \Theta_t \hat{T}_t + (1 - \Theta_t) \hat{T}_t, \]

and (iii) beliefs that are updated by (2.13) using \( \{P_t^a, P_t^b, \hat{T}_t\} \).

**2.3 Parameterization**

Parameter values used in the benchmark simulations are listed in Table 2.1. Each low-frequency period is interpreted as one quarter. The values of \( l \) and \( h \) result in the average growth rate of one percent per quarter.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.99</td>
</tr>
<tr>
<td>( T )</td>
<td>100</td>
</tr>
<tr>
<td>( Z )</td>
<td>100</td>
</tr>
<tr>
<td>( l )</td>
<td>0.99</td>
</tr>
<tr>
<td>( h )</td>
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</tr>
<tr>
<td>( \lambda )</td>
<td>0.9</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.3</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 2.1: Parameters values used in benchmark simulations.
If LC does not discover the true value of the fundamental by the end of the quarter, the model would generate a large price correction after the last trading period. Since dividend announcements can be interpreted as news in this model, and since news does not move stock prices (Cutler, Poterba and Summers, 1989, and Romer, 1993), I set parameters $T$ and $v$ so that, on average, prices in the final trading period reflect the value of the asset at the start of the following quarter. Higher values of $T$ allow the signal strength to be lower; however, setting $T$ above 100 does not change the qualitative results while making it more difficult to identify the information lag.

For clarity of exposition, I initially let LC represent 30% of the market; however, in Section 2.4.2 I test the full range of this parameter, $\theta \in [0, 1]$.

Without loss of generality, I set the starting dividend level $D_0$ and wealth ratio $\omega_0$ both to one.

2.4 Results

2.4.1 Benchmark Simulation

The purpose of this chapter is to show how markets learn about endogenous macroeconomic shifts of fundamentals. To this end, Figure 2.1 presents an example of the model simulated over one quarter as the dividend growth rate falls from $h$ in the first quarter to $l$ in the second quarter. The fundamental value of the asset (dashed line) is found using LC’s expectations given the signal history $S^t$. Notice that the fundamental value of the asset drops between trading periods $t \in [10, 20]$, while the asset price does not make a sustained drop until approximately period $t \approx 45$.

The key result of the model is that such drops occur without an exogenous shock having to immediately precede them. Rather, the timing and the severity of the price drop are determined endogenously by the interaction of MM with his two trading
partners. In many cases, the market may behave efficiently, i.e., prices will move with the fundamental; however, it is possible for prices to become detached from their fundamental value, which may subsequently lead to an abrupt and sizable price adjustment.

To understand the mechanism driving these results, we need to examine the actions of each of the players in the market. As LC learns that the dividend growth rate will fall next quarter, she starts to exchange the risky asset for the risk-free bond. Initially, FI randomly picks up a large portion of the excess supply of the risky asset, so MM is unable to learn much from the mixed transaction history of buys and sells. Only when FI also begins (randomly) selling his holdings of the local asset on a consistent basis can MM infer that the local asset is likely to have a low payout. Over long enough periods of time, random noise from the actions of FI cancels out and the true signal emerges. Thus, we should expect to see sharp price drops preceded by capital flow from LC to FI (capital flight). Froot, O’Connell, and Seasholes’s (2001) study of portfolio flows indicate that large asset price drops are not accompanied by foreign portfolio flow reversals, but instead by a modest lessening of flows into emerging markets. My results appear to corroborate this finding.

Because the results in the benchmark simulation are driven by the random behavior of the FI, the same simulation is run 100 times for robustness. The average price and fundamental value are shown in the top panel of Figure 2.2. The asset price lags the risky asset’s fundamental value as information is not immediately released into the market. The bottom panel of Figure 2.2 plots the average difference between the price and fundamental (which I call the information gap). The gap is the largest

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10These results clearly hinge on the random actions of the foreign investor; a series of sells would result in information entering the market much faster. In future research, I would like to endogenize the actions of FI.
around $t = 30$. Eventually though, information is released into the market and prices move closer to their fundamental value.

2.4.2 Robustness

Since many of the parameters used in the model are unobservable and therefore cannot be calibrated, in this section I test the robustness of the results for different values of $\{\theta, \lambda, l, h, v, z\}$.

As the percentage $\theta$ of LC’s in the market increases, information is released into the market more quickly. The information gap does not get as large with a high $\theta$ and dissipates rapidly. When $\theta = 1$, information is common between the traders and
Figure 2.2: The top panel shows the average price and fundamental value over 100 simulations, given the benchmark parameters. The information gap is graphed in the bottom panel.

MM will not be able to operate; when $\theta = 0$, there is no private information and MM has no need to adjust the price during trading periods.

The persistence $\lambda$ of the risky asset’s dividend growth rate affects the importance of the signal, as shown in Figure 2.3. As the growth rate becomes more persistent (right panels), it takes a larger number of low shocks to convince LC that the asset’s growth rate will fall. Thus the information gap peaks later in the trading periods.
(t ∈ [50, 80]) for a more persistent shock and earlier (t ∈ [15, 45]) for a less persistent one (left panels). Because markets in countries with a higher persistence of shocks learn more slowly, they are more likely not to learn the true value of an asset before news is release (i.e., the start of the following quarter when dividend earnings are announced). As mentioned in the Section 2.3, this feature of a market leaves it more vulnerable to a sudden drop in prices when unexpectedly low dividends are realized.

As can be seen in the lower panels of Figure 2.3, one standard deviation above the average information gap in the last trading period is approximately twice as large when λ = 0.99 as opposed to when λ = 0.8. In cases with such a large information gap so late in the quarter, a news shock will cause a significant price drop.

The growth rates \{l, h\} of the dividend have little effect on the nature of learning process during trading periods, except to change the price and fundamental value levels.

The strength of LC’s private signal \(v\) determines how fast both LC and the market learn about the future dividend growth rate. Additionally, as the signal becomes stronger, the gap between LC’s information and the market’s information widens more quickly, but drops just as quickly.

The size of the asset blocks \(Z\) have no noticable impact on the trading period results.

2.5 Conclusions

When looking back on the turbulent experience of the global financial markets over the last several decades, economists usually get the sense that, in the countries that have experienced currency crises, shifts in the fundamental health of the economy typically occur earlier than the crisis. What governs the duration of time between
Figure 2.3: The top left panel shows the price and fundamental value for $\lambda = 0.8$, while the bottom left panel shows the corresponding information gap. The right panels plot the same variables for $\lambda = 0.99$.

such a shift and its effects on the asset markets? Moreover, not all economies suffer crises following a shift in their fundamentals. What features of the economy and its asset markets determine whether a readjustment will occur gradually or instead cause a crisis?

In order to answer these questions, I must consider the day-to-day workings of the financial markets in which asset prices are determined. In this chapter, I develop
a high-frequency model of the exact timing of price drops. Since local and foreign investors do not trade directly with each other, I examine the workings of the asset market by explicitly modeling a market maker, an agent who facilitates trade. In addition, I introduce information asymmetry between local and foreign traders, and study the flow of this information from the fundamentals through the traders to the asset prices. This model setup can generate asset price histories with both gradual and sudden price movements.

One of the main contributions of this chapter lies in endogenizing the timing of asset price drops; moreover, I demonstrate that these drops depend critically on the interaction between the number of informed traders in the market and the quality of their private signal about the state of the economy. Healthy economies with transparent information systems and many informed traders have a far better chance of experiencing a smooth adjustment to a shift in its fundamentals; the ones with opaque markets and few informed traders run a much higher risk of asset price crashes. Since locals are typically more informed than foreigners, the model predicts that the opening up of asset markets in emerging nations increases the volatility of price movements.

The model predicts that the time it takes for the market to absorb new information is lengthened in cases where fundamentals are more persistent. Because of this, the information gap is larger and last longer in these markets. The occasional release of news into such markets has a more pronounced effect on the prices.

My results are driven in part by the assumption that foreign investors behave as noise traders. I would like to build an extension of this model in which the foreign investor instead behaves as a rational agent.
CHAPTER 3

BANK STRUCTURE AND INTERNATIONAL CAPITAL FLOWS

3.1 Introduction

The recent financial crisis has highlighted the importance of international financial linkages for macroeconomic health of individual countries and brought issues such as global portfolio imbalances, bank bailouts, and systemic risk to the forefront of economic research. However, neither the mechanisms that link global financial system to national economic performance, nor policy proscriptions to better stabilize the financial markets can be meaningfully analyzed in models without an international banking sector. In this chapter, I introduce an optimizing global banking system into a two-country DSGE model with incomplete financial markets, multiple assets and endogenous portfolio choice and show that it can explain some of the recent developments in global finance.

The trends in international portfolio allocations resulting from bank deregulation can be succinctly summarized by the following four stylized facts (also illustrated in Figure 3.1).¹

¹The following is based on portfolio data from Lane and Milessi-Feretti (2007), along with the Heritage Foundation’s Financial Freedom Index. Lane and Milessi-Feretti’s (2007) dataset includes gross holdings of international debt and equity assets and liabilities for 179 countries between 1970 to 2007. The Heritage Foundation’s Financial Freedom Index measures the level of overall regulation of the banking sector on a scale from 0 to 100, with 100 being the most open, for 183 countries from 1995 to 2011. Combining the two sources, I am able to analyze data from 155 countries between 1995 and 2007 (1863 data points).
1. There exists a positive 26% correlation between a country’s gross holdings of international assets and liabilities and its level of financial deregulation.\(^2\) In other words, the more free is a country’s banking system, the more likely that country is to hold large international portfolio positions.

2. There is a positive 5% correlation between net equity holdings and deregulation, suggesting that deregulated countries tend to hold a larger portion of their portfolios in equity.\(^3\)

3. Conversely, the correlation between net debt holdings and deregulation is negative 12%. Thus, countries with more heavily regulated banking sectors tend to hold a larger portion of their portfolios in debt.

4. Finally, there is a negative 13% correlation between a country’s net foreign assets (NFA) and deregulation. This result shows that countries which have liberalized their banking system tend to be net debtors in international financial markets.

Most of the existing literature on global imbalances focuses on the U.S. experience, illustrated in Figure 3.2 and quite in line with the above four observations. The U.S. began deregulating its banking sector in the 1980s and has been steadily accumulating both equity and debt ever since. While the U.S. NFA position has worsened considerably since 1984, the equity and debt subcomponents of U.S. assets and liabilities have behaved quite differently. As the U.S. has accumulated negative net debt position, it has simultaneously been building up a positive net foreign position in equity. Explanation for these events generally centers on the United States’ "exorbitant privilege" of borrowing at a discount because the dollar is the global reserve currency, as explained in Gourinchas and Rey’s (2007) influential work.

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\(^2\)All reported statistics are significant at the 1% level.  
\(^3\)Here I measure equity as a combination of portfolio equity and FDI.
Figure 3.1: Scatter plots indicate the relationships of net equity, net debt, and gross and net foreign assets with the Heritage Fund’s Financial Freedom Index over all countries and years in the sample. The horizontal axis shows billions of dollars.

This mechanism, however, cannot explain the experience of the other countries in my sample, which do not have the luxury of printing the reserve currency. Notice in Figure 3.2 that the U.K.’s portfolio shifts have been very similar to the U.S. experience. As a counter-point to the experiences of countries with liberalized financial systems, I demonstrate in Figure 3.3 that China had the opposite experience. In this chapter, I propose a new explanation of the above trends in capital flows, which can be applied to any country and centers on the links between financial sector liberal-
ization and portfolio allocation decisions of households. To do so, I must explicitly model the international financial sector, which, to the best of my knowledge, has not been done in the literature.

