

ESSAYS ON THE POLITICAL ECONOMY OF  
TRADE POLICY AND TRADE AGREEMENTS

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ABSTRACT

The first two chapters of this dissertation study the impact of the design of GATT/WTO trade agreements on the organization of domestic interest groups and explain several relevant empirical puzzles. Chapter 1 posits a model of trade policy formation, featuring endogenous firm participation in competing lobbies, and show that weak tariff bindings, as mandated by the GATT/WTO, serve to increase participation in the domestic pro-trade lobby and decrease applied tariffs. In the model the tariff ceiling works as an external constraint that reduces the off-the-equilibrium-path joint payoff of the government and the anti-trade lobby, thus reducing the contribution necessary for the pro-trade lobby to achieve a given applied tariff and thereby encouraging greater participation. The model explains the empirical puzzle, especially in small developing countries, that tariff caps even strictly above the applied tariffs tend to reduce the latter. We then extend the model to show that when a small country enters the negotiations towards a trade agreement, the pro-trade lobby group expands; and when the tariff cap stipulated by the trade agreement is imposed, the pro-trade lobby group expands further. Chapter 2 extends the model in the previous chapter to a setting with large countries. In the model countries can negotiate over a tariff cap that indirectly results in a desired level of applied tariff lower than the cap, and thereby eliminate the terms-of-trade externality with a positive “binding overhang”. Furthermore, larger countries have lower binding overhangs. The model

reconciles the discrepancy between the central role terms-of-trade motives should have played in designing GATT/WTO trade agreements and the implication of previous models that countries set unilateral optimal tariffs when the applied tariffs fall below the bound tariffs.

The third chapter of this dissertation builds the firm selection channel in Melitz (2003) into the “protection for sale” model and yields novel predictions on the relationship between a sector’s degree of firm heterogeneity and level of trade protection. We assume heterogeneous firms lobby the government for protection in the unilateral setting (as well as liberalization in the cooperative setting). A lower domestic tariff imposed on a sector will raise prices of the imported varieties and drive out relatively weaker foreign exporting firms, but will allow some less productive (and thus smaller) domestic ones to survive. In each sector, lobbying activities are (endogenously) dominated by larger firms that face the trade-off between driving out weaker foreign competitors (with a higher tariff) and weeding out less efficient domestic competitors (with a lower tariff). We are able to derive explicit formulas for the protection structures across different sectors, in both the unilateral and the cooperative setting. In particular, we link the “curvatures” of the productivity distributions of both domestic and foreign firms in a sector to the sector’s endogenous tariff level. We find that how a sector’s domestic firm heterogeneity impacts its protection level depends on whether the political economy consideration is dominant, and that how a sector’s foreign firm heterogeneity affects its protection level hinges on whether the home government sets tariffs unilaterally or cooperatively.

INDEX WORDS: Lobbying, Trade Agreements, Endogenous Trade Policy, Firm Heterogeneity

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## CHAPTER 1

### A MODEL OF LOBBY PARTICIPATION AND TARIFF BINDING OVERHANG

#### 1.1 INTRODUCTION

The history of multilateral trade liberalization since the end of the World War II has been closely linked to the General Agreement on Tariffs and Trade (GATT) and its successor the World Trade Organization (WTO). Freer trade has been achieved through tariff bindings above which contracting parties at GATT/WTO commit not to raise their tariffs in the future. Countries, especially small developing ones, often enter a trade agreement specifying a tariff cap, i.e., an upper limit on a tariff, that is far above the one they already apply, a phenomenon termed “tariff binding overhang”. A country can typically raise the applied tariff by some amount while not violating the rules of GATT/WTO. Despite the leeway for governments to adjust their tariffs upward, such weak tariff bindings are widely deemed as valuable. Furthermore, tariff caps, even with positive binding overhangs, tend to reduce the applied tariffs. Beshkar, Bond and Rho (2012) observe that following the agreements of the Uruguay Round, there is a significant reduction in applied tariffs that are already under the bindings. Bacchetta and Piermartini (2011) document the “taming effect” of tariff caps in a sample with 119 countries: when binding overhang is below 30 percent, the probability of a decrease in the applied tariff is on average nearly four times higher than that of an increase. When the overhang is above 30 percent, this “taming effect” of tariff caps largely disappears.

An emerging literature has proposed various models featuring tariff caps that are sometimes strictly above applied tariffs. When contracting over contingencies is too costly, the rigidity of trade agreements entails an upper bound on applied tariffs, so that governments may set the tariff below the bound when it unilaterally wishes to do so, as in Horn, Maggi and Staiger (2010). Or when the government holds private information regarding the optimal tariff level, a trade agreement with a tariff cap can be incentive-compatible and strictly improve upon one with a rigid tariff, as in Bagwell and Staiger (2005), Beshkar, Bond and Rho (2012) and Amador and Bagwell (2012). Amador and Bagwell (2012) further derive conditions under which the tariff cap is indeed the optimal incentive-compatible contract.

While these theories, with the assumption of uncertainty, provide interesting and realistic explanations for the design the GATT/WTO trade agreements, they nonetheless lead to the result that a tariff cap reduces the applied tariff *only* when the former coincides with the latter. As is noted by Maggi (2013)<sup>1</sup>,

(I)f tariff ceilings were due solely to the presence of non-contractible contingencies, we would expect that, for a given product in a given country, the applied tariff is sometimes at the bound level and sometimes below it, but it is not clear from existing evidence that this is actually the case in reality.

The existing theories are not able to explain the empirical pattern that tariff caps tend to reduce the applied tariff even with positive binding overhangs, and thus cannot fully account for the merits of weak tariff bindings that constitute a fundamental feature of international trade agreements.

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<sup>1</sup>The comment in this quote remains true if tariff caps were due to private information of the contracting governments.

This paper proposes a model in which a tariff ceiling strictly above the applied tariff can help the pro-trade lobby group in a small country get more “organized” relative to anti-trade one, and therefore indirectly lead to a lower applied tariff. In the model the applied tariff, as well as the binding overhang, co-moves with the bound tariff. Thus we provide the first model, as far as we know, in which a tariff cap can reduce the applied tariff even if the latter never hits the former.

We invoke four key assumptions as follows. (1) *Tariff bindings as effective commitment devices*: we assume the GATT/WTO trade agreement is an efficient commitment device for the small country’s government, that is, in the future the country cannot raise the applied tariff above the bound level. (2) *Lobbying as quid pro quo*: as in Grossman and Helpman (1994), interest groups incentivize the government with political contribution schedules, and the government trades off social welfare relative to political contributions. (3) *Lobby competition*: there are losers and winners from a lower tariff on the particular good to be considered, and the players in both groups may participate in lobbying. In particular, we work with a modified economic structure from Gawande and Olarreaga (2012), in which an upstream (anti-trade) industry benefits from a higher tariff on its good while a downstream (pro-trade) industry wants a lower tariff on the upstream intermediate good. (4) *Collective action problems*: following the fundamental insight of Olson (1965), we assume each industry faces the free rider problem, with the degree of free riding (endogenously) depending on the size distribution of firms in the industry. In particular, we assume firms *voluntarily* participate in the subsequent lobbying game, with the logic as in Dixit and Olson (2000).

First we show how a weak tariff binding can allow the pro-trade lobby group to grow relative to the anti-trade one and thus result in a lower applied tariff. Building on the contribution of Laussel and Le Breton (2001), we show in equilibrium the level

of contribution of each lobby group (whether firms in it lobby collectively or individually) depends on the joint payoff that the government and the other competing lobby group would achieve if the first lobby did not exist. Now suppose the government commits not to set its applied tariff above a bound level  $\bar{t}$ , which is higher than the tariff already in place but lower than  $t_{-1}$ , the tariff that would be chosen if the pro-trade lobby were absent. This bound tariff restricts joint payoff of the government and the anti-trade lobby group *out of equilibrium*. When the sizes of the two lobby groups are fixed, the pro-trade lobby needs to contribute less in equilibrium, because absent this group, under the constraint imposed by the trade agreement the government can set the tariff only at  $\bar{t} < t_{-1}$ , and the joint payoff of the government and the anti-trade lobby is now reduced. The equilibrium contribution of the anti-trade lobby remains the same because there is no lower limit on the applied tariff. At this point, the equilibrium applied tariff will not change although the pro-trade lobby group can “buy” a lower tariff more cheaply, for in the “truth equilibria” of the common agency game, the outcome will maximize the joint payoff of all of the players. However, we assume there is a stage *prior to* the lobbying game where firms decide whether to participate in lobbying or to free-ride. In this stage, a firm weighs the benefit of political participation (i.e., a change in the applied tariff in the favorable direction) against the cost of political contribution. As a result, each sector is only partial “organized” in that only a fraction of firms choose to attend the lobbying game. A tariff ceiling with a substantial amount of “binding overhang” can reduce the contribution that a pro-trade firm expects to pay, while not hurting its benefit from lobby participation. Meanwhile, in the model an anti-trade firm’s incentive to participate is not affected by the tariff ceiling. Therefore with the imposition of a bound tariff (or with a lower tariff cap) the size of pro-trade lobby group tends to grow while the size of anti-trade one remains fixed, leading to a lower equilibrium applied tariff in the common agency

game. We further show that the binding overhang also co-moves with the tariff ceiling. Moreover, given a tariff cap that binds off-the-equilibrium-path, the binding overhang increases with the degree of concentration of the (pro-trade) downstream sector. Intuitively, when the number of firms in the downstream industry goes to infinity, the free rider problem of this industry becomes very severe and it is as if lobby competition (a key assumption of our model) does not exist.

We then extend our model to a setting where at the stage leading to the trade agreement, firms also voluntarily participate in lobbying (for the tariff ceiling to be imposed). We envision a scenario where the small country was sitting at an equilibrium without any tariff bindings, and then the opportunity for signing a trade agreement exogenously arises. We show that at the stage leading to the trade agreement, the pro-trade lobby group will become more organized, and when the tariff ceiling is imposed, it will become further organized. In contrast, the fraction of firms in the anti-trade industry that participate in lobbying stays the same. Thus our model is able to explain not only why some sectors are more politically organized than others, as the previous literature did, but also why certain interest groups may *sometimes* appear more organized. The key assumption is that the small country takes as given that the only feasible form of the trade agreement is one that gives the government downward discretion, i.e., an upper limit on tariff. When the small country's government is negotiating the trade agreement, a fundamental asymmetry between the two competing lobby groups' bargaining positions arises: since the available commitment device for the government is an upper limit of tariff, not a lower one, in the absence of the anti-trade lobby group the government would be able to promise the pro-trade lobby a very low applied tariff in the future (by signing a trade agreement with a very low bound tariff), while without the pro-trade lobby, no matter how large the anti-trade lobby group is, it would be impossible for the government to guarantee a

very high future applied tariff, since the choice that would please the anti-trade group the most is only an arbitrarily high tariff ceiling (or no commitment at all), which will just lead to the applied tariff the same as the one in an equilibrium without any tariff binding. This constraint on the off-the-equilibrium-path joint payoff of the government and the anti-trade lobby saves political contribution for the pro-trade lobby, and thus facilitates participation of the pro-trade firms *ex ante*, just like the tariff ceiling induces more downstream firms to participate *ex post*. Moreover, we show when the (pro-trade) downstream industry is more concentrated, the trade agreement will lead to a greater tariff cut, due to less severe free-rider problems of the pro-trade industry *ex post* and hence a more stringent constraint on the joint payoff of the government and the anti-trade lobby group in the *ex ante* lobbying game.

As detailed in the main text, our model embodies Becker (1983)'s general theory on competing interest groups with micro-foundations of the Grossman-Helpman model and voluntary participation games, in the specific context of tariff binding overhangs. Becker (1983) proposed that when a competing interest group is more efficient at organizing itself, it will produce greater political pressure and thus acquire more favorable policies. In our model, the tariff ceiling is a tool for the downstream industry to better solve its collective action problem. The assumption of lobby competition over the tariff level is a realistic one. Gawande and Olarreaga (2012) find robust and quantitatively important effects of lobby competition between upstream and downstream sectors over trade policy in a sample covering over 40 countries. Furthermore, the story that *under the tariff bindings stipulated by GATT/WTO trade agreements* upstream and downstream firms influence the government and compete over the levels of the applied tariffs appears directly relevant in the real world. Ludema, Mayda and Mishra (2013) reports that the US regularly eliminates tariffs on some products (and thus exercises downward discretion under the WTO commitments) through “tariff

suspensions” which are usually initiated by a proponent in the downstream industry that intends to avoid paying tariffs on intermediate goods. The lobbying expenditure of the downstream proponent tends to increase the probability of a successful suspension while the messages and lobbying expenditures of the upstream firms tend to lower the probability<sup>2</sup>.

In viewing trade agreements as commitment devices vis-a-vis domestic interest groups, this paper is in line with the literature that emphasizes the domestic commitment motive for trade agreements. Contributions in this literature that are broadly linked with the theme of our paper include Maggi and Rodriguez-Clare (1998, 2007), Mitra (2002) and Limao and Tovar (2011). Our focus is different in that we abstract from the resource misallocation problem stemming from the lack of commitment of the government, and assumption of common agency game (with efficient transfers from lobbies to the government) implies that the government needs not “tie its hands” to enhance its bargaining position. Clearly, our concern is that the degree to which certain interest groups can overcome the collective action problem can vary in different circumstances and may depend on external constraints imposed on the country (i.e., tariff ceilings, or even the prospect of signing a trade agreement) that shape the off-the-equilibrium-path outcome.

As a by-product, we combine the logic of Dixit and Olson (2000) and the “protection for sale” model to endogenize the lobby participation decision of firms. In this way we adapt and extend the work of Kvintradze (2004). The literature has proposed various other approaches to modeling the organization of interest groups. The related papers include Bombardini (2008) and Bombardini and Trebbi (2012), which we will review in detail in the main text, Mitra (1999), in which a sector may incur

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<sup>2</sup>That paper focuses on lobbying as a means of transmitting information. With respect to lobbying expenditure, their empirical approach cannot disentangle its roles of quid pro quo and informing the government.

a fixed cost to get organized, and Pecorino (1998) and Magee (2003), in which the organization of an interest group is modeled as repeated interaction among firms.

The chapter is organized as follows. Section 1.2 lays out the baseline model that takes the tariff ceiling as given and shows it enables the pro-trade lobby to grow larger. Section 1.3 extends the model to incorporate ex ante lobby participation and endogenize the choice of the tariff ceiling, and shows the pro-trade lobby group can sequentially get more organized in each stage. Section 1.4 concludes.

## 1.2 THE BASELINE MODEL

### 1.2.1 THE ECONOMIC FRAMEWORK

A small open economy is populated with a continuum of residents with identical preferences represented by the utility function:

$$U = c_0 + u_1(c_1) + u_2(c_2),$$

where 0 denotes the numeraire and 1 and 2 denote the two goods that will be the focus of our analysis.

The total labor endowment of the economy is  $L$ . The numeraire good 0 is produced one-to-one with labor.  $L$  is large enough so that the numeraire good is always produced in equilibrium, and wage will be equal to 1.

Sector 1 is the final good sector, which will be referred to as the downstream sector (with respect to sector 2). To produce good 1, firms in this sector need to use the intermediate good 2, in addition to the sector-specific factor and labor. Specifically, good 1 is produced competitively with the Leontief technology:

$$y_1 = \min\left\{k_1, \frac{x_{21}}{\alpha}, l_1\right\},$$

where  $y_1$ ,  $k_1$ ,  $x_{21}$  and  $l_1$  respectively denote a sector 1's firm's output, the amounts of sector-specific capital, of the intermediate good 2, and of labor used in production.

Firms in sector 2 have access to the production technology:

$$y_2 = \min\{k_2, l_2\},$$

where  $y_2$ ,  $k_2$ , and  $l_2$  respectively denote output, the amounts of sector-specific capital and of labor.

Only the intermediate good 2 is traded. Let  $p^*$  denote the international price of good 2, and  $t$  be the specific import tariff when it is positive and specific import subsidy when it is negative. The trade policy  $t$  drives a wedge between good 2's domestic price  $p$  and its international price  $p^*$ :  $p = p^* + t$ .