Starting with Bernanke, Gertler and Gilchrist (1999), the banking sector in two-country models is only introduced to create a wedge between borrowing and lending interest rates and acts purely as a traditional bank which holds deposits and issues loans. In contrast, the financial intermediary described in this chapter is more closely
Figure 3.3: China’s net foreign asset position, broken down into categories of equity and debt, assets and liabilities, and total net foreign wealth, over the past four decades.

tied to the Kareken and Wallace (1978) and Diamond and Dybvig (1983) papers and the subsequent literature where the banks aggregate risk and provide liquidity. However, none of the papers in this field study the implications of this banking structure on international investment decisions.

A growing body of research finds that a substantial portion of international transactions is being handled by non-traditional financial institutions (money market funds, hedge funds and investment banks), and that there has been a shift in the scope
of operations of traditional banks, so that many of them now handle local operations across many countries. Baklanova (2010) reports that global money market funds in the first quarter of 2009 controlled an estimated $5.8 trillion in assets. Jotikasthira, Lundblad and Ramadora (forthcoming) explore the ties between the behavior of the investors in mutual and hedge funds and the behavior of the funds themselves, finding that investor decisions to withdraw capital from a fund can lead to fire sales of assets from countries in which the funds are invested. The growing influence of mutual and hedge funds in the international arena, the authors claim, is an important component in explaining contagion of financial crises among apparently unconnected countries. Meanwhile, Stein (2010) describes the efforts of the World Bank to encourage foreign banks to open branches in the developing world and expose it to the global banking system. This phenomenon is further supported by McCauley, McGuire and von Peter (2010), who explore the shift of global banking from international banks which specialize in cross-border transactions to multinational banks which conduct local transactions in many countries. These observations, however, are not reflected in the existing structural models of the banking sector.

These changes in the international financial environment call for a new generation of economic models that explicitly feature the non-traditional banking sector and can differentiate between debt and equity portfolio holdings of agents. While endogenous portfolio choice models have been gaining prominence in the literature, no work has been done on endogenizing the financial sector in an international setting. This chapter fills this gap by describing a two-country multiple asset model in which a global financial intermediary is capable of trading in multiple assets and issuing its own stock.

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4A non-exhaustive list includes Devereux and Sutherland (2011), Evans and Hnatkovska (2012), Pavlova and Rigobon (2010), and Tille and van Wincoop (2010).
I assume that all international asset transactions of households must go through the financial intermediary ("bank"). Households must purchase shares of the bank if they wish to diversify their portfolio holdings to include foreign equity. Because the bank does not consume its income (which it instead pays out as dividends), its stochastic discount factor is a weighted average of the stochastic discount factors of its shareholders.

The model is simulated for two specifications: a baseline setup with no financial frictions, and an asymmetric parameterization in which the foreign households must pay a fee to access the bank’s equity. This fee represents the regulatory burden placed on a country through government interference in financial markets; removal of the fee, therefore, represents financial deregulation. The results of the simulations indicate that the presence of the banking fee can explain the first two of the above stylized facts: that deregulation leads to increased international portfolio imbalances, and that less regulated countries hold positive net equities. The mechanism is quite simple, since the more costly access to the bank causes the foreign household to place her entire portfolio in her own equity and bonds. This means the home household is left holding almost all of his own equity, while maintaining his foreign equity positions through the financial intermediary.

The rest of the chapter is organized as follows. Section 3.2 introduces the economic environment and describes the setup of the financial intermediary. Section 3.3 briefly outlines the solution method. Calibration and the theoretical mechanisms of the model are discussed in section 3.4. I analyze the impact of financial frictions on the patterns of international capital flows in section 3.5. Finally, section 3.6 concludes and lists several extensions for future research.
3.2 Model

The main goal of this chapter is to analyze the impact of financial liberalization on global capital flows. Therefore, the model must include at least one debt and two equity assets, incomplete financial markets, and a banking sector that trades in debt and equity internationally.

The basic framework is closely related to Stepanchuk and Tsyrennikov (2011); I augment their model by incorporating an optimizing financial sector. There are three masses of agents (each of measure one): the home (H) and foreign (F) households and the financial intermediary (E). The two economies have the same symmetric structure in the baseline setup; therefore, most equations will be presented only for the home household. As a matter of notation, lower case letter denote individual variables, while capital letters stand for aggregate quantities.

3.2.1 Output Allocation

To reduce the number of state variables, I assume an endowment economy. Production in each country is determined exogenously by a two-state Markov process. Home output, $Y_t^h$, can be either low, $Y_t^{hl}$, or high, $Y_t^{hh}$, and evolves according to the following transition matrix:

$$
P(Y_t^h = Y_{t-1}^{hl} | Y_t = Y_t^{hl}) = P(Y_t^h = Y_{t-1}^{hh} | Y_t = Y_t^{hh}) = \lambda_h.
$$

Foreign output follows the same process with parameters $\rho_{Y_t^f}$. Additionally, the parameter $\rho$ controls the correlation between home and foreign outputs.\(^5\)

Home output is divided into the household’s wages $W^h_t$ and dividends $D^h_t$:

$$W^h_t = (1 - \mu_h) \frac{Y_t^h + Y_t^f}{2}$$

\(^5\)See the technical appendix for the construction of the combined Markov matrix for $\{Y_t^h, Y_t^f\}$.\(\)
and

\[ D_t^h = Y_t^h - W^h. \]

For modeling simplicity, I assume that not all of the country’s production is capitalized. Additionally, wages are non-stochastic; this assumption is supported by the observation that wage income tends to be less volatile than dividend income.

Rights to the future dividend realizations can be held by any of the three agents, who can purchase shares of home production. These shares are denoted \( a_t^{hh}, a_t^{hf} \) and \( a_t^{he} \) when they are held by the home household, foreign household, and the bank, respectively.

3.2.2 Home household

The population of the home economy is represented by the home household, who maximizes

\[ V_t^h \equiv \max_{c_t^h, b_t^h, a_t^{hh}, a_t^{hf}} \sum_{i=0}^{\infty} \beta^i E \left[ \frac{(c^h_{t+i})^{1-\gamma}}{1-\gamma} \right]. \]

The home household’s demand for home and foreign goods is given by a standard CES aggregator:

\[ c_t^h = \left( \phi_t^\eta \left( c_t^{hh} \right)^{\frac{n-1}{\eta}} + (1 - \phi_t) \frac{1}{\eta} \left( c_t^{hf} \right)^{\frac{n-1}{\eta}} \right)^{\frac{\eta}{n-1}}, \]  

where \( \eta \) measures the elasticity of substitution between home and foreign goods, and \( \phi_t > 0.5 \) indicates the degree of consumption home bias. To ensure that financial markets are incomplete, I add a symmetric two-state Markov shock to the demand for home and foreign goods, \( \phi_t \).

The price (which can be thought of as the Consumer Price Index, or CPI) of the consumption aggregate \( c_t^h \) is given by

\[ P_t^h = \left[ \phi_t \left( P_t^{hh} \right)^{1-\eta} + (1 - \phi_t) \left( P_t^{hf} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}, \]
where $P_{th}^h$ and $P_{tf}^f$ are the home and foreign Producer Price Indices (PPI), respectively. The demands for home and foreign goods are $c_{th}^h = \phi_t \left( \frac{P_{th}^h}{P_t} \right)^{-\eta} c_t^h$ and $c_{tf}^f = (1 - \phi_t) \left( \frac{P_{tf}^f}{P_t} \right)^{-\eta} c_t^f$, respectively. Due to arbitrage and the lack of transportation costs, the Law of One Price holds. Although the model represents a real economy, in order to maintain symmetry, a numéraire is not chosen. Instead, equations are written in terms of a common currency, i.e., the nominal exchange rate is assumed to be one. As a result, because a numéraire price is not eliminated, goods prices are normalized as follows:

$$P_{th}^h + P_{tf}^f = 2.$$ 

The household faces the following intertemporal budget constraint

$$P_t c_t^h + Q_t b_t^h + Q_t a_t^h + a_t^h = P_t W_t^h + b_{t-1}^h + (Q_t^h + P_t D_t^h) a_{t-1}^h + T_t^h.$$ 

(3.2)

I assume that the home household must rely on the bank for all asset transactions. This feature of my model is based on the observation that individual consumers no longer purchase assets directly from corporations, but rather utilize financial intermediaries (be it local banks, large hedge funds or online trading services). To this end, the household can choose to buy a riskless global bond $b_t^h$, claims on the home dividend stream $a_t^h$, and bank investment $a_t^e$, which is an indirect way of acquiring claims on future foreign production.\(^6\) Variables $Q_t^h$ and $Q_t^h$ denote the price of bonds and home equity, respectively, and $R_t^e$ stands for the return on bank investment.

A small fraction of the consumers’ investment in the bank is assumed to be taken by the government as the cost of regulation. Thus, an agent who invests $a_t^e$ in the bank in period $t$ will only receive $R_{t+1}^e (1 - \tau_h) a_t^e$ the following period. Because the

\(^6\)A risk-free bond pays out one unit of the common currency in the subsequent period. Because the payout is in common currency rather than in real home consumption, this investment is risky due to CPI fluctuations. The assumption that bond payouts are in common currency is made to guarantee model symmetry.
government has no other role in the economy, the regulation fee is returned to the home household in the form of a lump-sum transfer \( T_t^h \). Income not spent on the three assets is consumed \((c_t^h)\).

As I explain in section 3.3, the solution method requires that the state space be compact; therefore, household borrowing is limited to the lowest future wage realization, and she cannot short equities.

\[
\begin{align*}
    b_t^h &\geq - \min \left( P_{t+1}^h W_t^h \right) \\
    a_t^{hh}, a_t^{eh} &\geq 0
\end{align*}
\]

The restriction on the minimum holdings of home equity \( a_t^{hh} \) can be lowered to an arbitrary but fixed value to ensure compactness. The bank investment cannot be shorted as this would allow home households to reduce bank equity below zero. The household’s first order conditions can be found in the technical appendix.

### 3.2.3 Financial Intermediary

The financial intermediary, which will henceforth be called the "bank," is meant to represent not merely the traditional banking sector, but instead the complete international financial structure that includes mutual funds, hedge funds, etc. To this end, the banking sector not only provides the global risk-free bond, but it also invests in both home and foreign dividend streams. Additionally, consumers have the option of lending to the bank, or financing it directly. This setup allows for a more thorough analysis of the mechanism through which risk is dispersed throughout the global economy, including the ramifications of endogenous changes in the bank’s capital holdings. A frictionless version of the model, one with no financial regulations,

\footnote{Future extensions of this model in which the bank can fail may incorporate government bailout of the bank, in which case the fee would not be directly transferred to the household.}
produces the same portfolio choices as a standard model without the bank where consumers can purchase both countries’ equities freely. Thus, the bank only exists in the model to act as a target for financial regulations, and is not a friction in and of itself.

The bank possesses a technology to exchange claims on future production across national borders and thus acts as the entity responsible for allocating risk. The bank does not purchase stocks for personal consumption; all claims to output owned by the bank are passed along as returns to the two household groups at the beginning of each period. The two households’ net investments in the bank, $(1 - \tau_h) a_t^{eh}$ and $(1 - \tau_f) a_t^{ef}$, become its time $t$ capital:

$$k_t^e = (1 - \tau_h) a_t^{eh} + (1 - \tau_f) a_t^{ef}.$$  

The future rate of return on bank capital is given by

$$r_{t+1}^e = \frac{b_t^e + \sum_{i=\{h,f\}} (Q_{t+1}^i + P_{t+1}^i D_{t+1}^i) a_t^{ie}}{k_t^e}.$$ (3.5)

Banks optimally invest their capital each period in bonds $b_t^e$ and home and foreign equity, $a_t^{he}$ and $a_t^{fe}$. As noted in the introduction, all banking transactions are becoming increasingly global; therefore, I model only one representative financial intermediary, rather than having one bank for each country.

I model the stochastic discount factor of the bank as a Cobb-Douglas combination of the discount factors of the two consumers, where the weight on each factor depends on the aggregate ownership of the banking sector. Thus, the stochastic discount factor (SDF) is given by:

$$M_{t+1}^e = \exp \left\{ \frac{(1 - \tau_h) A_{t+1}^{eh}}{K_t^e} \ln \left( M_{t+1}^h \right) + \frac{(1 - \tau_f) A_{t+1}^{ef}}{K_t^e} \ln \left( M_{t+1}^f \right) \right\},$$

There is a large literature beginning with Dreze (1985) on the nature of the firm’s maximization problem in incomplete markets with multiple firm owners. Robustness checks indicate that the exact specification of the bank’s discount factor has no impact on the main results of the model.
where $M^h_{t+1}$ and $M^f_{t+1}$ are defined in the technical appendix.

The bank maximizes the expected discounted value of its next period returns by optimally choosing its portfolio position:

$$\max_{b_t^e, a_t^h, a_t^f} E \left[ M^e_{t+1} r^e_{t+1} k^e_t \right]$$

subject to the bank’s budget constraint

$$Q^h_t b_t^e + Q^h_t a_t^h + Q^f_t a_t^f = k^e_t.$$  

Additionally, the bank faces borrowing and shorting constraints

$$b_t^e \geq -\min \left[ (Q^h_{t+1} + P^h_{t+1} D^h_{t+1}) a_t^h + (Q^f_{t+1} + P^f_{t+1} D^f_{t+1}) a_t^f \right]$$

$$a_t^h, a_t^f \geq 0,$$

which ensure that the bank will always pay a positive return and so guarantee compactness of the state space. Similar to the restriction (3.4) on the home household, shorting restriction are introduced to ensure state-space compactness, but never bind in equilibrium since the bank will always wish to invest in equity. Constraint (3.6) prevents the bank from borrowing more than the minimum future value of its risky assets; this way, the bank can only borrow what it is guaranteed to be able to pay back in the future. Knowing that the bank has no means of acquiring further wealth, consumers would not be willing to lend any more than this amount. The second constraint prevents the bank from shorting either country’s equity.

The bank’s Euler equations can be found in the technical appendix.

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9 Alternatively, the bank could be allowed to default. This would cause the bank’s problem to become non-differentiable and prohibit the use of first order conditions in the solution method. I leave the exploration of that scenario to future research.
3.2.4 Market Clearing

All output is perishable and must be consumed immediately. This is a general equilibrium model, so all of the assets markets must clear. The stocks are assumed to be in unit supply and the risk-free bond is in zero net supply. Thus,

\[ A^{hh}_t + A^{he}_t = 1 \]  
(3.8)  
\[ A^{ff}_t + A^{fe}_t = 1 \]  
(3.9)  
\[ B^h_t + B^f_t + B^e_t = 0 \]  
(3.10)  
\[ C^{hh}_t + C^{hf}_t = Y^h_t \]  
(3.11)  
\[ C^{fh}_t + C^{ff}_t = Y^f_t. \]  
(3.12)

Agents are of measure one, so all aggregate variables equal individual variables in equilibrium.

3.3 Solution Method

The most widely used methods of solving incomplete market models with multiple assets, due to Devereux and Sutherland (2011) and Evans and Hnatkovska (2012), rely on the assumption of a steady state wealth level. This approach, however, cannot be applied to my model. One of the goals of my research is to track the changes in a country’s net foreign wealth (NFW) as a result of asymmetric financial frictions. This means that the international wealth ratio need not stay fixed over time, and so I cannot approximate the model dynamics around its arbitrarily chosen level. Second, one of the reasons for introducing a bank is to allow for endogenously occurring corner solutions where the bank’s borrowing or solvency constraints bind. Linearized models cannot deal with such inequalities and, in fact, rely on avoiding corner solutions in their methodology.
Therefore, I utilize a (more computationally demanding) global solution technique used in Stepanchuk and Tsyrennikov (2011), who employ an iterated grid search to find the agents’ optimal decision rules. I refer the interested reader to the above paper for technical details of this solution methodology, and use the rest of this section to briefly outline its two key features.

The first issue concerns reducing the dimension of the state space (since a grid search is employed in the solution method, a small state space is critical to solving the model). In the model described in the previous section, the vector of state variables consists of the asset positions of all agents \( \{A^h_{t-1}, A^{eh}_{t-1}, A^{ff}_{t-1}, A^{ef}_{t-1}, B^h_{t-1}, B^f_{t-1}\} \) as well as the exogenous state of nature \( \{Y^h_t, Y^f_t, \phi_t\} \). To simplify the solution algorithm and save on computing time, the state space can be reduced to four variables: a wealth ratio, the two exogenous output realizations, and the home bias shock.

Secondly, due to the presence of multiple assets in the household problem, the model must be solved using homotopy, as outlined in Schmedders (1998).

3.3.1 State Space

Consumers care primarily about the level of their income rather than its source (wages, debt or equity). Therefore, the model state space can be reduced as follows. I define the tradable income of each agent in period \( t \) as:

\[
I^h_t \equiv P^h_t W^h + b^h_{t-1} + (Q^h_t + P^h_t D^h_t) a^{hh}_{t-1} + R^e_t (1 - \tau_h) a^{eh}_{t-1} \\
I^f_t \equiv P^f_t W^f + b^f_{t-1} + (Q^f_t + P^f_t D^f_t) a^{ff}_{t-1} + R^e_t (1 - \tau_f) a^{ef}_{t-1}
\]

Government transfers are not included in these definitions since they are only determined once investment decisions have been made; thus, future wealth would be dependent on future portfolio allocations, which would complicate the state space. The two tradable incomes are aggregated and combined with equation (3.5) to calculate the
total tradable income in the economy:

\[ I_t \equiv I_t^h + I_t^f = \Pi_t^{hh}W^h + \Pi_t^{ff}W^f + Q_t^h + \Pi_t^{hh}D_t^h + Q_t^f + \Pi_t^{ff}D_t^f. \tag{3.13} \]

The second equality results from the application of the asset market clearing conditions (3.8)-(3.10).

From these definitions, the new state variables (tradable wealth ratios) are defined as follows:

\[ \omega_t^h \equiv \frac{I_t^h}{I_t} \in [0,1] \]
\[ \omega_t^f \equiv \frac{I_t^f}{I_t} \in [0,1] \]

where the restrictions on their values comes from the portfolio constraints (3.3), (3.4), (3.6) and (3.7). Combining these definitions with (3.13) implies that only one wealth ratio, \( \omega_t^h \), is needed to describe the endogenous state space; the second wealth ratio is a function of the first.

\[ \omega_t^f = 1 - \omega_t^h \]

Thus, I can solve the model over the state space where

\[ \omega_t^h \in [0,1] \]

\[ (Y_t^h, Y_t^f, \phi_t) = \begin{cases} (Y_t^h, Y_t^f, \phi_t), (Y_t^h, Y_t^f, \phi_t), (Y_t^h, Y_t^f, \phi_t), (Y_t^h, Y_t^f, \phi_h), (Y_t^h, Y_t^f, \phi_h), (Y_t^h, Y_t^f, \phi_h), \end{cases} \]

To solve the model, I define the state space on an \( n_h \times 8 \) grid with step sizes for the home wealth ratio being \( \frac{1}{n_h} \).\(^{10}\)

The home household’s budget constraint (3.2) now becomes:\(^{11}\)

\[ P_t^{jh}c_t^h + Q_t^{hh}b_t^h + Q_t^{hh}a_t^{hh} + a_t^{eh} = \omega_t^h I_t + T_t^h. \]

\(^{10}\)The number of steps used in the code is \( n_h = 11 \).

\(^{11}\)Note that \( \omega_t^h = 0 \) is not an absorbing state, since the home household can still borrow against future wage income in order to consume and invest. The same argument holds for the foreign household when \( \omega_t^h = 1 \).
3.3.2 Homotopy

The standard approach to solving models with multiple assets and endogenous portfolio choice is to come up with a starting guess for all endogenous variables (in particular for the portfolio allocations), and then employ a Newtonian algorithm that minimizes the Euler equation errors to find an approximate solution. However, if asset prices are changing along with portfolio holdings, the returns on two assets may become collinear, a classic problem described in Schmedders (1998). At the point where collinearity is reached, the portfolio decision ceases to be continuous, and the Newtonian method fails. This issue can be avoided by employing a simple homotopy algorithm.

The idea of the algorithm is to penalize all but one agent in each market for deviating from a fixed portfolio position, thus creating a continuous minimization problem. Gradually, the penalty is lifted until the original Euler equation obtains.

For example, the home household’s maximization problem becomes:

$$V^h \equiv \max_{c^h, b^h_t, a^{hh}, a^{eh}} \sum_{i=0}^{\infty} \beta^i E \left[ \frac{(c^h_{t+1})^{1-\gamma}}{1-\gamma} \right]$$

$$- (1-\tau) \frac{1}{2} \left[ (b^h_t - b^{hs})^2 + (a^{hh}_t - a^{hs})^2 + (a^{eh}_t - a^{ehs})^2 \right],$$

where $b^{hs}$, $a^{hh}$ and $a^{eh}$ are judiciously chosen as good starting guesses, and $\tau$ determines the relative magnitude of the penalty (given by the term in brackets). The Euler equation for $b^h_t$ becomes:

$$0 = \tau \left\{ E \left[ M^h_t + v^{bh}_t - Q^b_t \right] - (1-\tau) (b^h_t - b^{hs}) \right\},$$

where $v^{bh}_t$ is a Lagrange multiplier. Starting at $\tau = 0$, the optimal portfolio is clearly $b^h_t = b^{hs}$, but as $\tau$ is increased to one, the original Euler equation emerges:

$$Q^b_t = E \left[ M^h_{t+1} \right] + v^{bh}_t$$
As $\tau$ is gradually increased from zero to one, the algorithm converges on the true solution while remaining continuous the entire time.

This penalty is applied to all but one portfolio decisions for each asset. The remaining portfolio holdings are determined by the market clearing conditions, and the extra Euler equations are used to determine asset prices.

3.4 Portfolio Allocation Decisions

Since the bank itself does not distort financial markets, the model with no financial frictions ($\tau_h = \tau_f = 0$) produces the same results as standard two-country models with no bank, in which agents can freely trade assets across borders. For ease of comparison with the existing literature, I compute the total holdings on the home household in the model with the bank. To do this, the agent’s direct holdings of an asset must be added to her indirect holdings via her investment in the bank:

\[
TA_{th}^h = A_{th}^h + A_{th}^h \left(1 - \tau_h\right) \frac{A_{t}^{eh}}{K_t^e} \\
TA_{tf}^h = A_{tf}^e (1 - \tau_h) \frac{A_{t}^{eh}}{K_t^e} \\
TB_{th}^h = B_{th}^h + B_{th}^e (1 - \tau_h) \frac{A_{t}^{eh}}{K_t^e}
\]

3.4.1 Parameterization

Each time period in the model represents one year. To isolate the effects of country-specific financial frictions, I assume that the two economies are symmetric in all other aspects; parameters (listed in Table 3.1) are chosen to match the U.S. data.