The total stocks of the specific factors in the industry 1 and 2 are  $K_1$  and  $K_2$  respectively. When both goods are produced, the above Leontief technologies imply that the total outputs of the two sectors are  $Y_1 = K_1$  and  $Y_2 = K_2$ . Good 1 is non-traded, with domestic price  $p_1$ . The sub-utility function  $u_i(\cdot)$  implicitly gives rise to the demand function  $d_i(p_i)$  for good  $i$  ( $i = 1, 2$ ). Since the supply of good 1 is fixed at  $K_1$ , good 1's price  $p_1$  will be fixed in equilibrium.

Furthermore, the total profits earned by the two industries' firms are given by

$$\Pi_1 = p_1 K_1 - K_1 - (p^* + t)\alpha K_1, \quad (1.1)$$

$$\Pi_2 = (p^* + t)K_2 - K_2. \quad (1.2)$$

We now make further assumptions on the sub-utility functions  $u_1(\cdot)$  and  $u_2(\cdot)$ . We assume they implicitly define linear demand functions. Given domestic prices  $p_1$  and  $p$ , the demand functions for good 1 and 2 are given by, respectively,  $d_1(p_1) = D_1 - b_1 p_1$  and  $d_2(p) = D_2 - b_2 p$ . The import demand for good 2 is thus  $m = d_2(p) + \alpha K_1 - Y_2 = D_2 - b_2 p + \alpha K_1 - K_2$ , which takes account of the  $\alpha K_1$  units of good 2 used as the intermediate good.

Social welfare can be represented as the sum of consumer surplus, producer's profits and tariff revenue. Consumer surplus from consuming the non-numeraire good  $i$  is  $S_i(p_i) = u_i(d_i(p_i)) - p_i d_i(p_i)$  ( $i = 1, 2$ ), where  $p_2 = p$  denotes the domestic price of good 2. Tariff revenue collected from sector 2 is given by  $TR = tm$ . Therefore, gross social welfare can be written as  $W = L + S_1 + S_2 + TR + \Pi_1 + \Pi_2$ .

### 1.2.2 COMMENT ON THE ECONOMIC STRUCTURE

Two features of the above economic structure are worth noting. First, given that we are interested in explaining why weak tariff bindings tend to reduce the applied tariff with positive “overhang”, the assumption of small country is a natural one, since such phenomena are observed mostly in small developing countries. Second, this simple framework (linear demand and Leontief production functions) will generate the result that, *absent external constraints* (i.e., tariff ceilings), the share of firms that participate in lobbying in a sector will depend *only* on the sector's firm size distribution. Thus the model intends to capture this very important factor that affects an industry's ability to overcome the collective action problem. The impact of an industry's firm size distribution on how well it is political organized has long been observed in the literature (Olson (1965) and Stigler (1974)), with the recent empirical support from Bombardini (2008) in the context of endogenous trade policy.

### 1.2.3 LOBBYING

We propose a framework where each sector is only partially “organized”, in that some firms do not participate in lobbying and thus free-ride on other firms' political

contributions. We formalize the logic of free-riding with a “voluntary participation” game, as in Dixit and Olson (2000)<sup>34</sup>.

The tariff-setting game  $\Gamma$  consists of three stages:

**Stage 1:** Each firm in both the upstream and the downstream industries simultaneously determines whether to enter the next stage.

**Stage 2:** In each industry, the firms that have in the previous stage decided to participate have a “meeting”, determine the contribution schedule  $Contr_i(t)$  they jointly offer to government (where  $i$  denotes the industry and  $t$  the tariff vector). Firms in each group share the political contributions according an exogenous rule  $\mathcal{R}$  to be defined later. Each meeting yields an efficient agreement (for all the participants in the relevant sector). As a result, *two lobbies with conflicting interests emerge*.

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<sup>3</sup>One alternative model that generates the relationship between firm size distribution and the size of the lobby group of an industry is the one in Bombardini (2008). In that model, firms are the principals offering the government contribution schedules, but in order to pay the first dollar of political contribution to the government, a firm must incur a fixed cost. Thus it is only efficient for firms above a certain size to contribute. When firm sizes are more heterogeneous, firms above this size cut-off will occupy a larger share (in terms of sector-specific capital) of the industry, so the industry will appear more “organized”. However, there is one potential problem: when the fixed cost political contribution goes to zero, all firms will be active in the political game and no matter how the firm sizes are distributed, every industry will appear fully organized! Yet in the context of private provision of a nonexcludable public good, one would expect that free-riding should always to some extent exist, even when there is no fixed costs of contribution. For example, Ludema and Mayda (2009)(2013) find substantial free-riding on the GATT/WTO’s MFN clause when countries negotiate trade agreements. So the Bombardini (2008) model, while generating interesting results with reasonable assumptions, fails to capture the deeper nature of free-riding. Again, the reason is that the “truthful equilibria” of common agency games do not exhibit free rider problems. Laussel and Le Breton (2001) point out that to use this solution concept is as if to assume the principals engage in “log-rolling deals”, but are subject to some credibility constraints. Our model generates free-riding with a voluntary participation stage *before* the common agency game.

<sup>4</sup>At this point, a natural concern is whether the main results in this paper can also be generated by a model based on the Bombardini (2008) model. This is unlikely, since if we do not have the stage of voluntary participation, given linear demand and fixed production, the tariff cap will change the applied tariff *only* when these two coincide. This further confirms the importance of free-riding in driving our results.

**Stage 3:** The government sets the tariff, trading off social welfare against political contributions, with the objective function being  $aW(t) + Contr(t)$ , where  $Contr(t) = \sum_{i=1,2} Contr_i(t)$  is the total amount of contribution it will receive from the two competing lobbies when choosing tariff  $t$ . Then production takes place, prices are determined under the tariff chosen, firms earn profits and consumers enjoy the surpluses.

To complete the definition of game  $\Gamma$ , we need to formally define the sharing rule  $\mathcal{R}$  in stage 2.

**Definition 1.1** Let  $S^n = \{(\gamma_1, \dots, \gamma_n) : \gamma_i > 0 (\forall i), \sum_{i=1}^n \gamma_i = 1\}$ . A *sharing rule*  $\mathcal{R}$  is a sequence of mappings  $\{f^n\}_{n=1}^\infty$ , with  $f^n : S^n \rightarrow S^n$ , that satisfy the following requirement. For any  $n$ , let  $\gamma = (\gamma_1, \dots, \gamma_n) \in S^n$  and  $f^n(\gamma) = (f_1^n(\gamma), \dots, f_n^n(\gamma))$ . If  $\gamma_i = \gamma_j$  ( $1 \leq i, j \leq n$ ), then  $f_i^n(\gamma) = f_j^n(\gamma)$ .

According to this definition, for any group with the share of each group member's size in the whole group given, a sharing rule  $\mathcal{R}$  will dictate the share of contribution for any group member. Moreover, we require that group members with the same sizes share the contribution equally. For example, a *proportional sharing rule*  $\mathcal{R}^P$  stipulates that a firm's share of contribution should be proportional to its size:  $f_i^n(\gamma) = \frac{\gamma_i}{\sum_{j=1}^n \gamma_j}$ , where  $1 \leq i \leq n$  and  $n = 1, 2, 3, \dots$ .

Following the literature, we focus on “truthful equilibria” as defined in Bernheim and Whinston (1986). With this solution concept, we assume that for any tariff chosen by the government, if a lobby makes a positive contribution, it will enjoy the same payoff as in equilibrium.

#### 1.2.4 THE EQUILIBRIUM TARIFF

We look for the subgame perfect equilibrium of game  $\Gamma$  and solve the game with backward induction. To simplify the analysis we make the following (sensible) assump-

tion: the owners of each industry's specific factor occupy a negligible share of the population. Therefore every lobby aims to maximize the total profit of its member firm(s), i.e., it does not take account of its role as consumers when lobbying.

Let the participation share of lobby  $i$  be  $\Theta_i$  ( $i = 1, 2$ ), that is,

$$\Theta_i = \frac{K_{Li}}{K_i},$$

where  $K_{Li}$  denotes the total amount of specific factor owned by lobby  $i$ 's members.

It is well known that in the small open economy, the marginal welfare change with respect to tariff is:

$$\frac{\partial W}{\partial t} = -\frac{dm}{dp}t = -b_2t.$$

Using equations in (1.1), the marginal change in the welfare (profit) of lobby 1's members with respect to the upstream tariff  $t$  is given by

$$\frac{\partial(\Theta_1\Pi_1)}{\partial t} = -\Theta_1\alpha K_1, \quad (1.3)$$

and the marginal change in the welfare (profit) of lobby 2's members with respect to  $t$  is

$$\frac{\partial(\Theta_2\Pi_2)}{\partial t} = \Theta_2 K_2. \quad (1.4)$$

Equations (1.3) and (1.4) highlight competition between the two lobbies : lobby 1's firms use intermediate good 2 and benefit from a lower  $t$ , while lobby 2's firms lose from freer trade of the good they produce.

In "truthful equilibria" the government will choose a tariff that maximizes the joint welfare of itself and two competing lobbies. The first order condition is  $\frac{\partial(aW+\Theta_1\Pi_1+\Theta_2\Pi_2)}{\partial t} = 0$ , where  $a$  is the weight the government places on social welfare relative to lobby contributions. Then the equilibrium tariff imposed on good 2 can then be viewed as a function of lobby participation shares  $\Theta_1$  and  $\Theta_2$

$$t^O(\Theta_1, \Theta_2) = \left(\frac{K_2}{ab_2}\right)\Theta_2 - \left(\frac{\alpha K_1}{ab_2}\right)\Theta_1. \quad (1.5)$$

### 1.2.5 LOBBY CONTRIBUTIONS

Without any external constraint on the applied tariff, if pro-trade lobby 1 did not exist, the government would have set the tariff at  $t_{-1}$  that maximizes the joint payoff of itself and the competing anti-trade lobby 2. Let  $\Theta_1 = 0$  in the equilibrium tariff condition (1.5), we can write  $t_{-1}$  as a function of the anti-trade lobby's size  $\Theta_2$ :

$$t_{-1}(\Theta_2) = \frac{K_2}{ab_2}\Theta_2. \quad (1.6)$$

Similarly, if anti-trade lobby 2 did not exist, the government would have chosen a (minus) tariff  $t_{-2}$  that maximizes the joint payoff of itself and the pro-trade lobby 1. Let  $\Theta_2 = 0$  in (1.5), we have

$$t_{-2}(\Theta_1) = -\frac{\alpha K_1}{ab_2}\Theta_1. \quad (1.7)$$

We now use Laussel and Le Breton (2001)'s result (Proposition 4.4) to pin down equilibrium contributions. In equilibrium, the political contribution of the anti-trade lobby 2 compensates for the joint welfare loss of the government and the pro-trade lobby 1 due to the anti-trade lobby's existence. Given the participation shares  $\Theta_1$  and  $\Theta_2$ , the equilibrium contribution of the anti-trade lobby 2 is

$$C_2^* = \underbrace{\max_t \{aW(\cdot) + \Theta_1 \Pi_1(\cdot)\}}_{\text{Joint payoff of the government and the pro-trade lobby 1 if the anti-trade lobby 2 were absent.}} - \underbrace{(aW(t^O(\Theta_1, \Theta_2)) + \Theta_1 \Pi_1(t^O(\Theta_1, \Theta_2)))}_{\text{Joint payoff of the government and the pro-trade lobby 1 in equilibrium.}}$$

This contribution turns out to be quadratic in the anti-trade lobby's size  $\Theta_2$ :

$$C_2^*(\Theta_2) = \int_{t^O(\Theta_1, \Theta_2)}^{t_{-2}(\Theta_1)} \left( a \frac{\partial W}{\partial t} + \frac{\partial(\Theta_1 \Pi_1)}{\partial t} \right) dt = \frac{1}{2} \left( \frac{K_2^2}{ab_2} \right) \Theta_2^2. \quad (1.8)$$

Analogously, the equilibrium contribution of lobby 1 is (without any tariff ceiling)

$$C_1^* = \underbrace{\max_t \{aW(\cdot) + \Theta_2 \Pi_2(\cdot)\}}_{\text{Joint payoff of the government and the anti-trade lobby 2 if the pro-trade lobby 1 were absent.}} - \underbrace{\left( aW(t^O(\Theta_1, \Theta_2)) + \Theta_2 \Pi_2(t^O(\Theta_1, \Theta_2)) \right)}_{\text{Joint payoff of the government and the anti-trade lobby 2 in equilibrium.}}$$

It also depends only on this lobby's size:

$$C_1^*(\Theta_1) = \frac{1}{2} \left( \frac{\alpha^2 K_1^2}{ab_2} \right) \Theta_1^2. \quad (1.9)$$

### 1.2.6 VOLUNTARY LOBBY PARTICIPATION

We now go back to Stage 1 of the tariff setting game  $\Gamma$ , where firms voluntarily choose whether to attend the lobbying game in the next stage. Given the decisions of other firms, the benefit of a firm's participation is the increase in its profit that comes from the change in tariff due to this firm's participation. We define this benefit to be a firm's *willingness to pay*. Formally, let  $k_i$  denote the amount of specific factor owned by a firm in sector  $i$  and  $\theta_i \frac{k_i}{K_i}$  denote the firm's share of specific factor in its sector ( $i = 1, 2$ ). Given the two sectors' participation shares  $\Theta_1$  and  $\Theta_2$ , a firm's willingness to pay in sector 1 and 2 can be respectively written as

$$w_1(\theta_1) = \theta_1 \Pi_1(t^O(\Theta_1, \Theta_2)) - \theta_1 \Pi_1(t^O(\Theta_1 - \theta_1, \Theta_2)) = \left( \frac{\alpha^2 K_1^2}{ab_2} \right) \theta_1^2, \quad (1.10)$$

$$w_2(\theta_2) = \theta_2 \Pi_2(t^O(\Theta_1, \Theta_2)) - \theta_2 \Pi_2(t^O(\Theta_1, \Theta_2 - \theta_2)) = \left( \frac{K_2^2}{ab_2} \right) \theta_2^2. \quad (1.11)$$

In this model with linear demand and fixed output, a firm's willingness to pay is quadratic in its size (represented by  $\theta_i$ ,  $i = 1, 2$ ). Relative to their sizes, larger firms have a greater willingness to pay, and therefore are more likely to participate in lobbying. This property will drive the result that in a more concentrated industry a greater fraction of firms will participate.

A firm  $f$  will enter Stage 2 of the game if its willingness to pay outweighs its political contribution in that stage:

$$w_i(\theta_f) \geq c_f, \quad (1.12)$$

where  $\theta_f$  denotes the firm's share of size in whole sector and  $c_f$  denotes the contribution the firms will pay when participating.

Let  $\mathcal{L}_i$  be a group of firms in sector  $i$  ( $i = 1, 2$ ). Define the group  $\mathcal{L}_i$ 's *total willingness to pay* as the sum of firms' individual willingness to pay in this group:

$$\mathcal{W}_i = \sum_{f \in \mathcal{L}_i} w_i(\theta_f). \quad (1.13)$$

Now suppose in equilibrium,  $\mathcal{L}_i$  is the set of participating firms of sector  $i$  ( $i = 1, 2$ ). No matter how the firms share their contribution, the lobby group's total willingness to pay  $\mathcal{W}_i$  must be no less than its equilibrium political contribution, otherwise there must be at least one firm for which equation (1.12) does not hold, and this firm would like to deviate by choosing to free ride in the first stage. Therefore the following conditions must hold in equilibrium:

$$\mathcal{W}_i \geq C_i^*(\Theta_i). \quad (1.14)$$

At this point, multiple equilibria of the model may exist. In section 1.2.8 we will assume that firms are identical and pin down the unique closed form solution. Now we keep the model general and ask the following question: how large can a lobby group get in equilibrium? We state the following proposition.

**Proposition 1.1** *Suppose there is no tariff cap.*

(1) *If  $\mathcal{L}_i$  ( $i = 1, 2$ ) is the set of sector  $i$ 's firms that participate in lobbying, then  $\sum_{f \in \mathcal{L}_i} (\frac{\theta_f}{\Theta_{\mathcal{L}_i}})^2 \geq \frac{1}{2}$  (where  $\Theta_{\mathcal{L}_i} = \sum_{f \in \mathcal{L}_i} \theta_f$  denotes the size of group  $\mathcal{L}_i$ ), that is, the Herfindahl index of the size distribution of  $\mathcal{L}_i$ 's members must be no less than  $\frac{1}{2}$ .*

(2) An equilibrium of game  $\Gamma$  can be constructed in the following way. For each sector, starting with an empty lobby group, we add new firms to the group while keeping the group's Herfindahl index weakly above  $\frac{1}{2}$ , until this is no longer possible<sup>5</sup>. Now we have created two lobby groups  $\mathcal{L}_1$  and  $\mathcal{L}_2$ . There exists a sharing rule  $\mathcal{R}$  such that associated with  $\mathcal{R}$ ,  $\mathcal{L}_1$  and  $\mathcal{L}_2$  are equilibrium lobby groups.

**Proof.** See Appendix A.1. ■

Part (1) of the above proposition gives us the necessary condition for  $\mathcal{L}_i$  to be the equilibrium lobby group. It says the concentration of *this group* (as is measured by its Herfindahl index) cannot be too low (otherwise some firm in the group would be better off with free-riding). This result is directly derived from (1.14). Part (2) concerns the largest equilibrium lobby group of a sector if we consider all possible sharing rules<sup>6</sup>. In a nutshell, it says that in equilibrium a lobby group of a sector can be as large as possible as long as its Herfindahl index does not drop below  $\frac{1}{2}$ .

### 1.2.7 THE TARIFF CEILING AS AN OFF-THE-EQUILIBRIUM-PATH CONSTRAINT

Suppose now the trade agreement of GATT/WTO imposes a tariff ceiling  $\bar{t}$ , such that the home country must set an applied tariff below  $\bar{t}$ . We now show for any given participation shares  $\Theta_1$  and  $\Theta_2$ , the tariff cap can reduce the political contribution of the pro-trade lobby group 1. This is because when lobby 1 is absent, the joint welfare loss of the anti-trade lobby 2 and the government will be constrained by  $\bar{t}$ .

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<sup>5</sup>Note when we drop firms into a group, the group's Herfindahl index will decrease.

<sup>6</sup>In a model with a single manufacturing sector, Kvintradze (2004) shows that under the *proportional sharing rule*  $\mathcal{R}^P$  we have defined, only the largest firm will participate in lobbying for tariff if there is a single largest firm in the sector. Proposition 1.1 thus provides an extension to her result. For example, Proposition 1.1 immediately implies that if a sector has a firm that occupies 71% of the sector's specific factor, then there is an equilibrium (with a sharing rule different from  $\mathcal{R}^P$ ) where this sector is fully organized, i.e., all firms participate in lobbying, no matter how large the number of firms in this sector is and how the sizes of these firms are distributed.

Suppose  $\bar{t} \leq t_{-1}(\Theta_2)$  and  $\bar{t} \geq t^O(\Theta_1, \Theta_2)$ , i.e., the tariff ceiling  $\bar{t}$  would be low enough to constrain the off-the-equilibrium-path applied tariff chosen by the government if the pro-trade lobby group were absent, but is high enough to be above the equilibrium applied tariff. Then the pro-trade lobby 1 contributes

$$\begin{aligned} \tilde{C}_1 &= \underbrace{\max_{t \leq \bar{t}} \{aW(\cdot) + \Theta_2 \Pi_2(\cdot)\}}_{\text{Joint payoff of the government and lobby 2 if lobby 1 were absent, constrained by } \bar{t}.} \\ &\quad - \underbrace{\{aW(t^O(\Theta_1, \Theta_2)) + \Theta_2 \Pi_2(t^O(\Theta_1, \Theta_2))\}}_{\text{Joint payoff of the government and lobby 2 in equilibrium.}} \end{aligned}$$

The pro-trade contribution under the tariff ceiling can be decomposed as:

$$\begin{aligned} \tilde{C}_1 &= \underbrace{\left( \max_t \{aW(\cdot) + \Theta_2 \Pi_2(\cdot)\} - \{aW(t^O(\Theta_1, \Theta_2)) + \Theta_2 \Pi_2(t^O(\Theta_1, \Theta_2))\} \right)}_{\text{The contribution would have been paid without } \bar{t}} \\ &\quad - \underbrace{\left( \max_t \{aW(\cdot) + \Theta_2 \Pi_2(\cdot)\} - \max_{t \leq \bar{t}} \{aW(\cdot) + \Theta_2 \Pi_2(\cdot)\} \right)}_{\text{Reduction in the pro-trade political contribution due to } \bar{t}.} \\ &= C_1^*(\Theta_1) - \frac{ab_2}{2} (t_{-1}(\Theta_2) - \bar{t})^2. \end{aligned} \tag{1.15}$$

Comparing equation (1.15) with (1.9), we see that due to the tariff ceiling, the political contribution of the pro-trade lobby 1 (given its size) decreases by the amount  $\frac{ab_2}{2} (t_{-1}(\Theta_2) - \bar{t})^2$ . This is intuitive, since the lower  $\bar{t}$  is, the less joint payoff the government and lobby 2 can achieve absent lobby 1, and therefore the larger reduction in lobby 1's political contribution the tariff ceiling can induce. Also note that when the tariff cap is above  $t_{-1}(\Theta_2)$ , it will have no effect because even out of equilibrium it will not be binding.

Given  $\Theta_1$  and  $\Theta_2$ , the anti-trade lobby 2 will still contribute  $C_2^*(\Theta_2)$  (defined in (1.8)), which is the contribution level without the tariff ceiling. This is because there is no lower constraint on the applied tariff, and thus in the absence of lobby 2 the government can still choose the same off-the-equilibrium-path tariff  $t_{-2}(\Theta_1)$  and the

contribution of the anti-trade lobby 2 is still pinned down in the same way as in the case with no tariff cap.

Now we consider a group's total willingness to pay, namely, the sum of group members' gains from participating in this group. Consider a group of the anti-trade sector 2's firms denoted by  $\mathcal{L}_2$ . There is no reason that  $\mathcal{L}_2$ 's total willingness to pay should be different from that defined in (1.13) and therefore in the anti-trade sector 2 firms' incentives to participate in lobbying will never be hurt by the tariff ceiling .

For the pro-trade sector 1 we need the following condition to guarantee the tariff ceiling not affect the firms' incentive to join the lobby group.

**Condition 1.1** *For given participation shares  $\Theta_1$  and  $\Theta_2$  and a set of industry 1's firms  $\mathcal{L}_1$  with  $\sum_{f \in \mathcal{L}_1} \theta_f = \Theta_1$ ,  $t^O(\Theta_1 - \theta_f, \Theta_2) \leq \bar{t}$ ,  $\forall f \in \mathcal{L}_1$ . ( $\theta_f = \frac{k_f}{K_1}$  denotes a firm  $f$ 's share of the specific factor).*

This condition says if any member in the pro-trade lobby group  $\mathcal{L}_1$  had chosen to stay out in the first stage, given the participation of other group members, the equilibrium tariff would not change so much as to hit  $\bar{t}$ . When this is satisfied, we can still use the (1.13) to represent group  $\mathcal{L}_1$ 's total willingness to pay as in the case without any tariff cap. We will see this condition is satisfied when firms are identical.

Let the set of the pro-trade sector 1's firms that have chosen to participate be  $\mathcal{L}_1$ . Then it must be that their total willingness to pay is no less than the lobby's equilibrium contribution:

$$\mathcal{W}_1 \geq \tilde{C}_1. \quad (1.16)$$

With (1.15) that pins down the pro-trade lobby 1's contribution level under the tariff ceiling, equation (1.16) is reduced to

$$\sum_{f \in \mathcal{L}_1} \left( \frac{\theta_f}{\Theta_1} \right)^2 \geq \frac{1}{2} - \frac{1}{\left( \frac{\alpha^2 K_1^2}{ab_2} \right) \Theta_1^2} \frac{ab_2}{2} (t_{-1}(\Theta_2) - \bar{t})^2. \quad (1.17)$$

The above analysis has led to

**Proposition 1.2** *Let  $\mathcal{L}_1$  and  $\mathcal{L}_2$  denote two groups in sectors 1 and 2 with sizes  $\Theta_{\mathcal{L}_1}$  and  $\Theta_{\mathcal{L}_2}$ . Suppose condition 1.1 is satisfied. The trade agreement of GATT/WTO imposes a tariff ceiling  $\bar{t}$  on the upstream good 2.*

(1) *Let  $\mathcal{L}_1$  ( $\mathcal{L}_2$ ) be the set of pro-trade (anti-trade) firms that participate in lobbying in equilibrium. Then the Herfindahl indices must satisfy  $\sum_{f \in \mathcal{L}_1} \left( \frac{\theta_f}{\Theta_{\mathcal{L}_1}} \right)^2 \geq \frac{1}{2} - \frac{1}{\left( \frac{\alpha^2 K_1^2}{ab_2} \right) \Theta_{\mathcal{L}_1}^2} \frac{ab_2}{2} (t_{-1}(\Theta_{\mathcal{L}_2}) - \bar{t})^2$  and  $\sum_{f \in \mathcal{L}_2} \left( \frac{\theta_f}{\Theta_{\mathcal{L}_2}} \right)^2 \geq \frac{1}{2}$ .*

(2) *An equilibrium can be constructed in the following way. For the pro-trade sector 1 (anti-trade sector 2), starting with an empty lobby group, we add new firms in this sector to the group while keeping the group's Herfindahl index weakly above  $\frac{1}{2} - \frac{1}{\left( \frac{\alpha^2 K_1^2}{ab_2} \right) \Theta_1^2} \frac{ab_2}{2} (t_{-1}(\Theta_2) - \bar{t})^2$  ( $\frac{1}{2}$  for the anti-trade sector 2), until this is no longer possible. Now we have created two lobby groups  $\mathcal{L}_1$  and  $\mathcal{L}_2$ . There exists a sharing rule  $\mathcal{R}$  such that associated with  $\mathcal{R}$ ,  $\mathcal{L}_1$  and  $\mathcal{L}_2$  are the equilibrium pro-trade and anti-trade lobby groups, respectively.*

Proposition 1.2 needs to be understood in contrast to proposition 1.1, in which there is no external constraint imposed on the country. When the country's government commits not to set the tariff above  $\bar{t}$ , the necessary condition for the Herfindahl index of the equilibrium pro-trade lobby group 1 is now *relaxed*, while that for the anti-trade one remains the same. Also, starting from an equilibrium with no tariff ceiling, we can now drop some additional firms into the pro-trade lobby group while keeping the anti-trade lobby group unchanged, and in this way come up with an equilibrium under the tariff ceiling. This implies pro-trade lobby tends to grow relatively stronger than the anti-trade one.

The result that the pro-trade sector 1 can better solve its collective action problem echoes Becker (1983)'s theory on competition among interest groups (although our model sharply focuses on weak tariff bindings). Specifically, he proposed (pp.379):

If a group became more efficient at producing pressure, perhaps because of greater success at controlling free riding or at using television and other media, its optimal production of pressure would be raised for any level of pressure by the other group.

Surprisingly, the tariff ceiling on the upstream good 2 serves as a tool to make the pro-trade downstream sector 1 “more efficient at producing pressure”, because it lowers the cost of pro-trade political participation while does not hurt a pro-trade firm's gain from joining the relevant lobby group (or the “willingness to pay”), under the fairly general condition 1.1. And precisely as Becker (1983) has predicted, given the strength of the anti-trade lobby 2, the pro-trade lobby 1 tends to produce a greater level of pressure (represented by a larger size of this lobby).

Proposition 1.2 also implies that the tariff ceiling on good 2 does not change the pressure produced by the anti-trade sector 2. This is intuitive:  $\bar{t}$  has nothing to do with the off-the-equilibrium-path joint payoff of the government and the pro-trade lobby 1, so it does not affect the cost of lobbying of the anti-trade lobby 2, and thus the necessary and (almost) sufficient condition for the group size of lobby 2 is not affected.

At this point it is straightforward to see, from the equilibrium tariff equation (1.5), that the tariff ceiling can result in a lower applied tariff on the upstream good 2, due to the change in the relative strengths of the competing lobbies. Note this is achieved with a positive “binding overhang”! This result again embodies Becker (1983)'s general idea about the influence of lobby competition on the government's policy choice: A

group that becomes better at organizing itself relative to the competing group will be able to obtain more favorable policies<sup>7</sup>.

### 1.2.8 CLOSED FORM SOLUTION WITH IDENTICAL FIRMS

In this section we pin down the closed form solution with identical firms. We show how the tariff ceiling affects the sizes and contributions of the competing lobbies, the applied tariff and the binding overhang.

**Assumption 1.1** *Sector  $i$  consists of  $N_i$  identical firms ( $i = 1, 2$ ). Also we allow the measure of firms to be continuous.*

Since firms are of the same sizes in each sector, according to definition 1.1, any sharing rule requires that the participating firms in a lobby share the contribution evenly.

#### EFFECT OF THE TARIFF CAP ON LOBBY PARTICIPATION

Suppose lobby group  $i$  consists of  $n_i$  firms ( $i = 1, 2$ ). The participation share of sector  $i$  is therefore  $\Theta_i = \frac{n_i}{N_i}$ . A total willingness to pay of a sector  $i$ 's lobby group  $\mathcal{W}_i$  defined in (1.13) can be written as a function of its size  $\Theta_i$ :

$$\mathcal{W}_1(\Theta_1) = n_1 \left( \frac{1}{N_1} \Pi_1(t^O(\Theta_1, \Theta_2)) - \frac{1}{N_1} \Pi_1(t^O(\Theta_1 - \frac{1}{N_1}, \Theta_2)) \right) = \left( \frac{\alpha^2 K_1^2}{ab_2} \right) \frac{\Theta_1}{N_1}, \quad (1.18)$$

$$\mathcal{W}_2(\Theta_2) = n_2 \left( \frac{1}{N_2} \Pi_2(t^O(\Theta_1, \Theta_2)) - \frac{1}{N_2} \Pi_2(t^O(\Theta_1, \Theta_2 - \frac{1}{N_2})) \right) = \left( \frac{K_2^2}{ab_2} \right) \frac{\Theta_2}{N_2}. \quad (1.19)$$

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<sup>7</sup>The proposition 1 of the Becker (1983) paper states:“group that becomes more efficient at producing political pressure would be able to reduce its taxes or raise its subsidy.” The following corollary states:“ The political effectiveness of a group is mainly determined not by its absolute efficiency-e.g., its absolute skill at controlling free riding-but by its efficiency relative to the efficiency of other groups.”

Given participation share  $\Theta_i$  (and keeping the total amount of the sector  $i$ 's specific factor fixed), a lobby group's total willingness to pay  $\mathcal{W}_i$  decreases in  $N_i$ , the number of firms in this sector. When a group is composed of more firms that are each smaller, every firm has less stake and will have a smaller influence over the tariff if it participates. These two factors combined reduce the total willingness to pay of the group.

By assuming the measure of firms to be continuous, we also allow the measure of participation shares in each sector to be continuous. Since the identical firms in the same lobby group share their political contribution evenly, there will be a unique equilibrium in terms of sizes of the competing lobbies. In the absence of any tariff ceiling, for sector  $i$ , we solve for the measure of equilibrium participation share  $\Theta_i^*$  by equating the lobby group's total willingness to pay  $\mathcal{W}_i(\Theta_i)$  with equilibrium contribution  $C_i^*(\Theta_i)$  ( $i = 1, 2$ ):

$$\mathcal{W}_i(\Theta_i^*) = C_i^*(\Theta_i^*). \quad (1.20)$$

The above equation leads to the solution for equilibrium participation shares without the tariff ceiling<sup>8</sup>

$$\Theta_i^* = \frac{2}{N_i}, \quad (1.21)$$

that is, the model predicts that the measure of participating firms in sector  $i$  is 2.