I set the intertemporal discount factor $\beta = 0.95$, so that the annualized real interest rate is 5%. I set $\gamma$, the constant of relative risk aversion, equal to two, which is a standard value in financial macroeconomic models.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Intertemporal discount factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>CRRA</td>
<td>2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of substitution for goods</td>
<td>0.9</td>
</tr>
<tr>
<td>$\lambda^h, \lambda^f$</td>
<td>Markov persistence for $Y^h_t, Y^f_t$</td>
<td>0.825</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$Corr\left(Y^h_t, Y^f_t\right)$</td>
<td>0.2</td>
</tr>
<tr>
<td>${Y^i_t, Y^i_h}$</td>
<td>Low and high output, $i = {h, f}$</td>
<td>${1, 1.04}$</td>
</tr>
<tr>
<td>$\lambda^\phi$</td>
<td>Markov persistence for home bias $\phi_t$</td>
<td>0.5</td>
</tr>
<tr>
<td>${\phi_t, \phi_h}$</td>
<td>Low and high home bias values</td>
<td>${0.84, 0.86}$</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>Capitalization ratio, $i = {h, f}$</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 3.1: Benchmark calibration.

There is still some debate in the literature about the value of $\eta$, the elasticity of substitution between home and foreign goods; various estimates suggest that it lies in the $[0.9, 1.5]$ range. If $\eta$ is less than one, the two goods are complements, a reasonable short-run assumption used in international trade and finance papers since it helps to match volatility of the terms of trade and the negative correlation between terms of trade and the trade balance. Stepanchuk and Tsyrennikov (2011) use $\eta = 0.83$, Corsetti, Dedola and Leduc (2008) let $\eta = 0.85$, and Heathcote and Perri (2002) set $\gamma = 0.91$. For the baseline model, I will assume that home and foreign goods are complements and that $\eta = 0.9$.

The parameters $\{\lambda^h, \lambda^f, \rho\}$ are calibrated to match the U.S. GDP persistence of 65% and to generate a 20% correlation between home and foreign output.\footnote{Rouwenhorst (1995) describes the process for matching a Markov matrix to a continuous first order autoregressive process.} I normalize $Y^h_t$ and $Y^f_t$, the low realizations of output, to one and then set $Y^h_h$ and $Y^f_h$ so that the volatility of output matches that of the U.S. since the beginning of the "Great Moderation" (approximately 2% per year).
The home bias process $\phi_i$ is independently and identically distributed with a mean of 85% to match the U.S. average trade to GDP ratio of 16% (to bring the model as close to the data as possible, I exclude government spending and investment from the computation of GDP in the data). The standard deviation of the home bias shock is unobservable, so I conservatively set it to 1%.

Finally, capitalization ratios $\mu_h$ and $\mu_f$ are chosen to match U.S. corporate profits to GDP ratio.

3.4.2 MECHANISMS

International capital flows (both theoretical and empirical) result from portfolio allocation decisions of individual consumers; therefore, before turning to the impact of financial frictions on the patterns of global finance, I elucidate the mechanisms that drive these allocations. The consumers’ portfolio decisions rest on three particular considerations: whether to save or to borrow, whether to hold a risky or a safe portfolio, and what fraction of equity to hold in home versus foreign assets. These considerations are tightly linked to the household’s preferences regarding intertemporal consumption smoothing, equity home bias and international risk-sharing. I begin by describing each of these three channels in the context of my model before turning to their combined effect contained in the impulse response functions.

**Consumption Smoothing**

Current account is defined as the sum of net exports and net foreign income receipts:

$$C A^h_t \equiv \left( P_t^{hh} C^h_t - P_t^{hf} C^f_t \right) \text{ Net Exports}$$

$$+(1 - Q^b_{t-1}) T B^h_{t-1} + \left( Q^f_t + P_t^{ff} D^f_t - Q^f_{t-1} \right) T A^f_{t-1} - \left( Q^h_t + P_t^{hh} D^h_t - Q^h_{t-1} \right) T A^h_{t-1} \text{ Net Foreign Income Receipt}.$$
Unfortunately, the state space used to solve the model does not allow for a simple computation of the current account, since starting portfolio holdings and last-period price levels are unknown. Instead, I calculate the current account from simulation data. The current account decisions of the home and foreign agents are governed by two standard consumption smoothing mechanisms.

First, the Permanent Income Hypothesis dictates that the country with a lower current output realization tends to borrow from abroad to smooth its consumption during the temporary downturn. However, due to the complementarity between home and foreign goods and the persistence of output shocks, home households borrow during good times, since home goods prices fall, making the value of current wages decline. Moreover, the debtor country tends to reduce debt internationally regardless of its current output realization. The interaction between these two channels (the current realization of the state versus the relative wealth ratio) affects the magnitude of the current account movement; thus, for example, a debtor country that has just experienced a low output realization may repay a portion of its debt, albeit not as much as if the output realization were high.

**EQUITY HOME BIAS**

The second aspect of portfolio allocation decisions concerns the portfolio’s riskiness. An agent who is relatively more risk-seeking would prefer a portfolio that is light in risk-free bonds and heavy in risky equity. To study this mechanism, I first calculate the relative value of equity, $RA^h_t$, in the home household’s portfolio:

$$RA^h_t \equiv \frac{Q^h_tTA^h_{th} + Q^f_tTA^f_{th}}{Q^h_tTB^h_t + Q^h_tTA^h_{th} + Q^f_tTA^f_{th}}.$$

---

13See Mykhaylova and Staveley-O’Carroll (2012) for an in-depth description of the interaction of elasticity of substitution with international savings decisions.

14In the model simulations, the value of the current account under symmetric starting conditions ranges from -0.19% to 0.19% of the world GDP.
When $RA^h_t$ is less than one, the home agent is investing in a mix of equity and debt assets; values of $RA^h_t$ greater than one indicate that the agent is leveraging her wealth in order to invest in a riskier portfolio. The relative value of equity can be negative in some states; this simply indicates that the agent is borrowing so much that her total portfolio holdings are negative. Since both home and foreign agents are assumed to have the same degree of risk aversion, the only determinant of the relative riskiness of their portfolios in my model is equity home bias, which arises due to two hedging features of the risky assets.

First, home equity can serve as an inflation hedge for the home household’s non-stochastic wage income. Episodes of high payout of the home equity (due to favorable output realization) are accompanied by a reduction in the price of the home good (PPI deflation). This lowers the purchasing power of the home wage income at the same time as the value of the home dividend income is rising.

Second, the model assumes a stochastic process in the demand for goods. In those states of nature when agents prefer to consume more of the local good ($\phi_t$ increases), the price of both goods rises due to the presence of home bias (see equation 3.1). The effect of this price increase on the consumers’ purchasing power can be offset by owning more shares of the asset that produces the local good.

For these two reasons, the home household wishes to hold a long position in the home equity, which requires adjusting the risk profile of her investment portfolio. When the home household is indebted to the foreign country, she must leverage her wealth (borrow in bonds) to invest in her home equity; in such cases, $RA^h_t > 1$. When, on the other hand, the home household is a lender, she adopts a mixed portfolio that lowers risk by going long in bonds, $0 < RA^h_t < 1$. 

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INTERNATIONAL RISK-SHARING

The final consideration that affects the agents’ portfolio positions is the desire to share risk internationally. When financial markets are complete, risk is perfectly shared across countries, causing the wealth ratio to remain constant regardless of the realization of the state. In my model, consumers cannot completely share equity risk internationally.

The home agent wishes to hold more of those assets that are positively correlated with her SDF $M_{t+1}^h$; the payout of these assets is high in those states of nature when the agent discounts the future the least, i.e., when future inflation and consumption are low. Recall that

$$M_{t+1}^h = \beta \left( \frac{c_{t+1}^h}{c_t^h} \right)^{-\gamma} \frac{P_t}{P_{t+1}^h}.$$ 

As discussed in the previous section, home PPI falls when the home asset payout is high. On the other hand, due to the presence of home bias in equity, the home asset’s future return is strongly positively correlated with future consumption. The overall correlation of the home asset return with the home agent’s SDF is therefore ambiguous. In my model it turns out that in some states of nature, the foreign asset’s return is more positively correlated with $M_{t+1}^h$ than the return of the home asset. When this is the case, the home agent wishes to purchase foreign equity in order to share risk internationally.

It follows that lower volatility of home PPI makes foreign equity relatively more attractive to the home household. Inflation variance is much higher in states characterized by high wealth imbalance, since goods prices are much more reactive to the effects of demand shocks (the rich country produces heavily asymmetric demand).
Consequently, consumers buy more foreign equity when the two countries’ wealth is balanced.\footnote{Additionally, an agent may demand more of the foreign asset simply because she has reached the limit of her home equity holding (equation 3.4 is binding).}

**Impulse Response Functions**

Because the model’s stochastic process is a discrete Markov chain, I must translate the exogenous states $X_t = \{Y^h_t, Y^f_t, \phi_t\}$ into a continuous AR(1) process in order to generate impulse responses. To this end, I simulate the model for a particular starting wealth ratio $\omega^h_t$, and estimate the law of motion of $X_t$ as follows:\footnote{To generate data for various starting wealth ratios, I run 100 simulations, each 100 periods long, for every starting value of the wealth ratio. I aggregate the data from each simulation before running regressions.}

$$X_t = A_0 + A_1 X_{t-1} + \varepsilon_t,$$

where

$$A_0 = \begin{bmatrix} \bar{Y}^h \\ \bar{Y}^f \\ \bar{\phi} \end{bmatrix}, \quad A_1 = \begin{bmatrix} 2\lambda_h - 1 & 0 & 0 \\ 0 & 2\lambda_f - 1 & 0 \\ 0 & 0 & 2\lambda_\phi - 1 \end{bmatrix} \quad \text{and} \quad \Omega = \frac{\sum_{i=1}^{N-3} \varepsilon_i \varepsilon_i'}{N - 3} = \begin{bmatrix} \sigma^2_{y_h} & \rho \sigma_y \sigma_{yf} & 0 \\ \rho \sigma_{yh} \sigma_{yf} & \sigma^2_{yf} & 0 \\ 0 & 0 & \sigma^2_{\phi} \end{bmatrix}.$$

Finally, I regress endogenous variables on $X_t$:

$$Y_t = B_0 + B_1 X_t.$$
Figure 3.4 shows the impulse response functions of several home household’s control variables to a one standard deviation shock to home output for three values of the starting wealth ratio $\omega_t^h$. The responses of consumption, real interest rate and PPI are quite intuitive. In accordance with the three channels of portfolio allocation outlined above, in the benchmark case of $\omega_t^h = 0.5$, the household borrows internationally to smooth consumption (not shown), increases her overall exposure to risk (both total home and total foreign equity increase) and shifts her equity holdings towards the foreign asset (international risk sharing). The details of these portfolio changes, which show the split of total asset holdings into their direct and bank subcomponents, are presented in the bottom two rows of the figure.

Higher starting wealth of the home agent, $\omega_t^h = 0.6$, only affect the change in total holdings of the foreign asset. Inflation variance increases due to larger relative wealth imbalance, thus decreasing the attractiveness of foreign equity to the home household.

When I set $\omega_t^h = 0.4$, the equity home bias mentioned above generates an additional effect on the behavior of the home household in response to stochastic output realizations. When the current realization of home output is high, $Y_t^h = Y_t^h$, Markov persistence ($\lambda^h > 0.5$) implies an increase in the value of the risky home equity; thus, $Q_t^h$ rises. Moreover, the Euler equations of the consumers require the values of the other assets to increase as well ($Q_t^b$ and $Q_t^f$ rise). The resulting lower real interest rate makes it cheaper for debtors (home consumers) to leverage their investments, thus lowering the overall risk level of their portfolios. The symmetry of the model implies that the extra risk must therefore appear in the lenders’ portfolios.

17 Due to space considerations, I do not show the impulse response functions of home variables following a shock to $Y_t^f$, which follow the same patterns outlined here.
Figure 3.4: Impulse responses to a positive one standard deviation shock to home output are shown for home consumption, PPI and the real interest rate, as well as the home household’s total bond and equity holdings. The third and fourth row break down the total holdings into direct and indirect investments. The vertical axis represents deviations from the mean.
Additionally, the higher wealth level of the home household implies a lower future inflation variance so the demand for international risk sharing increases. As a result, the home household holds more of the foreign equity.

Finally, the impulse response functions appear to imply that the model has a steady state, which is not the case. Because a large portion of income derives from wages (which are fixed), positive shocks to output in the home country elicit two opposing wealth effects. First, home-bias in equity means the home country enjoys a positive wealth effect from higher dividend earnings. Second, falling prices of local goods cause the purchasing power of wages to fall (an effect that is exacerbated by the complementarity of home and foreign goods), which produces a negative wealth effect. These two effects may offset each other, which makes the model appear stationary under certain starting conditions. The two opposing wealth effects do not always offset, and shocks to consumption home-bias, together with changes in the wealth ratio, can cause one effect to outweigh the other, generating non-stationarities in the endogenous variables.

3.5 Bank Frictions and International Capital Flows

The model is first run for 10,000 periods with an initial wealth ratio of 50% to find state independent starting values for the simulation. The economy is then simulated for one million periods.

3.5.1 Frictionless Financial Markets

Table 3.2 lists the average portfolio positions for the three agents. Aggregating these portfolios together according to (3.14)-(3.16) gives the total assets listed in Table 3.3.
Table 3.2: Average (across the simulated time frame) portfolio holdings of home and foreign consumers and the bank in the frictionless specification of the model. Standard deviations are listed in parentheses. The total amount of bonds and bank equity are not normalized.

<table>
<thead>
<tr>
<th></th>
<th>$B_t$</th>
<th>$A^h_t$</th>
<th>$A^f_t$</th>
<th>$A^e_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home</td>
<td>-0.31</td>
<td>0.82</td>
<td>0.70</td>
<td>(0.25)</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.19)</td>
<td>(0.47)</td>
<td></td>
</tr>
<tr>
<td>Foreign</td>
<td>-0.31</td>
<td>0.82</td>
<td>0.70</td>
<td>(0.25)</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.19)</td>
<td>(0.47)</td>
<td></td>
</tr>
<tr>
<td>Bank</td>
<td>0.61</td>
<td>0.18</td>
<td>0.18</td>
<td>(0.48)</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.19)</td>
<td>(0.19)</td>
<td></td>
</tr>
</tbody>
</table>

Due to the symmetry of the model, the average wealth ratio remains at 50%, and total bond holdings of both consumers are zero. The model generates equity home bias since the home household holds on average over 95% of her own asset and only 4.4% of the foreign asset. This is a standard result in international financial literature attributed to a combination of the agent’s home bias in consumption and a desire to hedge against the decreased value of wages paid out in home goods whose relative price decreases when the payout of the home dividend is high.

Table 3.3: Average (across the simulated time frame) total portfolio holdings of home and foreign consumers in the frictionless specification of the model. Standard deviations are listed in parentheses. The total amount of bonds and bank equity are not normalized.

<table>
<thead>
<tr>
<th></th>
<th>$TB_t$</th>
<th>$TA^h_t$</th>
<th>$TA^f_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home</td>
<td>0</td>
<td>0.956</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>(0.0023)</td>
<td>(0.0027)</td>
<td>(0.0027)</td>
</tr>
<tr>
<td>Foreign</td>
<td>0</td>
<td>0.044</td>
<td>0.956</td>
</tr>
<tr>
<td></td>
<td>(0.0023)</td>
<td>(0.0027)</td>
<td>(0.0027)</td>
</tr>
</tbody>
</table>

I find that, in the frictionless economy, the bank does not leverage its capital but instead lends money to the consumers who then leverage their portfolios themselves.
(first column of Table 3.2). This somewhat counter-intuitive result can be attributed to the lack of frictions which make equity investment in the back relatively cheap. This mechanism has no impact on the total bond holdings $TB_t$.

3.5.2 Financial Friction

I next solve the model for $\tau_h = 0$ and $\tau_f = 0.01$. In this setup, the home country can be thought of as having completely liberalized its banking sector, while the foreign country maintains some degree of financial regulation. Simulation results are presented in Tables 3.4 and 3.5 for the disaggregated and total portfolio holdings, respectively.

<table>
<thead>
<tr>
<th></th>
<th>$B_t$</th>
<th>$A^h_t$</th>
<th>$A^f_t$</th>
<th>$A^e_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home</td>
<td>0.027</td>
<td>0.997</td>
<td>0.015</td>
<td>(0.066)</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.037)</td>
<td>(0.037)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>Foreign</td>
<td>−0.0034</td>
<td>0.987</td>
<td>0.00001</td>
<td>0.00005</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.0098)</td>
<td>(0.0098)</td>
<td>(0.0098)</td>
</tr>
<tr>
<td>Bank</td>
<td>−0.023</td>
<td>0.003</td>
<td>0.013</td>
<td>(0.071)</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.037)</td>
<td>(0.037)</td>
<td>(0.037)</td>
</tr>
</tbody>
</table>

Table 3.4: Average (across the simulated time frame) portfolio holdings of home and foreign consumers and the bank in the specification of the model with financial frictions. Standard deviations are listed in parentheses. The total amount of bonds and bank equity are not normalized.

The introduction of financial frictions causes an increase in equity home bias vis-à-vis the frictionless case, providing a third explanation for its existence in the model. The home agent now holds more of his own equity because the foreign agent demands less of it (due to the banking fee), and the foreign agent holds more of her own equity because it is cheaper than investing abroad. Thus, the asymmetric financial friction works in both countries to increase equity home bias.

The bank now finances its risky investment with leveraged funds (first column of Table 3.4). Once the foreign household reduces her investment in the bank, the
Table 3.5: Average (across the simulated time frame) total portfolio holdings of home and foreign consumers in the specification of the model with financial frictions. Standard deviations are listed in parentheses. The total amount of bonds and bank equity are not normalized.

<table>
<thead>
<tr>
<th></th>
<th>$TB_t$</th>
<th>$TA^h_t$</th>
<th>$TA^f_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home</td>
<td>0.0035</td>
<td>0.99994</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.031)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>Foreign</td>
<td>-0.0035</td>
<td>0.987</td>
<td>0.00006</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.016)</td>
<td>(0.031)</td>
</tr>
</tbody>
</table>

bank’s discount factor coincides almost perfectly with that of the home household. Consequently, the latter no longer has any motivation to put more money into the bank than is required to purchase foreign equity.

An examination of the impact of the banking fee on impulse responses in figure 3.5 shows two changes in the home household’s behavior. First, since the home household holds nearly all of the home equity, this variable ceases to respond to shocks. Second, the reactions of bond and foreign equity holdings are reversed relative to the frictionless case. Without frictions, the home household wishes to hold more home equity when home output experiences a positive shock. To share the risk associated with that equity, the home household buys more of the foreign equity as well. However, when the home household already holds all of its own equity, there is no need to increase risk-sharing. In fact, because the home equity is now expected to have a higher future payout, less risk-sharing foreign equity is required. The opposite argument holds for a positive foreign output shock.

Finally, the impact of shocks to home bias is magnified when the financial friction is introduced. This stems from the lower degree of international risk-sharing when one country has a regulated financial system. The foreign country holds very little
of the home equity, but the home country also holds less of the foreign asset, since it has become more valuable to the foreign household.

3.5.3 Stylized Facts

This chapter focuses on four main variables of interest to try to match the four stylized facts listed in the introduction: gross portfolio holdings, net equity investment, net debt investment and NFW, all as percent of GDP. In the context of the model, these...
variables are defined as

\[
\text{Gross}^h_t = \frac{Q^h_t |TB^h_t| + Q^h_t |TA^h_f| + Q^f_t |TA^h_t|}{P_{t}^{hh}Y^h_t}
\]

\[
\text{NetEquity}^h_t = \frac{Q^f_t TA^h_f - Q^h_t TA^h_f}{P_{t}^{hh}Y^h_t}
\]

\[
\text{NetDebt}^h_t = \frac{Q^h_t TB^h_t}{P_{t}^{hh}Y^h_t}
\]

\[
\text{NFA}^h_t = \frac{Q^h_t TB^h_t + Q^f_t TA^h_f - Q^h_t TA^h_f}{P_{t}^{hh}Y^h_t}
\]

By introducing a small financial fee on bank transactions of the foreign household, I am able to explain the first two of the four trends in global finance outlined in the introduction.

In the frictionless model, the home household holds an average of 20% of GDP in gross international assets and liabilities per year. When a banking fee is imposed on the foreign economy, this number falls to 6% of GDP. Thus, the model predicts, in accordance with fact (1), that lower levels of financial regulation increase gross international portfolio positions. The mechanism behind this finding is the increased home bias in equity that results from the addition of the financial friction.

The model also offers an explanation for fact (2), that countries with liberalized banking systems (for example, the U.S. and the U.K.) hold a positive net position in international equity. In the symmetric frictionless model neither country holds a net position in international equity. However, when the foreign friction is introduced, the home household’s net position in international equity increases on average to 3% of GDP. Due to asymmetry in the ease of access to the financial intermediary, home household’s investment abroad is much larger than the foreign household’s holdings of home equity.

The model is less successful in explaining the third stylized fact, that more heavily financially regulated countries hold a positive position in net debt. Two counteracting
mechanisms are at work here. On the one hand, the foreign household is discouraged by the fee from investing in the bank, which increases her demand for the other two assets, foreign equity and the riskless bond. This, in turn, causes in the foreign country to become a net lender in international bond markets, supporting fact (3). On the other hand, the foreign household is not diversifying risk as optimally as she does in the frictionless setup. Consequently, her NFA deteriorates over time against that of the home country (in the simulations the average home wealth ratio $\omega^h$ is 50.7%) through a combination of equity sales and bond shorting. This channel explains the tendency of the foreign country to hold a negative net debt position.

The interaction of these two mechanisms is quantitatively ambiguous. Table 3.5 indicates that the standard deviation of debt holdings is an order of magnitude higher than average debt holdings of either country; in the simulations the foreign country in a debtor almost as frequently as it is a lender.

The model gives a counterfactual prediction regarding fact (4) since the home household’s average NFA is 3% of GDP. Since this is the first paper to introduce the optimizing financial sector in a two-country framework, the model relies on several simplifying assumptions, one of them being the ad hoc treatment of financial regulations. In reality, banking restrictions are put in place to reduce financial market imperfections (such as moral hazard) and limit imprudent lending. One can imagine an extension of the model that introduces credit constrained hand-to-mouth consumers who react to financial liberalization by increasing their (and therefore their country’s) net debt. I leave the exploration of this mechanism for future research.

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3.6 Summary and Extensions

In this chapter, I introduce an optimizing international financial intermediary into a two-country DSGE model with incomplete markets and multiple assets. I show that this model can be used to explain several facts about global capital imbalances over the last several decades. I do so without invoking the often used "exorbitant privilege" explanation that relies on a country being able to borrow in its own currency, which makes it inapplicable to most economies. Instead, I use my model to illustrate how the process of financial deregulation can lead to an increase in a country’s gross international asset holding with emphasis on foreign equity. While the model does not match all of the stylized facts reported in the introduction, I outline a path for future extensions that can explain the observation that financially deregulated countries are net borrowers.