We now consider the effect of the tariff ceiling  $\bar{t}$ . Suppose for the moment that condition 1.1 is satisfied in equilibrium, so that  $\bar{t}$  does not change any pro-trade firm's and thus the pro-trade lobby group's willingness to pay. The equilibrium contribution of the pro-trade lobby should now be  $\tilde{C}_1$  in (1.15) instead of  $C_1^*$  in (1.9), since the former takes account of the effect of the constraint  $\bar{t}$  out of equilibrium. Equating the

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<sup>8</sup>Note the free-riding effect generated by this model seems too severe. This is because we assume firms voluntarily participate in lobbying and abstract from other mechanism that may punish free-riders. Saijo and Yamato (1999) and Dixit and Olson (2000) also find very serious free rider problems in models of voluntary participation.

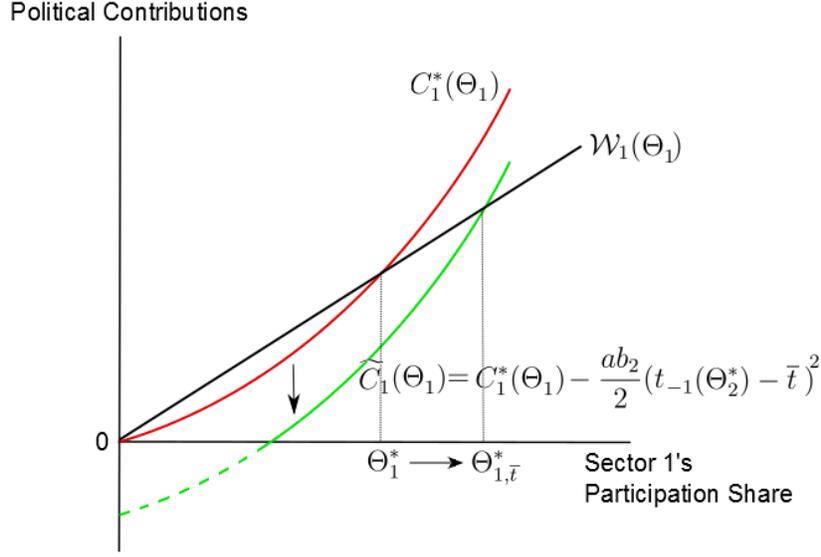


Figure 1.1: Effects of the tariff ceiling on the lobby participation of pro-trade firms.

pro-trade lobby 1's total willingness to pay  $\mathcal{W}_1$  with its contribution  $\tilde{C}_1$ , we solve for the equilibrium pro-trade participation share under the tariff ceiling  $\Theta_{1,\bar{t}}^*$  :

$$\mathcal{W}_1(\Theta_{1,\bar{t}}^*) = \tilde{C}_1, \quad (1.22)$$

which can be written as

$$\left(\frac{\alpha^2 K_1^2}{ab_2}\right) \frac{\Theta_{1,\bar{t}}^*}{N_1} = \frac{1}{2} \left(\frac{\alpha^2 K_1^2}{ab_2}\right) \Theta_{1,\bar{t}}^{*2} - \frac{ab_2}{2} (t_{-1}(\Theta_2^*) - \bar{t})^2, \quad (1.23)$$

where  $\Theta_2^* = \frac{2}{N_2}$  as in (1.21).

When the GATT/WTO trade agreement imposes a tariff ceiling, what happens to the lobby participation of firms in the pro-trade sector 1 can be summarized in Figure 1.1. Without the tariff ceiling, the equilibrium participation share is at the intersection of the curves  $\mathcal{W}_1(\cdot)$  and  $C_1^*(\cdot)$ . For example, when the participation share  $\Theta_1$  is such that the  $\mathcal{W}_1(\cdot)$  curve is above  $C_1^*(\cdot)$ , i.e., the group's total willingness to

pay exceeds the its political contribution, there must be some firm outside the group that would be better off by choosing to join the group in stage 1 of game  $\Gamma$ . When the home country commits not to set the tariff above  $\bar{t}$  at GATT/WTO, given any participation share  $\Theta_1$ , the pro-trade lobby 1 ends up paying less political contribution (if it contributes a positive amount), and by (1.15) the reduction in lobby expenditure is  $\frac{ab_2}{2}(t_{-1}(\Theta_2^*) - \bar{t})^2$ , which represents the distance of the downward shift from  $C_1^*(\cdot)$  to  $\tilde{C}_1(\cdot)$ . Moreover, when condition 1.1 is satisfied, the total willingness to pay given a certain group size is still  $\mathcal{W}_1(\cdot)$ . As a result, a greater measure of firms in the pro-trade sector 1 participate in lobbying and the participation share moves right from  $\Theta_1^*$  to  $\Theta_{1,\bar{t}}^*$ . Meanwhile, there is no reason that the corresponding curves for sector 2,  $\mathcal{W}_2(\cdot)$  and  $C_2^*(\cdot)$ , should be affected by  $\bar{t}$  in any way. This is because  $\bar{t}$  is an upper constraint on tariff, and it has nothing to do with the off-the-equilibrium-path tariff if the anti-trade 2 lobby were not present. Therefore the equilibrium participation share of sector 2 always satisfies equation (1.20), which yields  $\Theta_2^* = \frac{2}{N_2}$ .

Now we show with identical firms, condition 1.1 holds when the applied tariff co-moves with the tariff ceiling. Let

$$\tilde{t} = t_{-1}(\Theta_2^*) - \left( \frac{\alpha K_1}{ab_2} \right) \cdot 1,$$

$$\tilde{\tilde{t}} = t_{-1}(\Theta_2^*) - \left( \frac{\alpha K_1}{ab_2} \right) \left( \frac{(N_1 - 1)^2 - 1}{N_1^2} \right)^{1/2} > \tilde{t}.$$

In the model,  $\tilde{t}$  is the equilibrium tariff when all firms in pro-trade sector 1 participate in lobbying, and  $\tilde{\tilde{t}}$  is the level of tariff ceiling that induces this full participation of sector 1's firms.

We show in the Appendix the following lemma:

**Lemma 1.1** *When  $\tilde{\tilde{t}} \leq \bar{t} \leq t_{-1}(\Theta_2^*)$ , the participation shares  $\Theta_2^*$  and  $\Theta_{1,\bar{t}}^*$  satisfy condition 1.1 (and thus represent the equilibrium sizes of the lobby groups).*

Lemma 1.1 says as  $\bar{t}$  becomes lower, before the pro-trade sector 1 gets fully organized, the difference between  $\bar{t}$  and the equilibrium applied tariff will be large enough so that when a pro-trade firm quits the lobby group, the tariff will not hit  $\bar{t}$ . Thus it is indeed the case that the total willingness pay of an the pro-trade lobby group 1 with size  $\Theta_{1,\bar{t}}^*$  is still given by  $\mathcal{W}_1(\Theta_{1,\bar{t}}^*)$ , and change in lobby participation (and tariff) only results from the downward shift of pro-trade lobby 1's political contributions as in Figure 1.1.

We summarize this section's result in the following proposition:

**Proposition 1.3** *Under assumption 1.1, when the tariff ceiling  $\bar{t}$  satisfies  $\tilde{t} \leq \bar{t} \leq t_{-1}(\frac{2}{N_2})$ , the size of the anti-trade lobby group 2 (represented by the share of firms in sector 2 that participate in lobbying) is fixed:  $\Theta_2^* = \frac{2}{N_2}$ . In contrast, the size of the pro-trade lobby 1  $\Theta_{1,\bar{t}}^*$  grows larger with a lower  $\bar{t}$  and is implicitly given by the equation  $\left(\frac{\alpha^2 K_1^2}{ab_2}\right) \frac{\Theta_{1,\bar{t}}^*}{N_1} = \frac{1}{2} \left(\frac{\alpha^2 K_1^2}{ab_2}\right) \Theta_{1,\bar{t}}^{*2} - \frac{ab_2}{2} (t_{-1}(\Theta_2^*) - \bar{t})^2$ .*

#### EFFECT OF THE TARIFF CAP ON THE EQUILIBRIUM APPLIED TARIFF

Let  $t_{-1}^*$  be the out-of-equilibrium tariff in the absence of the pro-trade lobby 1:

$$t_{-1}^* = t_{-1}(\Theta_2^*) = \left(\frac{K_2}{ab_2}\right) \left(\frac{2}{N_2}\right).$$

When  $\tilde{t}_2 \leq \bar{t}_2 \leq t_{-1}^*$ , we can pin down the closed form solution for equilibrium *applied* tariff  $t^*$  and the “binding overhang”  $h$ , given the tariff ceiling  $\bar{t}$ :

$$t^* = t_{-1}^* - \left( \frac{1}{N_1} \left( \frac{\alpha K_1}{ab_2} \right) + \left( \frac{1}{N_1^2} \left( \frac{\alpha K_1}{ab_2} \right)^2 + (t_{-1}^* - \bar{t})^2 \right)^{1/2} \right), \quad (1.24)$$

and

$$h = \bar{t} - t^* = \left( \frac{1}{N_1} \left( \frac{\alpha K_1}{ab_2} \right) + \left( \frac{1}{N_1^2} \left( \frac{\alpha K_1}{ab_2} \right)^2 + (t_{-1}^* - \bar{t})^2 \right)^{1/2} \right) - (t_{-1}^* - \bar{t}). \quad (1.25)$$

We are now ready to completely characterize the relationship between the tariff ceiling  $\bar{t}$  and the equilibrium applied tariff  $t^*$  with the following proposition.

**Proposition 1.4** *Under assumption 1.1, the relationship between the bound tariff  $\bar{t}$  and the equilibrium applied tariff  $t^*$  is as follows:*

when  $\bar{t} \geq t_{-1}^*$ ,  $t^* = t^O(\frac{2}{N_1}, \frac{2}{N_2})$ ;

when  $\tilde{t} \leq \bar{t} \leq t_{-1}^*$ , both  $t^*$  and the binding overhang  $h$  decrease with  $\bar{t}$  ;

when  $\tilde{t} \leq \bar{t} \leq \tilde{t}$ ,  $t^* = \tilde{t}$  and the binding overhang  $h$  decreases with  $\bar{t}$ ;

when  $\bar{t} \leq \tilde{t}$ ,  $t^* = \bar{t}$ .

Figure 1.2 summarizes the content of proposition 1.4. The solid curves of both panels show the response of the equilibrium applied tariff to the tariff ceiling  $\bar{t}$ . When  $\bar{t} > t_{-1}^*$ , even if the pro-trade group were not present and the tariff were set to maximize the joint welfare of the government and lobby 2,  $\bar{t}$  would not be binding, so it has no effect on the equilibrium tariff. Starting from  $t_{-1}^*$ ,  $\bar{t}$  begins to constrain the tariff out of equilibrium, so a lower  $\bar{t}$  leads to more participation of the pro-trade firms and thus a lower applied tariff, as is shown in Figure 1.1 and Proposition (1.3). When  $\bar{t}$  reaches  $\tilde{t}$ , all the firms in the pro-trade sector choose to participate in lobbying, resulting in an applied tariff  $\tilde{t}$ . When  $\bar{t}$  moves from  $\tilde{t}$  to  $\tilde{t}$  the applied tariff cannot be lowered anymore due to the full participation of the pro-trade sector. When the  $\bar{t}$  becomes smaller than  $\tilde{t}$ , there would be no pro-trade firms lobbying in equilibrium and the applied tariff coincides with  $\bar{t}$ .

A natural concern is that when  $\bar{t}$  is above but very close to  $\tilde{t}$ , if a firm in pro-trade lobby chose to free ride, the tariff would hit  $\bar{t}$ , so the lobbying firms' total willingness to pay is no longer given by (2.10). Specifically, when  $\tilde{t} \leq \bar{t} \leq t_{-1}^* - \left(\frac{\alpha K_1}{ab_2} \frac{N_1-1}{N_1}\right)$ , if a lobbying firm in sector 1 chose to shirk, its loss in profit would only be  $\frac{1}{N_1} \alpha K_1 (\bar{t} - \tilde{t})$ . However, in the appendix we show that although in this case a pro-trade firm loses

less from free-riding and firms' total willingness to pay is smaller than that implied by (2.10), full participation is still an equilibrium, because the pro-trade contribution is also smaller and this dominates effect of the reduced willingness to pay. Thus we have rigorously established that the solid curve should be flat everywhere from  $\tilde{t}$  to  $\tilde{\tilde{t}}$ .

Figure 1.2 (upper panel) also shows that a larger  $N_1$  leads to a smaller overhang  $h$ , which is a direct implication of equation (1.24). When the downstream industry is less concentrated, firms tend to free ride more on each other and therefore in equilibrium they are able to get a smaller binding overhang below the tariff ceiling. And as  $N_1 \rightarrow \infty$ , the applied tariff will be infinitely close to  $\bar{t}$  (when  $\bar{t} \leq t_{-1}^*$ ), i.e., the binding overhang vanishes.

**Corollary 1.1** *Given any  $\bar{t}$ , as  $N_1$ , the number of firms in sector 1, increases, the binding overhang  $h$  strictly decreases when the applied tariff  $t^*$  co-moves with  $\bar{t}$ .*

Figure 1.2 (lower panel) shows the effect of an decrease in the number of firms in the anti-trade sector 2. The curve depicting the relationship between the tariff ceiling and the applied tariff shifts northeast, due to greater concentration of sector 2's firms and thus their enhanced ability to solve the collective action problem.

#### EFFECT OF THE TARIFF CAP ON LOBBIES' CONTRIBUTION SCHEDULES

We now look at how the tariff ceiling changes the domestic lobbies' contribution schedules. In a "truthful equilibrium", lobbies adjust their contributions off the equilibrium path so they remain the same well off as in the equilibrium (as long as the contribution is nonnegative). When the participation shares are  $\Theta_1$  and  $\Theta_2$ , lobby 2's

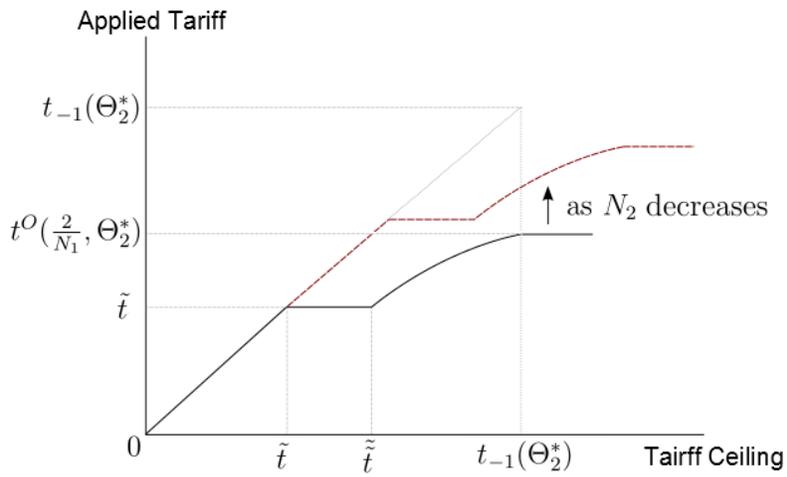
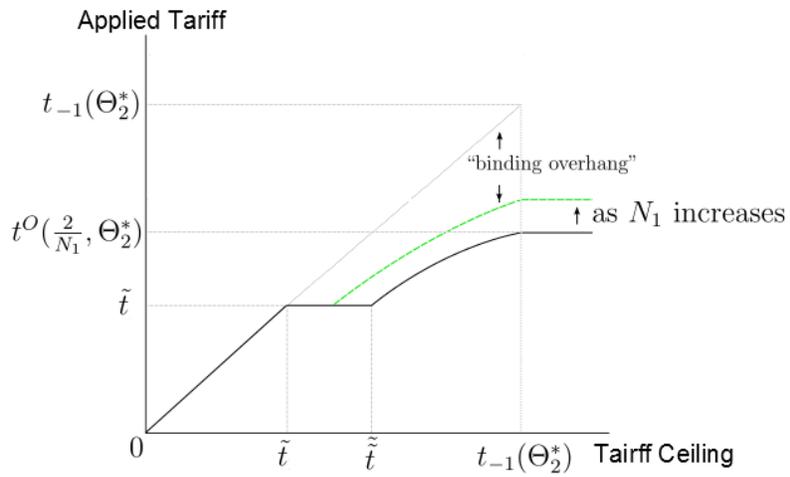