The incorporation of the bank into the standard international macroeconomic model opens the door to a host of other fascinating research questions. My framework allows for examination of the impact of specific national financial regulations (such as restrictions on portfolio risk and capital requirements) on portfolio holdings. The bank may be allowed to fail, which would motivate the exploration of issues related to systemic risk and bailouts versus regulation.
Appendix A

Chapter Two Appendix

A.1 Low-Frequency Equilibrium

The follow equations describe the equilibrium of the model:

\[
\tilde{A}(D, i) = \frac{1}{\theta + (1 - \theta)\omega_0} \\
\hat{A}(D, i) = \omega_0\tilde{A}(D, i) \\
\tilde{B}(D, i) = \tilde{B}(D, i) = 0 \\
\tilde{C}(D, i) = D\tilde{A}(D, i) \\
\hat{C}(D, i) = \omega_0\tilde{C}(D, i) \\
P(D, i) = \frac{\beta D}{(1 - \beta)} \\
Q(D, i) = \beta \sum_{j=1}^{2} A_{ij} (\Delta D_j)^{-1}.
\]

Additionally, wealths are given by

\[
\tilde{W} = \frac{\tilde{C}(D, i)}{(1 - \beta)} \\
\hat{W} = \omega_0\tilde{W}.
\]

A.2 The Signal: Derivations

Here I offer more detail in the derivation of the probabilities associated with LC’s private signal. These calculations are used to solve the trading period problem of LC and MM.

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A.2.1 Stationary Distributions of Dividend Shock

To calculate the unconditional probability of each dividend realization, I must first find their stationary distributions\(^1\)

\[
\begin{pmatrix}
\Lambda_1 \\
\Lambda_2
\end{pmatrix} = 
\begin{pmatrix}
\Lambda_{11} & \Lambda_{12} \\
\Lambda_{21} & \Lambda_{22}
\end{pmatrix}
\begin{pmatrix}
\Lambda_1 \\
\Lambda_2
\end{pmatrix}
\]

given

\[
\Lambda_1 + \Lambda_2 = 1.
\]

This implies:

\[
\Lambda_j = \Lambda_{jj} \Lambda_j + \Lambda_{ji} (1 - \Lambda_j) = \frac{\Lambda_{ji}}{1 - \Lambda_{jj} + \Lambda_{ji}} = \frac{\Lambda_{ji}}{\Lambda_{ij} + \Lambda_{ji}}, \quad \forall i \neq j.
\]

Using this formula, the symmetry of the Markov probability matrix

\[
\Lambda = \begin{pmatrix}
\lambda & 1 - \lambda \\
1 - \lambda & \lambda
\end{pmatrix}
\]

implies:

\[
\Pr(D_{\tau+1} = j) = \Lambda_j = \frac{1 - \lambda}{2(1 - \lambda)} = \frac{1}{2} \quad j \in \{l, h\}.
\]

(A.1)

A.2.2 Unconditional Probability of the Signal

Using the Law of Total Probability, I can calculate the unconditional probability of receiving a signal. First set

\[
\Pr(D_{\tau+1} = i) = \sum_j \Pr(D_{\tau+1} = i|S_\tau = j) \Pr(S_\tau = j).
\]

\(^1\)\(\Lambda\)'s with a single subscript denote stationary probabilities, while \(\Lambda\)'s with two subscripts denote conditional probabilities.
Since there are only two states of nature, this can be written in matrix notation:

\[
\begin{pmatrix}
\Pr(D_{r+1} = l) \\
\Pr(D_{r+1} = h)
\end{pmatrix}
= \begin{pmatrix}
\Pr(D_{r+1} = l|S_t = l) & \Pr(D_{r+1} = l|S_t = h) \\
\Pr(D_{r+1} = h|S_t = l) & \Pr(D_{r+1} = h|S_t = h)
\end{pmatrix}
\begin{pmatrix}
\Pr(S_t = l) \\
\Pr(S_t = h)
\end{pmatrix}.
\]

Now solve for the unconditional signals:

\[
\begin{pmatrix}
\Pr(S_t = l) \\
\Pr(S_t = h)
\end{pmatrix}
= \begin{pmatrix}
\Pr(D_{r+1} = l|S_t = l) & \Pr(D_{r+1} = l|S_t = h) \\
\Pr(D_{r+1} = h|S_t = l) & \Pr(D_{r+1} = h|S_t = h)
\end{pmatrix}^{-1}
\begin{pmatrix}
\Pr(D_{r+1} = l) \\
\Pr(D_{r+1} = h)
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\gamma_{11} & \gamma_{12} \\
\gamma_{21} & \gamma_{22}
\end{pmatrix}^{-1}
\begin{pmatrix}
\Lambda_1 \\
\Lambda_2
\end{pmatrix}
\]

\[
= \frac{1}{\gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}}
\begin{pmatrix}
\gamma_{22} & -\gamma_{12} \\
-\gamma_{21} & \gamma_{11}
\end{pmatrix}
\begin{pmatrix}
\Lambda_1 \\
\Lambda_2
\end{pmatrix}
\]

From this I can find:

\[
\Pr(S_t = S_j) = \frac{\gamma_{ii}\Lambda_j - \gamma_{jj}\Lambda_i}{\gamma_{jj}\gamma_{ii} - \gamma_{ji}\gamma_{ij}} = \frac{1}{2} \left( \frac{2v - 1}{v^2 - (1 - 2v + v^2)} \right).
\]

Through cancelation I get

\[
\Pr(S_t = j) = \frac{1}{2} \quad j \in \{l, h\}.
\] (A.2)

A.2.3 Signal History

LC's signal contains information about the local risky asset's next dividend payout such that

\[
v \equiv \Pr(D_{r+1} = j|S_t = j) \quad \text{for} \quad j \in \{l, h\}.
\]

This can be written in matrix notation as

\[
\gamma_{kj} \equiv \Pr(D_{r+1} = D_k|S_t = S_j) \quad \text{for} \quad k, j \in \{1, 2\}
\]

where

\[
\Gamma = \begin{pmatrix}
v & 1-v \\
1-v & v
\end{pmatrix}. 
\]
Given (A.1) and (A.2), I can apply Bayes’ Law to get
\[
\Pr (S_t = S_j | D_{t+1} = D_k) = \frac{\Pr (D_{t+1} = D_k | S_t = S_j) \Pr (S_t = S_j)}{\Pr (D_{t+1} = D_k)} = \Pr (D_{t+1} = D_k | S_t = S_j).
\]
The result is that
\[
\Pr (S_t = S_j | D_{t+1} = D_k) = \gamma_{kj}.
\] (A.3)
The signal history is created by summing up all of the high signals received up to period \( t \):
\[
S^t = \sum_{i=1}^{t} I [S_i = h].
\]
Equation (A.3) clearly shows that the individual signals are statistically independent of each other as long as their probabilities are conditioned on next period’s dividend value. Therefore, the signal history is a binomial random variable:
\[
\Pr (S^t = j | D_{t+1} = D_k) = \frac{t! (\gamma_{k1})^{t-j} (\gamma_{k2})^j}{j! (t-j)!}.
\]
Using the Law of Total Probability I can find
\[
\Pr (S^t = j | D_r = D_i) = \sum_k \Pr (S^t = j | D_{t+1} = D_k) \Pr (D_{t+1} = D_k | D_r = D_i) = \sum_k \frac{t! (\gamma_{k1})^{t-j} (\gamma_{k2})^j}{j! (t-j)!} \lambda_{ki}.
\]
Now apply Bayes’ Law to get
\[
\Pr (D_{t+1} = D_k | S^t = j, D_r = D_i) = \frac{(\gamma_{k1})^{t-j} (\gamma_{k2})^j \lambda_{ki}}{\sum_k (\gamma_{k1})^{t-j} (\gamma_{k2})^j \lambda_{ki}}.
\]
LC’s information set can be summed up as \( \tilde{\Omega}_{tij} = \{D_r = D_i, S^t = j\} \) (written in the body of the chapter as \( \tilde{\Omega}_t \) for simplicity). Given this information set, LC forecasts next period’s local asset dividend as follows:
\[
\Pr (D_{t+1} = D_k | \tilde{\Omega}_{tij}) = \frac{\lambda_{ki} (\gamma_{k1})^{t-j} (\gamma_{k2})^j}{\sum_k \lambda_{ki} (\gamma_{k1})^{t-j} (\gamma_{k2})^j},
\] (A.4)
A.3 Market Maker

This section contains the derivations of MM’s beliefs, and how they are used to set prices, in the high-frequency periods.

MM enters trading period \( t \) with the following prior beliefs, which are derived from the history of trades he has witnessed:

\[
\pi_{s,t}^j = \Pr \left( S_{t-1}^j = j \mid \Omega_t \right) \\
\pi_{d,t}^k = \Pr \left( \Delta D_{t+1} = \Delta D_k \mid \Omega_t \right) \\
\text{s.t. } \Omega_t = \left\{ \Delta D_t = \Delta D_i, \left( T_j \right)_{j=1}^{t-1} \right\}.
\]

The initial beliefs when \( t = 1 \) are easily found:

\[
\pi_{s,1}^j = 1 \\
\pi_{d,1}^k = \Lambda_{ik} \text{ } \forall k.
\]

Writing out the explicit formulas for the ask and bid prices reveal which probabilities need to be found:

\[
P_a^t = \sum_k \Pr \left( \Delta D_{t+1} = \Delta D_k \mid B_t, \Omega_t \right) \Delta D_k \left( P_t + D_t \right)
\]

\[
P_b^t = \sum_k \Pr \left( \Delta D_{t+1} = \Delta D_k \mid S_t, \Omega_t \right) \Delta D_k \left( P_t + D_t \right).
\]

Because MM deals with both LC and FI, whose information and reasons for trading are entirely different, the necessary probabilities must be split up based on who MM might be trading with weighted by the chance of trading with each agent type:

\[
\Pr \left( \Delta D_{t+1} = \Delta D_k \mid B_t, \Omega_t \right) = \Pr \left( \Delta D_{t+1} = \Delta D_k \mid \hat{B}_t, \Omega_t \right) \Pr \left( \hat{B}_t \mid B_t, \Omega_t \right) \\
+ \Pr \left( \Delta D_{t+1} = \Delta D_k \mid \hat{B}_t, \Omega_t \right) \Pr \left( \hat{B}_t \mid \hat{B}_t, \Omega_t \right)
\]

\[
\Pr \left( \Delta D_{t+1} = \Delta D_k \mid S_t, \Omega_t \right) = \Pr \left( \Delta D_{t+1} = \Delta D_k \mid \hat{S}_t, \Omega_t \right) \Pr \left( \hat{S}_t \mid S_t, \Omega_t \right) \\
+ \Pr \left( \Delta D_{t+1} = \Delta D_k \mid \hat{S}_t, \Omega_t \right) \Pr \left( \hat{S}_t \mid \hat{S}_t, \Omega_t \right)
\]
Since trades with FI contain no information,

\[
\Pr \left( \Delta D_{\tau+1} = \Delta D_k | \tilde{T}_t, \Omega_t \right) = \pi_{k,t}^d \quad \text{for } \tilde{T}_t \in \left\{ \tilde{S}_t, \tilde{S}_t \right\}
\]

Since trades with LC contain information about the private signal,

\[
\Pr \left( \Delta D_{\tau+1} = \Delta D_k | \tilde{T}_t, \Omega_t \right) = \sum_{j=0}^{t} \Pr \left( \Delta D_{\tau+1} = \Delta D_k | S^t = j, \tilde{T}_t, \Omega_t \right) \Pr \left( S^t = j | \tilde{T}_t, \Omega_t \right).
\]

The first piece of this equation comes from (A.4) in the previous section; however, MM must use the trade history to infer the probability of each signal history, \( \Pr \left( S^t = j | \tilde{T}_t, \Omega_t \right) \).