Figure 1.2: The relationship between bound and applied tariffs.

contribution at an applied tariff  $t$  would be

$$\begin{aligned} \text{Contr}_2(t; \Theta_1, \Theta_2) &= \max\{C_2^*(\Theta_2) + \int_{t^O(\Theta_1, \Theta_2)}^t \frac{\partial(\Theta_2 \Pi_2)}{\partial t} dt, 0\} \\ &= \max\{\Theta_2 K_2 \left( t + \frac{\alpha K_1}{ab_2} \Theta_1 - \frac{1}{2} \frac{K_2}{ab_2} \Theta_2 \right), 0\}. \end{aligned} \quad (1.26)$$

Without  $\bar{t}$ , lobby 1's contribution schedule is

$$\begin{aligned} \text{Contr}_1(t; \Theta_1, \Theta_2) &= \max\{C_1^*(\Theta_1) + \int_{t^O(\Theta_1, \Theta_2)}^t \frac{\partial(\Theta_1 \Pi_1)}{\partial t} dt, 0\} \\ &= \max\{\Theta_1 \alpha K_1 \left( -t + \frac{K_2}{ab_2} \Theta_2 - \frac{1}{2} \frac{\alpha K_1}{ab_2} \Theta_1 \right), 0\}. \end{aligned} \quad (1.27)$$

If  $t^O(\Theta_1, \Theta_2) \leq \bar{t}$ , i.e., in equilibrium lobby 1 contributes positive amounts so that the applied tariff does not hit the tariff ceiling, the pro-trade group's contribution schedule becomes (using (1.15))

$$\begin{aligned} \text{Contr}_1(t; \Theta_1, \Theta_2) &= \max\{C_1^*(\Theta_1) - \frac{ab_2}{2} (t_{-1}(\Theta_2) - \bar{t})^2 + \int_{t^O(\Theta_1, \Theta_2)}^t \frac{\partial(\Theta_1 \Pi_1)}{\partial t} dt, 0\} \\ &= \max\{\Theta_1 \alpha K_1 \left( -t + \frac{K_2}{ab_2} \Theta_2 - \frac{1}{2} \frac{\alpha K_1}{ab_2} \Theta_1 \right) - \frac{ab_2}{2} (t_{-1}(\Theta_2) - \bar{t})^2, 0\}. \end{aligned} \quad (1.28)$$

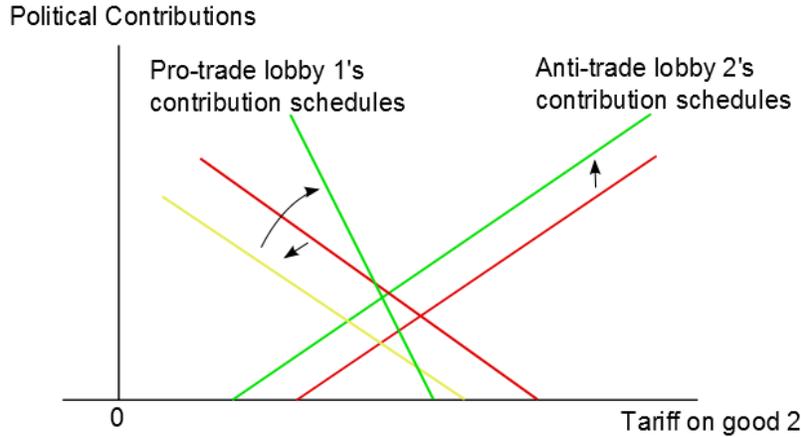


Figure 1.3: Changes in contribution schedules when the tariff ceiling is imposed.

Figure 1.3 highlights how the tariff ceiling, as an external constraint to the home country, changes how special interest groups interact with the government. When  $\bar{t}$  is

imposed, at first there is a downward parallel shift of the pro-trade lobby's contribution schedule. However, as the cost of lobbying for lower tariff is reduced and the gains from participation unaffected, more firms in the downstream industry 1 participate in lobbying and the pro-trade group grows larger. As a result, the pro-trade lobby offers the government a steeper contribution schedule, rewarding the latter with a greater increase in political contributions due to a marginal reduction in tariff. The upstream industry's firms' participation decisions are unaffected and the anti-trade lobby's size unchanged. Now in equilibrium, the anti-trade lobby pays the same amount of political contribution as before, but "buys" a lower tariff since it now faces a stronger rival, so its payoff is reduced. Since the contribution schedules are "truthful", the anti-trade lobby's contribution schedule must shift upward, i.e., it will contribute more to the government for the same level of tariff.

#### THE BINDING OVERHANG

It still remains to show that the binding overhang  $h$  decreases with  $\bar{t}$  when  $\bar{t} \leq \bar{t} \leq t_{-1}^*$  in order to fully establish proposition 1.4. One can prove this directly by differentiating (1.25). However, we show it more intuitively by writing the model in a slightly different way. With a little abuse of notation, let  $\tilde{C}_1(h)$  be the contribution of lobby 1 as in (1.15), but now it is a function of binding overhang  $h = \bar{t} - t^O(\Theta_1, \Theta_2^*)$ . With (1.15),

$$\begin{aligned}
\tilde{C}_1(h) &= \{aW(\cdot) + \Theta_2^* \Pi_2(\cdot)\}|_{t^O(\Theta_1, \Theta_2^*)}^{\bar{t}} \\
&= C_1^*(\Theta_1) - \frac{ab_2}{2}(t_{-1}(\Theta_2^*) - \bar{t})^2 \\
&= \frac{1}{2}ab_2h^2 + ab_2(t_{-1}(\Theta_2^*) - \bar{t})h.
\end{aligned} \tag{1.29}$$

We can think  $\tilde{C}_1(h)$  as, given  $\bar{t}$ , the amount of political contribution needed to induce the government to set an applied tariff with binding overhang  $h$ . Note when  $h = 0$ ,

$\tilde{C}_1(h) = 0$ , i.e., the pro-trade lobby can get an applied tariff equal to  $\bar{t}$  at no cost since we assume the tariff cap stipulated by the GATT/WTO is an effective commitment device.

Then what is the total willingness to pay of lobby 1 implied by  $h$ ? Because lobby 2's participation share  $\Theta_2^*$  is fixed and  $h = \bar{t} - t^O(\Theta_1, \Theta_2^*)$ , there is one-to-one mapping from  $h$  to lobby 1's participation share  $\Theta_1$ : a larger  $\Theta_1$  leads to a lower applied tariff and thus larger  $h$ . Lobby 1's total willingness to pay is a function of  $\Theta_1$  as in (2.10), so it can also be written as a function of  $h$ , which is derived from (1.5)(2.10):

$$\tilde{W}_1(h) = \frac{\alpha K_1}{N_1} (h + (t_{-1}(\Theta_2^*) - \bar{t})). \quad (1.30)$$

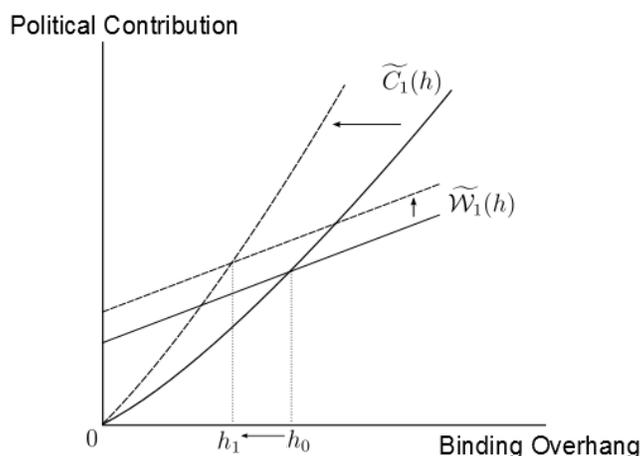


Figure 1.4: The change in the binding overhang  $h$  due to a lower  $\bar{t}$ .

When  $\tilde{t} \leq \bar{t} \leq t_{-1}^*$ , the equilibrium overhang is found by equating  $\tilde{C}_1(h)$  with  $\tilde{W}_1(h)$ , as in figure 1.4. At the intersection of the  $\tilde{C}_1(h)$  and the  $\tilde{W}_1(h)$  curves, a lobbying firm is indifferent between whether to participate, and this is the way we define the equilibrium. When the tariff ceiling  $\bar{t}$  is lower, to “buy” the same amount of overhang  $h$ , the pro-trade interest group need to pay more. This is because the joint welfare of the government and the anti-trade lobby is always maximized at  $t_{-1}^*$ ,

and due to the concavity of this maximization problem, any deviation from  $t_{-1}^*$  would result in a welfare loss that is convex in terms of this deviation. Given a lower  $\bar{t}$ , the same overhang  $h$  would correspond to a smaller applied tariff and hence a larger deviation from  $t_{-1}^*$ , and this implies that the joint payoff of the government and the anti-trade lobby decreases faster at a given  $h$ . So the  $\tilde{C}_1(h)$  moves from the solid line to the dashed line, implying more contribution needed to induce the same level of  $h$ .

As  $\bar{t}$  becomes lower, the  $\tilde{W}_1(h)$  curve moves up, as can be seen in equation (1.30), due to a larger  $\Theta_1$  implied by the same amount of  $h$ . However, this effect is dominated by the shift of the  $\tilde{C}_1(h)$  curve, and thus in our model a lower  $\bar{t}$  leads to a smaller overhang  $h$ .

### 1.2.9 WHAT IF FIRMS LOBBY INDIVIDUALLY?

The previous analysis assumes that there are two lobbies: the pro-trade lobby 1 and the anti-trade lobby 2. This is just for expository convenience, but not essential for our results. In this section we allow firms individually offer contribution schedules to the government, and show that our main results continue to hold<sup>9</sup>.

Specifically, we alternatively consider a game  $\Gamma'$  that only differs from game  $\Gamma$  in the second stage.

*Stage 2'*: All firms that have chosen to enter this stage lobby *individually*. Specifically, a firm  $f$  presents the politician with its own contribution schedule  $contr_f(t)$ .

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<sup>9</sup>A related paper is Bombardini and Trebbi (2012). They show both empirically and theoretically that in a sector with more substitutable goods, firms tend to lobby together. In their model, if a firm lobbies individually, it will target a import good that is the closest substitute for this firm's product. Our focus is different, since we fix a product (i.e., the upstream good 2), and consider whether the firms affected by this product (i.e., the producers and users) lobby individually and jointly.

**Definition 1.2** *The tariff setting game  $\Gamma'$  is the sequential game (Stage 1 ,Stage 2', Stage 3), where Stage 1 and Stage 3 is identical to those in the game  $\Gamma$  defined in section 1.2.3.*

The following proposition establishes the equivalence between games  $\Gamma$  and  $\Gamma'$  with symmetric firms.

**Proposition 1.5** *Consider the tariff setting game  $\Gamma'$ . Under assumption 1.1, given any tariff ceiling  $\bar{t}$ , the equilibrium outcomes, i.e., participation shares, political contributions, and the applied tariff, are identical to those derived in section 1.2.8. Proposition 1.4 and corollary 1.1 still hold.*

Appendix A.2 provides the detailed proof of proposition 1.5. The key is that we have been working with common agency games that satisfy the “two-sided” condition define in Laussel and Le Breton (2001). Intuitively, this condition says we can divide all the principals into two groups (i.e., two sides), such that given any two outcomes  $a$  and  $b$ , players in the same group rank  $a$  and  $b$  in the same way while players in different groups rank them differently. In the tariff setting game  $\Gamma$  we previously analyzed, this condition is trivially satisfied, since the upstream lobby wants a higher applied tariff and the downstream lobby desires a lower one. Yet a more subtle implication of the results in Laussel and Le Breton (2001) relating to “two-sided” common agency games is that even when we have many principals, (for example, if the *firms* lobby the government as in game  $\Gamma'$ ), as long as they are “two-sided”, each “side” acts *as if* they are in a group, with the equilibrium contribution of each side uniquely pinned down and multiplicity of equilibria arising only due to the allocation of contributions within each side. Thus the main results in this paper also generalize to the tariff setting game  $\Gamma'$ .

The above comment also suggests that the intuition of our analysis will continue to hold even in different economic structures with an arbitrary number of lobbies, as long as the lobbies are “two-sided”, i.e., one group of lobbies benefit from a higher tariff while the other group benefit from a lower one. The “two-sided” condition is plausible in the context of the political economy of trade policy, since one fundamental insight is that there are always losers and gainers from a lower trade barrier, and what shapes the trade policy in the real world is how well each group can overcome its free-rider problem.

### 1.3 EX ANTE LOBBY PARTICIPATION

In the previous section we have shown that the tariff cap serves as an instrument to alleviate the free rider problem of the pro-trade downstream industry. It is then natural to ask why governments choose to impose tariff ceilings on themselves in the first place and what factors determine the corresponding tariff reduction.

Following the literature, we assume when the governments are negotiating trade agreements, they are also influenced by domestic lobbies<sup>10</sup> and firms also voluntarily participate in lobbying. In the model to be considered, we shall show that in the small

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<sup>10</sup>In such settings, key differences arise between *ex ante* stages, which lead to the trade agreement, and *ex post* stages, where the policy (i.e., the applied tariff) is set. Amador and Bagwell (2013) assume political shocks are only revealed *ex post* and are private information to the relevant government. In their model, the key trade-off relating to the design of a trade agreement is giving the government more discretion and thus making better use of its private information on political shocks, versus restricting its tendency to use beggar-thy-neighbor policies. They characterize the conditions under which tariff caps are optimal. Without uncertainty, their model would have collapsed to a trivial result: the trade agreement just has to stipulate a particular tariff level, and the tariff cap will not be preferred in any way. Maggi and Rodriguez-Clare (2007) differentiate between *ex ante* and *ex post* lobbying. They show the tariff ceiling is preferred to an exact tariff commitment, because it leaves the discretion to adjust the tariff downward for the government, forces the protected industry to lobby *ex post* and thus alleviates the over-investment problem. In contrast, the model in this section will not only take into account both *ex ante* and *ex post* *lobbying*, but also both *ex ante* and *ex post* *lobby participation*.

country, when the firms know the the government may “tie its hands” with a tariff cap, the pro-trade lobby will become more organized; when the tariff cap is in place, the pro-trade lobby will get further organized. In contrast, the size of the anti-trade lobby remains fixed in each stage.

We propose the following game  $\Gamma_0$  in which the opportunity for the government to choose some  $\bar{t}$  arises exogenously:

**Stage 1:** Each firm in both industries voluntarily decides whether to participate in the next stage.

**Stage 2:** The government may enter a trade agreement that sets a tariff cap  $\bar{t} \in (-\infty, +\infty)$ . Those firms that have chosen to participate in stage 1 in sector  $i$  form a lobby  $\mathcal{L}_{0i}$  ( $i = 1, 2$ ), which in turn offers the government a contribution scheduling  $Contr_{0,i}(\cdot)$ , with political contributions depending on the government’s choice. Again, we assume this common agency game ends up with a “truthful equilibrium”.

**Later stages:** The events of  $\Gamma$  unfold, as described in section 1.2.3.

We solve the game  $\Gamma_0$  under assumption 1.1 of symmetric firms. Note in the stage 1 of  $\Gamma_0$  as well as in the stage 1 of  $\Gamma$ , although we can pin down the number of firms that participate in each industry, it is not clear which firms participate. We impose the following assumption:

**Assumption 1.2** (*following assumption 1.1*)

*At stage 1 of game  $\Gamma$ , each equilibrium occurs with the same probability. Therefore, (ex ante) each (symmetric) firm in the same industry has the same probability of participating in stage 2 of game  $\Gamma$ .*

In the game  $\Gamma$  we previously analyzed, the equilibrium outcome is that some non-participating firms free-ride on the lobbying efforts of the other participating firms

(and each firm's choice is a best response to others'), even when we assume identical firms in assumption 1.1. There is no way to predict which set of identical firms will participate in stage 2 of game  $\Gamma$ , leading to multiple equilibria. Assumption 1.2 says when firms calculate the costs and benefits in stage 1 and 2 of game  $\Gamma_0$ , they expect the world will fall into the above multiple equilibria with equal probabilities. This assumption makes the problem tractable.

Now suppose the government chooses some level of tariff cap  $\tilde{t} \leq \bar{t} \leq t_{-1}^*$  in stage 2 of game  $\Gamma_0$ . According to proposition 1.4, when  $\bar{t}$  belongs to this range, the equilibrium applied tariff will co-move with the tariff ceiling, as in Figure 1.4. The logic in Figure 1.1 applies, that is, the participation share of the pro-trade lobby 1 increases if  $\bar{t}$  becomes lower, but is still smaller than 1 until  $\bar{t}$  reaches  $\tilde{t}$ . The equilibrium contribution of lobby 1 will be equal to  $\mathcal{W}_1(\Theta_1)$ , i.e., the group's "total willingness to pay", because in equilibrium  $\Theta_1$  has reached such a value that firms in the group are indifferent between whether to participate. The total payoff of *sector* 1 in game  $\Gamma$  is

$$V_1 = \Pi_1(t) - \mathcal{W}_1(\Theta_1), \quad (1.31)$$

where  $t$  is the equilibrium applied tariff that will emerge in game  $\Gamma$ .

Similarly, the total payoff of *sector* 2 in game  $\Gamma$  is

$$V_2 = \Pi_2(t) - \mathcal{W}_2(\Theta_2). \quad (1.32)$$

When  $\tilde{t} \leq \bar{t} \leq t_{-1}^*$ , the equilibrium applied tariff  $t$  monotonically decreases with  $\bar{t}$ . Therefore when the government chooses the tariff cap  $\bar{t}$  in stage 2 of game  $\Gamma_0$ , *it is as if choosing the equilibrium applied tariff  $t$  in the subgame  $\Gamma$* , which will be determined by equation (1.24) that characterizes the equilibrium relationship between the bound and applied tariff. Note the participation share of industry 2 in game  $\Gamma$  will always be  $\Theta_2^* = \frac{2}{N}$ , and the equilibrium applied tariff will be  $t = \left(\frac{K_2}{ab_2}\right)\Theta_2^* - \left(\frac{\alpha K_1}{ab_2}\right)\Theta_1 =$

$t_{-1}^* - \left(\frac{\alpha K_1}{ab_2}\right) \Theta_1$  from equilibrium tariff condition (1.5). Using the above formula and the expression for  $\mathcal{W}_1(\Theta_1)$  with symmetric firms (equation (2.10)), we derive  $V_1$  as a function of game  $\Gamma$ 's equilibrium applied tariff  $t$  from equation (1.31):

$$V_1(t) = \Pi_1(t) - \left(\frac{\alpha K_1}{N_1} t_{-1}^* - \frac{\alpha K_1}{N_1} t\right). \quad (1.33)$$

Given sector 2's participation share in game  $\Gamma$   $\Theta_2^* = \frac{2}{N_2}$ , and using equation (2.11) which pins down lobby 2's total willingness to pay in  $\Gamma$   $\mathcal{W}_2(\Theta_2)$ , we have  $V_2$  as a function of the equilibrium tariff  $t$  from equation (1.32):

$$V_2(t) = \Pi_2(t) - \frac{1}{N_2} \left(\frac{K_2^2}{ab_2}\right) \Theta_2^*. \quad (1.34)$$

Equations (1.33)(1.34) lead to

$$\frac{\partial V_1}{\partial t} = \frac{\partial \Pi_1}{\partial t} + \frac{\alpha K_1}{N_1} = -\alpha K_1 + \frac{\alpha K_1}{N_1}. \quad (1.35)$$

And

$$\frac{\partial V_2}{\partial t} = \frac{\partial \Pi_2}{\partial t} = K_2. \quad (1.36)$$

The payoff of the government in game  $\Gamma$  (gross of political contributions) is (as a function of  $t$  induced by the corresponding tariff ceiling  $\bar{t}$ ) :

$$G_\Gamma(t) = aW(t) + \mathcal{W}_1(\Theta_1) + \mathcal{W}_2(\Theta_2^*) = aW(t) - \frac{\alpha K_1}{N_1} t + \frac{\alpha K_1}{N_1} t_{-1}^* + \frac{1}{N_2} \left(\frac{K_2^2}{ab_2}\right) \Theta_2^*. \quad (1.37)$$

Equations (1.31)-(1.37) only hold *locally* if the tariff ceiling chosen in stage 2 satisfies  $\tilde{t} \leq \bar{t} \leq t_{-1}^*$ , i.e., the tariff cap is low enough to be binding absent the pro-trade lobby, but high enough to not make the pro-trade lobby get fully organized.

Now we solve the complete game with backward induction. Suppose the number of firms in sector  $i$  that participate in stage 2 is  $\#\mathcal{L}_{0i} = n_{0i}$ , with participation share  $\Theta_{0i} = \frac{n_{0i}}{N_i}$  ( $i = 1, 2$ ). Since we assume symmetric outcomes for firms in the same

sector (from ex ante perspective) with assumption 1.2, the expected payoff of each firm in sector  $i$  in game  $\Gamma$  is captured by  $\frac{V_i}{N_i}$ , and the joint (expected) payoff of the members in group  $\mathcal{L}_{0i}$  will be  $n_{0i}\frac{V_i}{N_i} = \Theta_{0i}V_i$ . The lobby group  $\mathcal{L}_{0i}$  collectively offer the government a contribution schedule  $Contr_{0i}(\cdot)$ , defined over possible tariff ceilings. Suppose the government chooses some tariff cap  $\bar{t}^*$ , inducing the (future) equilibrium applied tariff  $t^*$ , then the choice must maximize the (expected) joint payoff of the itself and the two lobbies formed in stage 2 of game  $\Gamma_0$ :

$$\begin{aligned} t^* &= \arg \max_t \{G_\Gamma(t) + \Theta_{01}V_1(t) + \Theta_{02}V_2(t)\} \\ &= \arg \max_t \{aW(t) + \Theta_{01}\Pi_1(t) + \Theta_{02}\Pi_2(t) + (1 - \Theta_{01})\mathcal{W}_1(\Theta_1) + (1 - \Theta_{02})\mathcal{W}_2(\Theta_2^*)\}, \end{aligned} \tag{1.38}$$

where lobby 1's "willingness to pay" in game  $\Gamma$ ,  $\mathcal{W}_1(\Theta_1) = \frac{\alpha K_1}{N_1}t_{-1}^* - \frac{\alpha K_1}{N_1}t$ , can also be viewed as a function of  $t$ , as in the derivation of (1.33).

Equation (1.38) shows that the marginal change in game  $\Gamma$ 's equilibrium applied tariff  $t$  has two effects: (1) it affects social welfare and lobbying firms' profits, which is captured by  $(aW(t) + \Theta_{01}\Pi_1(t) + \Theta_{02}\Pi_2(t))$  in the above maximization problem; (2) a lower  $t$  must be induced by a lower tariff cap  $\bar{t}$ , and a lower  $\bar{t}$  increases participation share of lobby 1 in game  $\Gamma$ , leading to a higher equilibrium contribution  $\mathcal{W}_1(\Theta_1)$ . Specifically, there will be firms that *participate in the lobby group  $\mathcal{L}_1$  in game  $\Gamma$  but were not present in the lobby group  $\mathcal{L}_{01}$* . The players in the ex ante stage 2 has incentives to impose a low tariff cap to attract such "new" participants (and thus contributors).

The first order condition of maximization problem (1.38) is

$$\underbrace{\frac{\partial}{\partial t} (aW(t) + \Theta_{01}\Pi_1(t) + \Theta_{02}\Pi_2(t))}_{\text{Effect of the induced applied tariff on welfare and profits.}} + \underbrace{\frac{\partial}{\partial t} \left( (1 - \Theta_{01}) \left( \frac{\alpha K_1}{N_1} t_{-1}^* - \frac{\alpha K_1}{N_1} t \right) \right)}_{\text{Effect of the tariff ceiling on organization of lobby } \mathcal{L}_1.} = 0.$$

**Lemma 1.2** *Given the lobby participation shares  $\Theta_{01}$  and  $\Theta_{02}$  in Stage 2 of game  $\Gamma_0$ , the applied tariff that will emerge in the subgame  $\Gamma$  is*

$$t^* = \underbrace{\left( \frac{K_2}{ab_2} \Theta_{02} - \frac{\alpha K_1}{ab_2} \Theta_{01} \right)}_{\text{Same formula as in equation (1.5), taking account of social welfare and ex ante lobbies' profits in game } \Gamma.} - \underbrace{\frac{\alpha K_1}{ab_2} (1 - \Theta_{01}) \frac{1}{N_1}}_{\text{Tariff cap as an instrument to induce **additional** downstream firms to contribute ex post.}}, \quad (1.39)$$

$$\text{if } t^* \in \left( \frac{K_2}{ab_2} \frac{2}{N_2} - \frac{\alpha K_1}{ab_2} \cdot 1, \frac{K_2}{ab_2} \frac{2}{N_2} - \frac{\alpha K_1}{ab_2} \frac{2}{N_1} \right).$$

Note the absolute value of the second term in the RHS of (1.39) decreases with both  $N_1$  and  $\Theta_{10}$ . When  $N_1$  increases, the tendency of free-rider in the pro-trade industry becomes greater, and the tariff ceiling is a less efficient tool of making the pro-trade lobby stronger. When  $\Theta_{10}$  increases, sector 1's participation share in stage 2 of  $\Gamma_0$  is already quite high. Recall this term only captures the fact that the government and the pro-trade lobby benefit from the participation of “new” participants of the *ex post* lobby group  $\mathcal{L}_1$  of game  $\Gamma$ . When the *ex ante* lobby group  $\mathcal{L}_{01}$  already consists of a larger share of firms, the chance that a firm belongs to  $\mathcal{L}_1$  but *not*  $\mathcal{L}_{01}$  becomes smaller. If  $\Theta_{01} = 1$ , i.e., all pro-trade firms participated in stage 2 of  $\Gamma_0$ , the second term in equation (1.39) will simply disappear.

Now we go to stage 1 of game  $\Gamma_0$ , and ask: how many firms in each industry will voluntarily participate in stage 2 and influence the government's choice of the tariff ceiling? In other words, what are the sizes of the *ex ante* lobbies,  $\mathcal{L}_{01}$  and  $\mathcal{L}_{02}$ , which affect the trade agreements the home government commits to? And how the strengths of the *ex ante* lobbies compared with the *ex post* ones that lobby for the applied tariff under the government's commitment?

We determine the sizes of the *ex ante* lobbies with the same approach that we used to pin down the sizes the *ex post* ones, that is, to find the largest participation share at which the group’s “total willingness to pay” does not exceed its equilibrium contribution. Given the participation shares  $\Theta_{01}$  and  $\Theta_{02}$ , we can determine the induced applied tariff  $t^*$  (and thus the corresponding tariff ceiling) with equation (1.39). However, as we will see, complexity arises regarding the political contributions in stage 2 of  $\Gamma_0$ .

First we solve for the equilibrium  $\Theta_{02}$  under the following assumption:

**Assumption 1.3** *The measures of firms in the two industries satisfy:*

$$\left(1 - \frac{1}{N_1}\right) \left(1 - \frac{1}{N_1} - \left(\frac{1}{N_1^2} + \frac{1}{(N_1 - 1)^2}\right)^{\frac{1}{2}}\right) \geq \frac{K_2}{\alpha K_1} \frac{2}{N_2}. \quad (1.40)$$

Assumption 1.3 ensures that given the equilibrium downstream participation share  $\Theta_{01}^*$  that we will solve for, even if the upstream lobby  $\mathcal{L}_{02}$  were absent, the chosen tariff ceiling would still be (weakly) above  $\tilde{t}_2$ . In other words, at stage 2 of game  $\Gamma_0$ , without the influence of the *ex ante* upstream lobby, the *ex ante* downstream lobby would certainly induce the government to choose a lower tariff ceiling, but this tariff ceiling will not be so low as to reach  $\tilde{t}$ , which would result in full participation of the downstream industry *ex post* in game  $\Gamma$ . In Figure 1.2, assumption 1.3 guarantees that absent the *ex ante* upstream lobby, the government will still choose a tariff ceiling in the interval  $[\tilde{t}, t_{-1}^*]$ , where the applied tariff strictly decreases with the bound one, with some positive “binding overhang”.

In Appendix A.3 we prove the following lemma:

**Lemma 1.3** *Under assumptions 1.2 and 1.3, only two upstream firms participate in the *ex ante* lobby  $\mathcal{L}_{02}$ . The corresponding equilibrium *ex ante* participation share*

$$\Theta_{02}^* = \frac{2}{N_2}.$$

Lemma 1.3 says for the upstream industry, the ex ante participation share is exactly the same as the ex post one, and they are equal to  $\frac{2}{N_2}$ . The intuition is as follows. When choosing  $\bar{t}$  that satisfies  $\tilde{t} \leq \bar{t} \leq t_{-1}^*$ , the payoffs for the firms and the government are given by the system (1.31)-(1.37), where the players objective functions are defined over the applied tariff  $t$  (induced by the corresponding tariff cap). In such systems in which the derivative of the government objective function of tariff is linear and the derivative of lobbies' objective functions are constant, absent any external constraint, the participation share of an industry *only* depends on its *own* size distribution, as in our baseline model in section 2.2. Since the upstream industry is always composed of the  $N_2$  firms, we get exactly the same participation shares both ex ante and ex post. One concern is that if the upstream lobby  $\mathcal{L}_{02}$  were absent, the off-the-equilibrium path applied tariff derived from (1.39) (by letting  $\Theta_{02} = 0$ ) would fall out of the range that can be induced by tariff caps satisfying  $\tilde{t} \leq \bar{t} \leq t_{-1}^*$ . Assumption 1.3 ensures this not to happen.

Next we solve for the equilibrium  $\Theta_{01}$ , given the  $\Theta_{02}^*$  in lemma 1.3. In stage 2, let  $t_{-01}^*$  denote the applied tariff (induced by the corresponding tariff cap) chosen if lobby group  $\mathcal{L}_{01}$  were absent. We have mentioned that the system (1.31)-(1.37) hold locally around the equilibrium applied tariff. If they held everywhere, absent the pro-trade lobby  $\mathcal{L}_{01}$  in stage 2 of  $\Gamma_0$ , the government would have chosen a future applied tariff

$$\hat{t}_{-01} = \frac{K_2}{ab_2} \Theta_{02}^* - \frac{\alpha K_1}{ab_2} \frac{1}{N_1}, \quad (1.41)$$

which is derived by letting  $\Theta_{01} = 0$  in (1.39). However, this is impossible, because the highest applied tariff in game  $\Gamma$  that the government can “choose” in stage 2 of  $\Gamma_0$  is  $t^O(\frac{2}{N_1}, \frac{2}{N_2}) = \frac{K_2}{ab_2} \frac{2}{N_2} - \frac{\alpha K_1}{ab_2} \frac{2}{N_1}$ , by imposing no tariff cap or a very high one that will not bind in game  $\Gamma$  even out of equilibrium. Clearly, absent lobby group  $\mathcal{L}_{01}$  the level of induced applied tariff  $\hat{t}_{-01}$  is infeasible, and the joint welfare of the government

and  $\mathcal{L}_{02}$  would be maximized at the ex post applied tariff

$$t_{-01}^* = t^O\left(\frac{2}{N_1}, \frac{2}{N_2}\right), \quad (1.42)$$

which is induced by an arbitrarily high tariff ceiling, or no commitment at all.