This inference is made using Bayesian learning. The first step is to find \( \Pr \left( S^t = j | \Omega_t \right) \) by updating MM’s prior beliefs:

\[
\Pr \left( S^t = j | \Omega_t \right) = \pi_{j,t}^s \Pr \left( S^t = l | \Omega_t \right) + \pi_{j-1,t}^s \Pr \left( S^t = h | \Omega_t \right) \quad \text{for } j \in \{0,1,...,t\}
\]

The probability of a certain signal occurring \( \Pr \left( S^t = S_i | \Omega_t \right) \) for \( i = \{1,2\} \) can simply be found using the law of total probability:

\[
\Pr \left( S^t = S_i | \Omega_t \right) = \sum_k \Pr \left( S^t = S_i | D_{\tau+1} = D_k, \Omega_t \right) \Pr \left( D_{\tau+1} = D_k | \Omega_t \right)
\]

\[
= \sum_k \Pr \left( S^t = S_i | D_{\tau+1} = D_k \right) \Pr \left( D_{\tau+1} = D_k | \Omega_t \right)
\]

\[
= \sum_k \pi_{k,t}^d \pi_{k,t}^d.
\]

Refering to the model section of the chapter, we assume here that bounds \( g \) and \( h \) are used (if these happen to be the optimal bounds \( g_o \) and \( h_o \) then the resulting prices will be optimal). It follows that Bayes law gives \( \Pr \left( S^t = j | \tilde{T}_t, \Omega_t \right) \) for all \( j \). For example, if \( \tilde{T}_t = \tilde{S}_t \) then,
\[
Pr \left( S^t = j | \tilde{S}_t, \Omega_t \right) = \frac{Pr \left( \tilde{S}_t | S^t = j, \Omega_t \right) Pr \left( S^t = j | \Omega_t \right)}{\sum_{j=0}^{t} Pr \left( \tilde{S}_t | S^t = j, \Omega_t \right) Pr \left( S^t = j | \Omega_t \right)}
\]

\[
= \begin{cases} 
\frac{Pr \left( S^t = j | \Omega_t \right)}{\sum_{j=0}^{g} Pr \left( S^t = j | \Omega_t \right)} & \text{if } j \leq g \\
0 & \text{otherwise}
\end{cases}
\]

since \( Pr \left( \tilde{S}_t | S^t = j, \Omega_t \right) = 1 \) for \( j \leq g \) and \( Pr \left( \tilde{S}_t | S^t = j, \Omega_t \right) = 0 \) for \( j > g \). Similar steps are used to solve for \( \tilde{T}_t = \{ \tilde{B}_t, \tilde{P}_t \} \).

To set prices, MM still needs to find the conditional probability that he is trading with LC and FI given the trade observed.

There are three possible states shown in Table (A.1). The probabilities of all possible transactions can be calculated conditioned on these states, as shown in Table (A.2). These probabilities can then be combined using Bayes law to calculate \( Pr \left( T_t | T_t, \Omega_t \right) \) and \( Pr \left( \tilde{T}_t | T_t, \Omega_t \right) \). For example,

\[
Pr \left( \tilde{B}_t | B_t, \Omega_t \right) = \frac{Pr \left( B_t | \tilde{B}_t, \Omega_t \right) Pr \left( \tilde{B}_t | \Omega_t \right)}{Pr \left( B_t | \Omega_t \right)}
\]

where

\[
Pr \left( B_t | \tilde{B}_t, \Omega_t \right) = 1
\]

\[
Pr \left( \tilde{B}_t | \Omega_t \right) = \sum_{\tilde{Z} = 1}^{3} Pr \left( \tilde{B}_t | \tilde{Z}, \Omega_t \right) Pr \left( \tilde{Z} | \Omega_t \right)
\]

\[
Pr \left( B_t | \Omega_t \right) = \sum_{\tilde{Z} = 1}^{3} Pr \left( B_t | \tilde{Z}, \Omega_t \right) Pr \left( \tilde{Z} | \Omega_t \right) + \sum_{\tilde{Z} = 1}^{3} Pr \left( \tilde{B}_t | \tilde{Z}, \Omega_t \right) Pr \left( \tilde{Z} | \Omega_t \right).
\]

This result follows directly from the monotonicity of LC’s decision rule in \( S^t \).
I can calculate all probabilities this way, which gives the following results:

\[
\begin{align*}
\Pr\left(\mathbb{B}_t \mid \mathbb{B}_t, \Omega_t\right) &= \frac{\Pr\left(\mathbb{B}_t \mid \mathbb{B}_t, \Omega_t\right) \Pr\left(\mathbb{B}_t \mid \Omega_t\right)}{\Pr\left(\mathbb{B}_t \mid \Omega_t\right)} = \frac{\theta \sum_{j=h}^t \Pr\left(S^t = j \mid \Omega_t\right)}{(1 - \theta) / 2 + \theta \sum_{j=h}^t \Pr\left(S^t = j \mid \Omega_t\right)} \\
\Pr\left(\mathbb{S}_t \mid \mathbb{S}_t, \Omega_t\right) &= \frac{\Pr\left(\mathbb{S}_t \mid \mathbb{S}_t, \Omega_t\right) \Pr\left(\mathbb{S}_t \mid \Omega_t\right)}{\Pr\left(\mathbb{S}_t \mid \Omega_t\right)} = \frac{\theta \sum_{j=0}^q \Pr\left(S^t = j \mid \Omega_t\right)}{(1 - \theta) / 2 + \theta \sum_{j=0}^q \Pr\left(S^t = j \mid \Omega_t\right)} \\
\Pr\left(\mathbb{S}_t \mid \mathbb{S}_t, \Omega_t\right) &= \frac{\Pr\left(\mathbb{S}_t \mid \mathbb{S}_t, \Omega_t\right) \Pr\left(\mathbb{S}_t \mid \Omega_t\right)}{\Pr\left(\mathbb{S}_t \mid \Omega_t\right)} = \frac{(1 - \theta) / 2}{(1 - \theta) / 2 + \theta \sum_{j=0}^q \Pr\left(S^t = j \mid \Omega_t\right)} \\
\Pr\left(\mathbb{P}_t \mid \mathbb{P}_t, \Omega_t\right) &= \frac{\Pr\left(\mathbb{P}_t \mid \mathbb{P}_t, \Omega_t\right) \Pr\left(\mathbb{P}_t \mid \Omega_t\right)}{\Pr\left(\mathbb{P}_t \mid \Omega_t\right)} = 1 \\
\Pr\left(\mathbb{P}_t \mid \mathbb{P}_t, \Omega_t\right) &= \frac{\Pr\left(\mathbb{P}_t \mid \mathbb{P}_t, \Omega_t\right) \Pr\left(\mathbb{P}_t \mid \Omega_t\right)}{\Pr\left(\mathbb{P}_t \mid \Omega_t\right)} = 0.
\end{align*}
\]

<table>
<thead>
<tr>
<th>State</th>
<th>Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z = 1)</td>
<td>(T(S^t, Q_t, P_t) &gt; 0)</td>
</tr>
<tr>
<td>(Z = 2)</td>
<td>(T(S^t, Q_t, P_t) = 0)</td>
</tr>
<tr>
<td>(Z = 3)</td>
<td>(T(S^t, Q_t, P_t) &lt; 0)</td>
</tr>
</tbody>
</table>

Table A.1: Three states of nature represented by \(Z\): LC buys, sells or passes, given the opportunity.
<table>
<thead>
<tr>
<th>States</th>
<th>( Z = 1 )</th>
<th>( Z = 2 )</th>
<th>( Z = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Pr \left( \tilde{Z}</td>
<td>\Omega_t \right) )</td>
<td>( \sum_{j=h}^t \Pr (S' = j</td>
<td>\Omega_t) )</td>
</tr>
</tbody>
</table>

**Purchase Probabilities**

| \( \Pr \left( \tilde{E}_t | \tilde{Z}, \Omega_t \right) \) | \( \theta \) | 0 | 0 |
| \( \Pr \left( \tilde{E}_t | \tilde{Z}, \Omega_t \right) \) | \( \frac{1 - \theta}{2} \) | \( \frac{1 - \theta}{2} \) | \( \frac{1 - \theta}{2} \) |

**Sale Probabilities**

| \( \Pr \left( \tilde{S}_t | \tilde{Z}, \Omega_t \right) \) | 0 | 0 | \( \theta \) |
| \( \Pr \left( \tilde{S}_t | \tilde{Z}, \Omega_t \right) \) | \( \frac{1 - \theta}{2} \) | \( \frac{1 - \theta}{2} \) | \( \frac{1 - \theta}{2} \) |

**Pass Probabilities**

| \( \Pr \left( \tilde{P}_t | \tilde{Z}, \Omega_t \right) \) | 0 | \( \theta \) | 0 |
| \( \Pr \left( \tilde{P}_t | \tilde{Z}, \Omega_t \right) \) | 0 | 0 | 0 |

Table A.2: List of the probabilities of certain transactions occurring given state \( Z \).
Appendix B

Chapter Three Appendix

B.1 Markov Matrix

The three transition matrices are given by the parameters \( \{\lambda^h, \lambda^f, \lambda^\phi\} \) as

\[
\Lambda^h = \begin{bmatrix}
\lambda^h & 1 - \lambda^h \\
1 - \lambda^h & \lambda^h
\end{bmatrix}
\]

\[
\Lambda^f = \begin{bmatrix}
\lambda^f & 1 - \lambda^f \\
1 - \lambda^f & \lambda^f
\end{bmatrix}
\]

\[
\Lambda^\phi = \begin{bmatrix}
\lambda^\phi & 1 - \lambda^\phi \\
1 - \lambda^\phi & \lambda^\phi
\end{bmatrix}
\]

To find the transition matrix for the entire state space, the correlation of the home and foreign output must be added to the Kronecker product of the transition matrices.

\[
R = \begin{bmatrix}
\rho & -\rho & -\rho & \rho \\
\rho & -\rho & -\rho & \rho \\
\rho & -\rho & -\rho & \rho \\
\rho & -\rho & -\rho & \rho
\end{bmatrix}
\]

\[
\Lambda = (\Lambda^h \otimes \Lambda^f + R) \otimes \Lambda^\phi
\]

B.2 Model with Financial Intermediary

In this appendix, I present all of the equations used in the model with the financial intermediary. All of the variables are capitalized as each agents are representative.
To start with, there are a series of definitions for the lump sum transfers, bank capital, and bank returns.

\[
T^h_t \equiv \tau_h A^e_t
\]
\[
T^f_t \equiv \tau_f A^e_t
\]
\[
K^c_t \equiv (1 - \tau_h) A^e_t + (1 - \tau_f) A^e_t
\]
\[
R^e_{t+1} \equiv \frac{B^e_t + (Q^h_{t+1} + P^h_{t+1} D^h_{t+1}) A^he_t + (Q^f_{t+1} + P^f_{t+1} D^f_{t+1}) A^fe_t}{K^e_t}
\]

Additionally, the stochastic discount factors of the home and foreign households, and the bank are defined as:

\[
M^h_{t+1} \equiv \beta \left( \frac{C^h_{t+1}}{C^h_t} \right)^{\gamma} \frac{P^h_t}{P^h_{t+1}}
\]
\[
M^f_{t+1} \equiv \beta \left( \frac{C^f_{t+1}}{C^f_t} \right)^{\gamma} \frac{P^f_t}{P^f_{t+1}}
\]
\[
M^e_{t+1} \equiv \exp \left\{ \frac{(1 - \tau_h) A^e_t}{K^e_t} \ln \left( M^h_{t+1} \right) + \frac{(1 - \tau_f) A^e_t}{K^e_t} \ln \left( M^f_{t+1} \right) \right\}
\]