The above condition imposes a *constraint* on the possible joint payoff of the government and lobby group  $\mathcal{L}_{02}$  in stage 2 of  $\Gamma_0$ . Intuitively, without the anti-trade lobby  $\mathcal{L}_{02}$ , the government and pro-trade lobby  $\mathcal{L}_{01}$  faces *no* constraint, because the government can promise  $\mathcal{L}_{01}$  a low applied tariff by committing to a low tariff cap that induces a large share of downstream firms to participate ex post in game  $\Gamma$ . However, absent lobby group  $\mathcal{L}_{01}$ , there is no way for the government to promise  $\mathcal{L}_{02}$  a very high applied tariff in game  $\Gamma$ . The highest applied tariff in  $\Gamma$  the government can guarantee ex ante is  $t_{-01}^* = t^O\left(\frac{2}{N_1}, \frac{2}{N_2}\right)$ , which is achieved with an arbitrarily high tariff cap. Therefore, a constraint on the off-the-equilibrium-path joint payoff of the government and  $\mathcal{L}_{02}$  arises naturally (which we will call “natural constraint” henceforth). As we will see, this constraint increases the participation share of the downstream industry in the ex ante stage 2, just as the tariff ceiling raises the size of the downstream lobby in the ex post game  $\Gamma$ .

In equilibrium, lobby  $\mathcal{L}_{01}$  must compensate for the joint welfare loss of the government and the competing lobby due to the effect of its lobbying, taking account of the constraint mentioned above. We have shown lobby group  $\mathcal{L}_{01}$  makes the ex post applied tariff shift from  $t_{-01}^*$  to  $t^*$ . The equilibrium contribution of  $\mathcal{L}_{01}$  (as a function of  $\Theta_{01}$ ) is

$$\tilde{C}_{01} = \int_{t^*}^{t_{-01}^*} \left( \frac{\partial G_{\Gamma}(t)}{\partial t} + \Theta_{02}^* \frac{\partial V_2(t)}{\partial t} \right) dt,$$

Using equations (1.36)(1.37) and the definitions of  $t^*$ ,  $\hat{t}_{-01}$  and  $t_{-01}^*$  in (1.39)(1.41)(1.42), we derive  $\tilde{C}_{01}$  as a function of  $\Theta_{01}$ :

$$\begin{aligned}
\tilde{C}_{01}(\Theta_{01}) &= \underbrace{\int_{t^*}^{\hat{t}_{-01}} \left( -ab_2t - \frac{\alpha K_1}{N_1} + \Theta_{02}^* K_2 \right) dt}_{\text{Contribution would have been paid without the above "natural constraint"}.} \\
&\quad - \underbrace{\int_{t_{-01}^*}^{\hat{t}_{-01}} \left( -ab_2t - \frac{\alpha K_1}{N_1} + \Theta_{02}^* K_2 \right) dt}_{\text{The amount of contribution saved due to the "natural constraint"}.} \\
&= \frac{1}{2} ab_2 \left( \left( 1 - \frac{1}{N_1} \right) \frac{\alpha K_1}{ab_2} \right)^2 \Theta_{01}^2 - \frac{1}{2} ab_2 \left( \frac{1}{N_1} \frac{\alpha K_1}{ab_2} \right)^2. \tag{1.43}
\end{aligned}$$

Note the above “natural constraint” on the joint payoff of the government and the anti-trade group  $\mathcal{L}_{02}$  reduces the equilibrium pro-trade contribution of  $\mathcal{L}_{01}$  by the amount of  $\frac{1}{2} ab_2 \left( \frac{1}{N_1} \frac{\alpha K_1}{ab_2} \right)$ , which decreases in  $N_1$ . This is intuitive: when  $N_1$  increases, the downstream firms free-ride on each other more severely *ex post*, so in stage 2 of  $\Gamma_0$ , absent group  $\mathcal{L}_{01}$ , the government can promise  $\mathcal{L}_{02}$  a higher applied tariff (with an arbitrarily high tariff ceiling), which means the “natural constraint” is relaxed.

Next we pin down lobby group  $\mathcal{L}_{01}$ ’s “total willingness to pay”, denoted with  $\mathcal{W}_{01}$ . When a firm in group  $\mathcal{L}_{01}$  considers whether to shirk given other firms’ decisions, it realizes that the change in the equilibrium applied tariff will be small. Although when the whole *group*  $\mathcal{L}_{01}$  is absent, the “natural constraint” will be binding, no constraint will be hit if a single firm quits. So when deriving  $\mathcal{W}_{01}$ , the system of equations (1.31)-(1.37) still applies.

Consider a single firm with sector-specific capital  $\frac{K_1}{N_1}$ . If it chooses not to participate, equation (1.39) shows that equilibrium  $t^*$  in game  $\Gamma$  will shift by the amount  $\left( \frac{1}{N_1} \frac{\alpha K_1}{ab_2} \right) \left( 1 - \frac{1}{N_1} \right)$ , and from (1.35) we know this firm’s payoff in  $\Gamma$  will decrease by  $w_f = \left( \frac{1}{N_1} \frac{\alpha K_1}{ab_2} \right) \left( 1 - \frac{1}{N_1} \right) \cdot \frac{1}{N_1} \frac{\partial V_1}{\partial t} = \left( \frac{1}{N_1} \frac{\alpha K_1}{ab_2} \right) \left( 1 - \frac{1}{N_1} \right) \cdot \left( \frac{1}{N_1} \alpha K_1 \right) \left( 1 - \frac{1}{N_1} \right)$ . This  $w_f$  is a firm’s “willingness to pay” (in stages 1 and 2 of game  $\Gamma_0$ ). Then with  $n_{01}$  firms in group  $\mathcal{L}_{01}$ , the “total willingness to pay” can be written as a function of participation

share  $\Theta_{01}$ :

$$\mathcal{W}_{01}(\Theta_{01}) = n_{01} \cdot w_f = \Theta_{01} \frac{1}{N_1} ab_2 \left( \frac{\alpha K_1}{ab_2} \left( 1 - \frac{1}{N_1} \right) \right)^2. \quad (1.44)$$

The equilibrium is found by letting  $\tilde{C}_{01}(\Theta_{01}) = \mathcal{W}_{01}(\Theta_{01})$ , such that a firm in the group  $\mathcal{L}_{01}$  is indifferent between whether to quit or not. This leads to the following lemma:

**Lemma 1.4** *When  $N_1 \geq 3$ , the closed-form solution of ex ante downstream participation share is<sup>11</sup>*

$$\begin{aligned} \Theta_{01}^* &= \frac{1}{N_1} + \left( \frac{1}{N_1^2} + \frac{1}{(N_1 - 1)^2} \right)^{\frac{1}{2}} \\ &= \underbrace{\frac{2}{N_1}}_{\text{Participation share if there were no "natural constraint".}} \\ &\quad + \underbrace{\left( \left( \frac{1}{N_1^2} + \frac{1}{(N_1 - 1)^2} \right)^{\frac{1}{2}} - \frac{1}{N_1} \right)}_{\text{Increased participation due to the "natural constraint"}.} \end{aligned} \quad (1.45)$$

Equation (1.45) shows when  $N_1$  is larger, the equilibrium *ex ante* participation share  $\Theta_{01}^*$  becomes smaller for two reasons: (1) the downstream firms tend to free-ride on each other more badly *ex ante*, so  $\frac{2}{N_1}$ , the hypothetical participation share if there were no “natural constraint”, decreases. (2) the downstream firms free-ride more on each other *ex post* in game  $\Gamma$ , so at the *ex ante* stage 2, absent the downstream lobby  $\mathcal{L}_{01}$ , the government could promise the upstream lobby  $\mathcal{L}_{02}$  a higher applied tariff in the subgame  $\Gamma$  (with an arbitrarily high tariff cap), forcing  $\mathcal{L}_{01}$  to pay greater contributions in equilibrium and thus making the *ex ante* downstream lobby less attractive to downstream firms. The term  $\left( \left( \frac{1}{N_1^2} + \frac{1}{(N_1 - 1)^2} \right)^{\frac{1}{2}} - \frac{1}{N_1} \right)$ , which captures the increased participation share due to the “natural constraint”, also becomes smaller.

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<sup>11</sup>When  $N_1 = 2$ , assumption 1.3 is unlikely to be satisfied. So we only consider cases where  $N_1 \geq 3$ .

We have derived the equilibrium ex ante participation shares  $\Theta_{01}^*$  and  $\Theta_{02}^*$  and summarized the results in lemmas 1.3 and 1.4. Plugging the values of  $\Theta_{01}^*$  and  $\Theta_{02}^*$  into equation (1.39), we get the equilibrium applied tariff that will finally emerge in game  $\Gamma_0$ :

$$\begin{aligned}
t^* &= \frac{K_2}{ab_2} \frac{2}{N_2} - \frac{\alpha K_1}{ab_2} \left( \frac{1}{N_1} + \left( \frac{1}{N_1^2} + \frac{1}{(N_1 - 1)^2} \right)^{\frac{1}{2}} \right) \\
&\quad - \frac{\alpha K_1}{ab_2} \left( 1 - \left( \frac{1}{N_1} + \left( \frac{1}{N_1^2} + \frac{1}{(N_1 - 1)^2} \right)^{\frac{1}{2}} \right) \right) \frac{1}{N_1} \\
&\leq t^O\left(\frac{2}{N_1}, \frac{2}{N_2}\right). \tag{1.46}
\end{aligned}$$

Note if the competing groups just lobby for the tariff in a setting without any tariff cap, the equilibrium tariff would be  $t^O(\frac{2}{N_1}, \frac{2}{N_2})$ , with participation shares  $\frac{2}{N_1}$  and  $\frac{2}{N_2}$ . This tariff is greater than  $t^*$  in (1.46), verifying that the tariff cap associated with  $t^*$  would indeed be binding out of equilibrium.

What will be the reduction in (applied) tariff resulting from the trade agreement? The government, constantly influenced by the lobby groups, which are in turn in constant need of solving their free-rider problems, will commit to a tariff cap that reduces the applied tariff on good 2 from  $t^O(\frac{2}{N_1}, \frac{2}{N_2})$  (the equilibrium tariff absent any external constraint) to  $t^*$  defined in (1.46). The reduction in the applied tariff is

$$\begin{aligned}
t^O\left(\frac{2}{N_1}, \frac{2}{N_2}\right) - t_2^* &= \underbrace{\frac{\alpha K_1}{ab_2} \left( \Theta_{01}^* - \frac{2}{N_1} \right)}_{\text{Reduction in tariff due to an ex ante pro-trade participation share greater than } \frac{2}{N_1}.} \\
&+ \underbrace{\frac{\alpha K_1}{ab_2} (1 - \Theta_{01}^*) \frac{1}{N_1}}_{\text{Using tariff cap to induce additional pro-trade firms to contribute ex post.}} \tag{1.47}
\end{aligned}$$

Equation (1.47) shows that there are two reasons for imposing a tariff cap for this small country. First, there is a relatively stronger downstream lobby when the government chooses the tariff cap. As previously mentioned, because we assume the

only possible constraint the government may commit to is a tariff ceiling, absent the upstream lobby the government can choose a very low tariff ceiling that lead to a very low applied tariff *ex post*, but without the downstream lobby the highest tariff the government can promise the upstream lobby is naturally constrained - it is just  $t^O(\frac{2}{N_1}, \frac{2}{N_2})$ , the applied tariff that results from an arbitrarily high tariff ceiling and (*ex post*) voluntary lobby participation of firms in both industries. This “natural constraint” on the joint payoff of the government and the *ex ante* anti-trade lobby reduces the contribution needed by the *ex ante* pro-trade lobby, while does not hurt the downstream lobby group’s “willingness to pay”, because when a single firm calculates whether to participate or not, the change in the government’s choice just occurs locally and the constraint is unlikely to be hit. Thus the *ex ante* downstream participation share is raised to a level above  $\frac{2}{N_1}$ , which is the downstream participation share in game  $\Gamma$  without any tariff ceiling, as in Figure 1.5. The stronger *ex ante* downstream lobby demands a lower applied tariff, leading to the first term on the RHS of equation (1.47). The second term on the RHS of (1.47) reflects the fact that a lower tariff ceiling induces more downstream firms to contribute *ex post*, some of which do not belong to the *ex ante* downstream lobby  $\mathcal{L}_{01}$ , and an *ante* players jointly benefit from the contributions of these *additional* firms<sup>12</sup>.

We now state the following proposition.

**Proposition 1.6** *Suppose assumption 1.3 holds.*

(1) *In game  $\Gamma_0$ , the small country’s government will enter a trade agreement specifying a tariff ceiling that would be binding in the subgame  $\Gamma$  if the pro-trade lobby were absent.*

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<sup>12</sup>Note if  $\Theta_{01}^* = 1$ , this term would simply disappear.

(2) Let  $\Theta_{0i}^*$  be share of firms in sector  $i$  that participate in lobbying at Stage 2 of game  $\Gamma_0$ , and  $\Theta_i^*$  be the share of participating firms in sector  $i$  in subgame  $\Gamma$ . Then for the anti-trade sector  $\Theta_{02}^* = \Theta_2^* = \frac{2}{N_2}$ , while for the pro-trade sector  $\frac{2}{N_1} < \Theta_{01}^* < \Theta_1^*$ .

(3) Suppose  $N_1 \geq 3$ . When  $N_1$  is smaller (i.e., the downstream pro-trade sector is more concentrated), the trade agreement will lead to a greater reduction in the applied tariff.

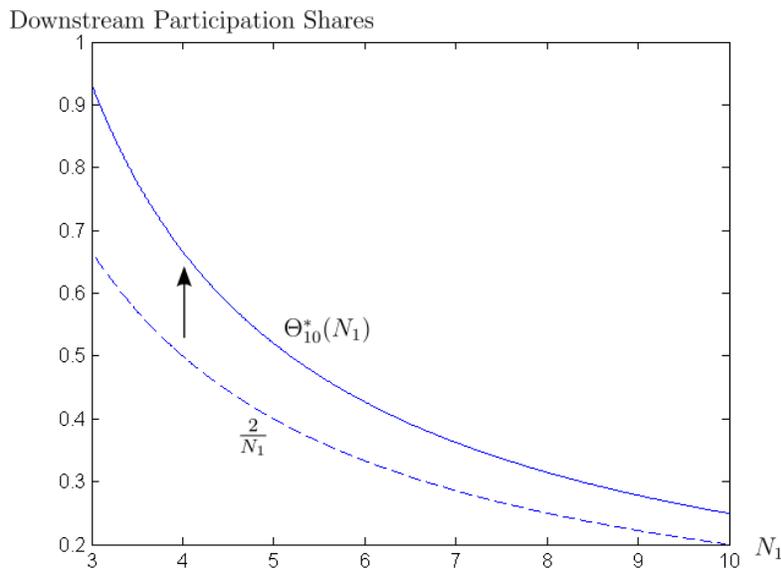


Figure 1.5: The change in the share of participating pro-trade firms when firms know the government is negotiating toward a trade agreement.

Part (3) of proposition 1.6 states the reduction in the upstream applied tariff in the small country is larger when the downstream industry is more concentrated. Note this is not simply because a more concentrated industry faces a less severe free-rider problem. When  $N_1$  is smaller, the downstream industry can *always* solve its collective action problem better, and even without tariff caps, the equilibrium tariff  $t^O(\frac{2}{N_1}, \frac{2}{N_2})$  will be lower. The mechanism that leads to the relationship between  $N_1$  and the tariff reduction is as follows. As we mentioned earlier, the main reason that the government

chooses to reduce its tariff is that when the chances to commit to an tariff cap arise, the pro-trade downstream lobby becomes stronger, i.e.  $\Theta_{01}^* \geq \frac{2}{N_1}$ . The first term of the RHS of (1.47) captures the result of the stronger  $\mathcal{L}_{01}$ . When the private sector knows that the government is negotiating a trade agreement, the downstream firms become more likely to lobby because there is the “natural constraint”: without the influence of the downstream lobby  $\mathcal{L}_{01}$ , the anti-trade lobby  $\mathcal{L}_{02}$  can only induce the government to choose an arbitrarily high bound tariff, which will only result in an applied tariff  $t^O(\frac{2}{N_1}, \frac{2}{N_2})$  in the subgame  $\Gamma$ . When  $N_1$  is smaller, the downstream firms free-ride less on each other *ex post*, resulting in a lower  $t^O(\frac{2}{N_1}, \frac{2}{N_2})$  and thus a more stringent *ex ante* constraint on the joint payoff of the government and the upstream lobby  $\mathcal{L}_{02}$ . Just like a lower tariff ceiling reduces the cost of lobbying for the *ex post* pro-trade lobby and increases its participation share as in section 2.2, this more stringent *ex ante* constraint makes lobbying easier for the group  $\mathcal{L}_{01}$  and more downstream firms participate in *ex ante* lobbying. This means the ex ante participation share  $\Theta_{01}^*$  will be further above  $\frac{2}{N_1}$  (as can be seen in Figure 1.5), and the first term of the RHS of (1.47) becomes larger. Note the second term of the RHS of (1.47) also changes with  $N_1$ , but this will always be dominated by the change in the first term<sup>13</sup>.