Finally, tradable income and the wealth ratios are defined as follows (the first two equations show how the endogenous state transition):

\[
I^h_{t+1} \equiv P^h_{t+1} W^h_{t+1} + B^h_t + (Q^h_{t+1} + P^h_{t+1} D^h_{t+1}) A^h_t + R^e_{t+1} (1 - \tau_h) A^e_t
\]
\[
I^f_{t+1} \equiv P^f_{t+1} W^f_{t+1} + B^f_t + (Q^f_{t+1} + P^f_{t+1} D^f_{t+1}) A^f_t + R^e_{t+1} (1 - \tau_f) A^e_t
\]
\[
I_t \equiv P^h_t W^h_t + P^f_t W^f_t + (Q^h_t + P^h_t D^h_t) + (Q^f_t + P^f_t D^f_t)
\]
\[
\omega^h_t \equiv \frac{I^h_t}{I_t}
\]
\[
\omega^f_t \equiv \frac{I^f_t}{I_t} = 1 - \omega^h_t
\]

Due to Walras law, only one of the transition equations is necessary to solve the model.
From the standard bundler problem, I find expressions for the home and foreign 
CPIs,
\[
P^h_t = \left[ \phi_t \left( P^{hh}_t \right)^{1-\eta} + (1 - \phi_t) \left( P^{ff}_t \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}, \\
P^f_t = \left[ \phi_t \left( P^{ff}_t \right)^{1-\eta} + (1 - \phi_t) \left( P^{hh}_t \right)^{1-\eta} \right]^{\frac{1}{1-\eta}},
\]
and the optimal split of home and foreign goods at a given consumption level by each 
household
\[
C^{hh}_t = \phi_t \left( \frac{P^{hh}_t}{P^h_t} \right)^{-\eta} C^h_t, \\
C^{ff}_t = (1 - \phi_t) \left( \frac{P^{ff}_t}{P^h_t} \right)^{-\eta} C^h_t, \\
C^{fh}_t = \phi_t \left( \frac{P^{ff}_t}{P^f_t} \right)^{-\eta} C^f_t, \\
C^{hf}_t = (1 - \phi_t) \left( \frac{P^{hh}_t}{P^f_t} \right)^{-\eta} C^f_t.
\]
In order to solve the model, prices must be normalized:
\[
2 = P^{hh}_t + P^{ff}_t.
\]
The budget constraints of the home and foreign households, and the bank are 
given by
\[
P^h_t C^h_t + Q^h_t B^h_t + Q^h_t A^{hh}_t + A^{ch}_t = \omega^h_t I_t + T^h_t \\
P^f_t C^f_t + Q^f_t B^f_t + Q^f_t A^{ff}_t + A^{cf}_t = \omega^f_t I_t + T^f_t \\
Q^h_t B^e_t + Q^h_t A^{he}_t + Q^f_t A^{fe}_t = K^e_t.
\]
Because of Walras law, one of the budget constraints is unnecessary for solving the 
model.
Due to the presence of shorting constraints and borrowing limits, I employ the Garcia-Zangwill trick to turn inequalities into equalities. The trick is a simple change of variable, i.e.,

\[\nu_t^{bh+} = \max(0, \nu_t^{bh}) \quad \text{and} \quad \nu_t^{bh-} = \max(0, -\nu_t^{bh}).\]

Note that the new variables meet the requirement of complementary slackness

\[\nu_t^{bh+} \geq 0, \nu_t^{bh-} \geq 0, \text{ and } \nu_t^{bh+} \times \nu_t^{bh-} = 0.\]

Employing this method, I find equality expressions for the home households Euler equations,

\[
\begin{align*}
Q^h_t &= E\left[M^h_{t+1}\right] + \nu_t^{bh-} \\
Q^h_t &= E\left[M^h_{t+1}(Q^h_{t+1} + P^h_{t+1}D^h_{t+1})\right] + \nu_t^{bh-} \\
1 &= E\left[M^h_{t+1}r^e_{t+1}(1 - \tau_h)\right] + \nu_t^{eh-},
\end{align*}
\]

where \(\nu_t^{bh-}\), \(\nu_t^{bh-}\), and \(\nu_t^{eh-}\) are Lagrange multipliers which must be greater than or equal to zero. Similar first order conditions exist for the foreign household,

\[
\begin{align*}
Q^f_t &= E\left[M^f_{t+1}\right] + \nu_t^{bf-} \\
Q^f_t &= E\left[M^f_{t+1}(Q^f_{t+1} + P^f_{t+1}D^f_{t+1})\right] + \nu_t^{ff-} \\
1 &= E\left[M^f_{t+1}r^e_{t+1}(1 - \tau_f)\right] + \nu_t^{ef-},
\end{align*}
\]

and the bank,

\[
\begin{align*}
Q^h_t &= \frac{Q^h_t}{E\left[M^h_{t+1}\right] + \nu_t^{be-}} \left\{ E\left[M^e_{t+1}(Q^h_{t+1} + P^h_{t+1}D^h_{t+1})\right] + \nu_t^{be-} \min\left(Q^h_{t+1} + P^h_{t+1}D^h_{t+1}\right) + \nu_t^{he-} \right\} \\
Q^f_t &= \frac{Q^f_t}{E\left[M^e_{t+1}\right] + \nu_t^{be-}} \left\{ E\left[M^e_{t+1}(Q^f_{t+1} + P^f_{t+1}D^f_{t+1})\right] + \nu_t^{be-} \min\left(Q^f_{t+1} + P^f_{t+1}D^f_{t+1}\right) + \nu_t^{fe-} \right\}.
\end{align*}
\]
The borrowing constraints act as expressions for the portfolio holdings of the home household,

\[ A_{th}^h = v_{th}^{h+} \]
\[ A_{eh}^h = v_{eh}^{h+} \]
\[ B_{t}^h = v_{t}^{h+} - \min \left( P_{t+1}^{hh} W_{t+1}^h \right) , \]

the foreign household,

\[ A_{tf}^f = v_{tf}^{f+} \]
\[ A_{ef}^f = v_{ef}^{f+} \]
\[ B_{t}^f = v_{t}^{f+} - \min \left( P_{t+1}^{ff} W_{t+1}^f \right) , \]

and the bank,

\[ A_{te}^h = v_{te}^{he+} \]
\[ A_{fe}^f = v_{fe}^{he+} \]
\[ B_{t}^e = v_{t}^{be+} - \min \left[ \left( Q_{t+1}^h + P_{t+1}^{hh} D_{t+1}^h \right) A_{te}^h + \left( Q_{t+1}^f + P_{t+1}^{ff} D_{t+1}^f \right) A_{fe}^f \right] . \]

Finally, the market clearing conditions are necessary to close the model.

\[ A_{th}^h + A_{te}^h = 1 \]
\[ A_{tf}^f + A_{te}^f = 1 \]
\[ B_{t}^h + B_{t}^f + B_{t}^e = 0 \]
\[ C_{t}^{hh} + C_{t}^{hf} = Y_{t}^{h} \]
\[ C_{t}^{fh} + C_{t}^{ff} = Y_{t}^{f} \]
B.3 Solution Method

B.3.1 Homotopy

The model must be solved using homotopy, which requires placing a penalty on the six of the nine Euler equations.

\[
0 = \tau \left\{ E \left[ M^h_{t+1} \right] + v^{bh}_t - Q^h_t \right\} - (1 - \tau) (v^{bh}_t - \bar{v}^{bh}) \\
0 = \tau \left\{ E \left[ M^h_{t+1} + P^{hh}_{t+1} D^h_{t+1} \right] + v^{-hh}_t - Q^h_t \right\} - (1 - \tau) (v^{-hh}_t - \bar{v}^{hh}) \\
0 = \tau \left\{ E \left[ M^f_{t+1} R^{e}_{t+1} (1 - \tau h) \right] + v^{-eh}_t - 1 \right\} - (1 - \tau) (v^{-eh}_t - \bar{v}^{eh}) \\
0 = \tau \left\{ E \left[ M^f_{t+1} + v^{-bf}_t - Q^b_t \right\} - (1 - \tau) (v^{-bf}_t - \bar{v}^{bf}) \\
0 = \tau \left\{ E \left[ M^f_{t+1} + P^{ff}_{t+1} D^f_{t+1} \right] + v^{-ef}_t - Q^f_t \right\} - (1 - \tau) (v^{-ef}_t - \bar{v}^{ef}) \\
0 = \tau \left\{ E \left[ M^f_{t+1} R^{e}_{t+1} (1 - \tau f) \right] + v^{-ef}_t - 1 \right\} - (1 - \tau) (v^{-ef}_t - \bar{v}^{ef})
\]

The values of \( \{ \bar{v}^{bh}, \bar{v}^{hh}, \bar{v}^{eh}, \bar{v}^{bf}, \bar{v}^{ff}, \bar{v}^{ef} \} \) are chosen to match the starting guess and are true solutions when \( \tau = 0 \). The model is solved for \( \tau = 0 \), and the solutions is used for the starting guess as \( \tau \) is slowly increases. Once \( \tau = 1 \), the model’s true solution has been found.

B.3.2 Iteration process

The state variables are \( X_t = \left\{ Y^h_t, Y^f_t, \phi_t, \omega^h_t \right\} \), where \( Y^h_t \in \{ Y^h_t, Y^h_t \} \), \( Y^f_t \in \{ Y^f_t, Y^f_t \} \), \( \phi_t \in \{ \phi_t, \phi_t \} \) and \( \omega^h_t \in [0, 1] \). I need to compute cubic B-splines for the following variables:

\[
\left\{ Q^h (X_t), Q^f (X_t), P^{hh} (X_t) \right\}.
\]

I calculate the initial coefficients for the splines \( \{ \hat{c}_0^{gh}, \hat{c}_0^{gf}, \hat{c}_0^{phh} \} \) by using initial guesses for the grids \( \left\{ \hat{Q}^h_0 (X_t), \hat{Q}^f_0 (X_t), \hat{P}^{hh}_0 (X_t) \right\} \). Once these initial guesses are made,
the model is solved for all of the grid points. This gives an updated set of grids:
\[ \{ \hat{Q}_h^1 (X_t), \hat{Q}_f^1 (X_t), \hat{P}_{hh}^1 (X_t) \} \].

1) Initial Guess

Choose a value \( \varepsilon > 0 \) for the cutoff point. Use two-period model values to find the initial B-spline coefficients:

\[ \{ \rho_{qh}^0, \rho_{qf}^0, \rho_{phh}^0 \} \]

2) Solve Model

Solve the model for all grid point combinations using the previous iteration results for \( \{ \rho_{qh}^{k-1}, \rho_{qf}^{k-1}, \rho_{phh}^{k-1} \} \) where \( k \) is the current iteration:

\[
Y^h_t \in \{ Y^h_t, Y^h_h \} \\
Y^f_t \in \{ Y^f_t, Y^f_h \} \\
\phi_t \in \{ \phi_t, \phi_h \} \\
\omega_t^h \in \left\{ 0, \frac{1}{n_h}, \frac{2}{n_h}, ..., 1 \right\} \quad \text{where} \ n_h + 1 \ \text{is the number of grid points}
\]

Given the solution, there exists a new set of grids \( \{ \hat{Q}_h^k (X_t), \hat{Q}_f^k (X_t), \hat{P}_{hh}^k (X_t) \} \).

3) Check and Update Guess

Check if the new grids are close enough to the previous grids. If

\[ \| \hat{Q}_h^k - Q_h^{k-1} \| + \| \hat{Q}_f^k - Q_f^{k-1} \| + \| \hat{P}_{hh}^k - P_{hh}^{k-1} \| < \varepsilon, \]

then the code has converged and the iteration process can end. Go to step (5). Otherwise, use the new grids to solve for new cubic B-spline parameters \( \{ \rho_{qh}^k, \rho_{qf}^k, \rho_{phh}^k \} \), iteration number becomes \( k + 1 \) and return to step (2).

4) End
Finish iteration sequence.

\[ Q^h (X_t) \approx \tilde{Q}_k^h (X_t) \]
\[ Q^f (X_t) \approx \tilde{Q}_k^f (X_t) \]
\[ P^{hh} (X_t) \approx \tilde{P}_k^{hh} (X_t) \]