#### 1.4 CONCLUDING REMARKS

We have studied the impact of the design of international trade agreements on the organization of competing domestic interest groups. We only considered a small country, since the phenomena of tariff binding overhang are most prominent in small

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<sup>13</sup>Actually, one can show that as  $N_1 \rightarrow \infty$ , this second term first increases and then decreases. This is because (1) when  $N_1$  is smaller,  $\Theta_{01}^*$  will be lower, so there is a smaller pool of downstream firms that are not in the ex ante lobby  $\mathcal{L}_{01}$  by may participate ex post, (2) but ex post they also tend to free-rider less on each other, making the tariff ceiling a more efficient tool to induce downstream lobby participation. When  $N_1$  is smaller, effect (1) dominates, while when  $N_1 \rightarrow \infty$ , effect (2) dominates.

developing countries. Future research may extend the model to a large country to analyze how the tariff caps and binding overhang vary with market power, in the spirit of Beshkar, Bond, and Rho (2012).

## CHAPTER 2

### MARKET POWER, LOBBY PARTICIPATION AND TARIFF BINDING OVERHANG

#### 2.1 INTRODUCTION

The GATT/WTO has played an important role in facilitating trade among its member countries for the past decades. There is now consensus among economists that the rationale of the GATT/WTO can be viewed as to constrain countries from pursuing beggar-thy-neighbor policies. In a two country tariff game, Johnson (1953) shows that in the absence of trade agreements, the two countries can be trapped in a prisoner's dilemma where both of them attempt to lower the international price of imports by setting trade barriers. Bagwell and Staiger (1999) interprets the principles of GATT/WTO as assisting countries in their efforts to escape from the terms-of-trade driven prisoner's dilemma. In reality, the GATT/WTO trade agreements stipulate "bound tariffs", which are upper bounds on tariffs, and a difference between the bound and applied tariffs, which are termed "binding overhang", may exist. Empirical studies have found most of the tariff lines, especially those set by small developing countries, are below the bound tariffs<sup>1</sup>. The theoretical literature on GATT/WTO has proposed fundamental explanations for the existence of tariff caps and binding overhangs in models featuring terms-of-trade externality and uncertainty regarding the environment in which the trade agreements are implemented: when governments possess private information at the implementation stage of the agreements (Bagwell

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<sup>1</sup>For example, see Bacchetta Piermartini (2011) and Beshkar, Bond, and Rho (2012).

and Staiger (2005), Amador and Bagwell (2013)), or there are complexity costs of negotiating and drafting the contracts with respect to various contingencies (Horn, Maggi, and Staiger (2010)), a tariff ceiling can improve upon a rigid tariff and when trade agreement is in place, the applied tariff can sometimes go below the bound one.

However, there is a subtle gap between the models featuring tariff ceilings in the terms-of-trade framework and the empirical fact that a large fraction of the tariffs lines of GATT/WTO members are below the bound tariffs. According to these models, when a tariff line falls below the tariff ceiling specified in the trade agreement, it is simply the unilateral optimal tariff of the government. It is quite striking to deduce from such theories that when binding overhangs exist, the government is just free to excise its market power and the GATT/WTO plays no role in curbing such terms-of-trade manipulation. Moreover, there is evidence that countries do cooperate under positive binding overhangs. Nicita, Olarreaga and Silva (2013) provide evidence that WTO members refrain from manipulating terms of trade in the presence of substantial binding overhangs. In their empirical verification of the terms-of-trade theory of GATT/WTO trade agreements, Ludema and Mayda (2013) use the data of *applied* tariffs, many of which are below the tariff ceilings, to show that while subject to the free-rider problems, the WTO do help countries to internalize terms-of-trade externality.

This paper presents a model in which countries sign trade agreements featuring positive binding overhangs, but the applied tariffs nevertheless eliminate the terms of trade externality. Furthermore, the model features the result that large countries have smaller binding overhangs, as is observed empirically<sup>2</sup>. We extend the model in Qu (2014) to a setting with large countries. Contrary to most models in the literature that incorporate political economy forces by attaching an extra weight on producer

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<sup>2</sup>For example, see Beshkar, Bond, and Rho (2012).

surplus in the government's objective function, we assume firms voluntarily participate in lobbying and in this way, the strength of a lobby group can change under different circumstances. Specifically, we consider a world with two countries, Home and Foreign. Home imports a good from Foreign, and the only policy instrument in the model is an import tariff employed by the Home government. There is an anti-trade group, consisting of firms that produce the imported good and thus benefit from a higher tariff, and a pro-trade group, consisting of firms that use the imported good as inputs and therefore benefit from a lower tariff. The pro-trade and anti-trade firms voluntarily participate in lobbying both in the *ex ante* stage<sup>3</sup> where the governments negotiate toward a trade agreement, and the *ex post* stage where the tariff ceiling stipulated by the trade agreement is in place and the Home government is allowed to set the applied tariff below it. Akin to the logic in Qu (2014), we show when the tariff ceiling is low enough, it exerts downward pressure on the applied tariff, because the tariff ceiling serves as the out-of-equilibrium outcome for the pro-trade lobby group if this group were absent, and a lower tariff ceiling reduces the political contribution the pro-trade lobby needs in order to achieve a certain level of tariff and therefore encourages more pro-trade firms to participate and strengthens the organization of the pro-trade group. As the pro-trade lobby grows stronger relative to the anti-trade one, the applied tariff decreases.

When the two countries are negotiating over a trade agreement, they treat the tariff ceiling as an instrument to achieve a certain level of applied tariff. They reach an agreement on the level of future applied tariff that reflects the producer surplus of the *ex ante* lobbies but not the market power of Home, and a corresponding tariff ceiling that will lead to this desired level of applied tariff. The model generates the stark

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<sup>3</sup>Maggi and Rodriguez-Clare (2007) also investigate the implications of *ex ante* lobbying for trade agreements.

result that the applied tariff that countries implicitly agree upon does not depend on the size or market power of country that sets the tariff. However, the binding overhang negatively relates to the size of the tariff-setting country. This is because the government of a larger country has a greater tendency to manipulate the terms of trade by increasing its tariff, and therefore in lobbying for a lower tariff, the pro-trade lobby needs to incur greater costs of political contributions. Since the trade agreement is intended to fully internalize the terms-of-trade externality and the desired applied tariff bears no relation to the size of the country, to offset the greater cost of lobbying faced by the pro-trade lobby group in a larger country, the tariff ceiling must be lower to improve the pro-trade lobby's out-the-equilibrium path outcome in the tariff-setting game. This leads to a lower binding overhang in equilibrium.

We also show that negotiating over tariff ceilings enable countries to achieve applied tariffs that are lower than if they had directly negotiated over precise applied tariffs. In addition to eliminating the terms-of-trade externality, a tariff ceiling can mobilize more pro-trade firms to participate and contribute ex post, some of which did not participate ex ante (i.e., when the government is negotiating toward a trade agreement). In the model, the ex ante players jointly benefit from the contribution of these new participants. In contrast, a trade agreement specifying a precise applied tariff would just shut down the ex post lobbying game. Therefore, when countries negotiate over tariff ceilings, they have greater incentives to reduce the future applied tariff.

The chapter is organized as follows. Section 2.2 lays out the basic model and characterizes the unilateral trade policy with endogenous lobby participation. Section 2.3 characterizes the trade agreement with endogenous lobby participation both ex ante and ex post. Section 2.4 concludes.

## 2.2 THE BASIC MODEL

### 2.2.1 BASIC STRUCTURE

There are two countries: Home (with \*) and Foreign (without \*). Three goods are produced. Good 0 is a numeraire good that is freely traded. Good 1 is a non-tradable good that is only produced in Home. Good 2 is produced in both countries and Foreign is a natural exporter of this good.

The total labor endowment of Home (Foreign) is  $L$  ( $L^*$ ), which is large enough so that the numeraire good is always produced in equilibrium. In both countries, firms use the Leontief technology to produce the tradable good 2:

$$y_2 = \min\{k_2, l_2\},$$

where  $y_2$  denotes output, and  $k_2$  and  $l_2$  denote the sector specific-capital and labor employed in production.

Firms in Home use good 2 as an intermediate good to produce good 1:

$$y_1 = \min\left\{k_1, l_1, \frac{x_{21}}{\alpha}\right\},$$

where  $y_1$  is the firm's output, and  $k_1$ ,  $l_1$ ,  $\alpha$  respectively denote the sector specific capital, labor and intermediate good 1 used in the production of good 2.

The demand relationships for the two non-numeraire goods in Home are given by  $d_1(p_1) = 1 - p_1$  and  $d_2(p_2) = 1 - p_2$ . The Leontief production functions imply fixed supply for the two goods:  $y_1 = K_1$  and  $y_2 = K_2$ , where  $K_i$  denotes the total stocks of the corresponding sector specific factor in sector  $i$  ( $i = 1, 2$ ).

Now we introduce a parameter  $\lambda > 0$  that measures the size of the Foreign. Specifically, the stock of specific sector for producing the tradable good 2 in Foreign is  $K_2^* = \lambda K_2$ , and therefore the production of good 2 in Foreign is given by  $y_2^* = \lambda y_2$ .

Moreover, the Foreign demand for good 2 is  $d_2^*(p_2) = \lambda d_2(p_2) = \lambda(1 - p_2)$ . Therefore,  $\lambda$  is inversely related to the size and, as we will see, the market power of Home.

Since good 2 is the only tradable good, we now drop the subscripts in variables relating to this good. The Home import demand (taking account of consumer demand and the use as an intermediate good) is

$$m(p) = d(p) + \alpha K_1 - K,$$

where  $p$  is the domestic price of good 2.

The Foreign export supply is

$$e(p^*) = K^* - d^*(p^*) = \lambda(K - d(p^*)),$$

where  $p^*$  is the Foreign price (or world price) of the good.

We assume that the only available trade policy instrument for Home is a specific import tariff  $t$ , and that the Foreign government does not intervene trade since this country exports the good we consider. The price relationship between domestic and world prices is therefore given by  $p = p^* + t$ . Equating Home import demand  $m(p)$  and Foreign export supply  $e(p^*)$ , the domestic and world prices are given by

$$p(t) = \frac{\lambda t + \alpha K_1}{1 + \lambda} + (1 - K), \tag{2.1}$$

$$p^*(t) = \frac{\alpha K_1 - t}{1 + \lambda} + (1 - K). \tag{2.2}$$

Note that a greater tariff  $t$  results in a lower world price  $p^*$ , that is, the tariff policy improves Home's terms of trade. This effect disappears as  $t \rightarrow \infty$ , i.e., Home is a small country and employs no market power.

Welfare of Home can be represented as the sum of consumer surplus<sup>4</sup>, producer profits and tariff revenue:

$$W(t) = CS(p(t)) + \Pi_1(p(t)) + \Pi_2(p(t)) + TR, \quad (2.3)$$

where  $CS$  is the consumer surplus derived from good 2,  $\Pi_i$  is the total profit of sector  $i$  ( $i = 1, 2$ ), and  $TR$  is the tariff revenue. Note  $CS$ ,  $\Pi_1$  and  $\Pi_2$  are functions of the domestic price  $p$ , which is in turn determined by tariff  $t$ .

Since we assume good 1 is only present in Home and only Home (which is the importing country) employs any trade policy, the welfare of Foreign can be written as

$$W^*(t) = CS^*(p^*(t)) + \Pi_2^*(p^*(t)), \quad (2.4)$$

where  $CS^*$  and  $\Pi_2^*$  denote Foreign consumer surplus and the profits of Foreign firms in sector 2.

### 2.2.2 UNILATERAL TRADE POLICY WITH ENDOGENOUS LOBBY PARTICIPATION

In this section we derive the unilateral trade policy set by the Home government. Suppose there are  $N_i$  identical firms in sector  $i$  ( $i = 1, 2$ ). We let the firms voluntarily participate in the lobbying activity, and therefore each sector is only partial “organized” (as opposed to the canonical Grossman-Helpman model where a sector must be either “organized” or not) in that in equilibrium only a fraction of firms participate and other firms will just free-ride. The degree of free-riding of a sector will be determined by the size distribution of firms which, in this case, is represented by the number of firms in this sector. The model will generate the classical result emphasized by Olson (1965) that a sector that is more concentrated will be better at solving its collective action problem.

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<sup>4</sup>we consider only the consumer surplus derived from good 2, since in this model the price of good 1 and therefore the corresponding consumer surplus are fixed.

Specifically, we propose the following tariff-setting game  $\Gamma$ :

**Stage 1:** Each firm in both sectors simultaneously decides whether to enter the next stage.

**Stage 2:** In each sector, the firms that have in the previous stage decided to participate have a “meeting”, which determine the contribution schedule  $Contr_i(t)$  they jointly offer to government (where  $i$  denotes the industry and  $t$  the tariff vector). Firms in each group share the political contributions evenly. Each meeting yields an efficient agreement (for all the participants in the group).

**Stage 3:** The government sets the tariff with the objective function being  $aW(t) + Contr(t)$ , where  $Contr(t) = \sum_{i=1,2} Contr_i(t)$  is the total amount of contribution it will receive from the two competing lobbies when choosing tariff  $t$ , and  $a$  is the weight the government places on social welfare. Then production and consumption take place under the tariff chosen by the government.

#### TARIFF WITH GIVEN SIZES OF THE LOBBY GROUPS

Suppose now  $n_i$  firms choose to participate in Stage 2 of the tariff-setting game  $\Gamma$ , and define the *participation share* of sector  $i$  ( $i = 1, 2$ ) as

$$\Theta_i = \frac{n_i}{N_i}.$$

Thus, a larger  $\Theta_i$  indicates that sector  $i$  is more successful in overcoming the free rider problem.

Stage 2 and 3 of game  $\Gamma$  constitute a common agency game. We invoke the solution concept of “truthful equilibrium” as defined in Bernheim and Whinston (1986), since such an equilibrium has two desirable properties: first, for any strategies chosen by other players, a player in the game can always respond with a “truthful strategy”;

second, the equilibrium is “coalition-proof”, i.e., it cannot be sabotaged by the deviation of a subgroup of players. The equilibristic trade policy will just maximize the joint payoff of all the players in the game, with the first order condition

$$\frac{\partial}{\partial t}(aW + \Theta_1\Pi_1 + \Theta_2\Pi_2) = 0.$$

The profits earned by the two sectors are  $\Pi_1 = p_1K_1 - K_1 - p\alpha K_1$  and  $\Pi_2 = pK - K$ , with the partial derivatives  $\frac{\partial\Pi_1}{\partial t} = -\alpha K_1 \frac{\partial p}{\partial t} = -\alpha K_1 \left(\frac{\lambda}{1+\lambda}\right)$  and  $\frac{\partial\Pi_2}{\partial t} = K \frac{\partial p}{\partial t} = K \left(\frac{\lambda}{1+\lambda}\right)$ . The marginal change of social welfare with respect to tariff  $t$  can be written as  $\frac{\partial W}{\partial t} = \frac{\partial}{\partial t}(\Pi_1 + \Pi_2 + CS + TR) = \frac{\partial p}{\partial t}(-\alpha y_1 + y - d) + m + t \frac{\partial m}{\partial t} = -m \frac{\partial p^*}{\partial t} + t \frac{\partial m}{\partial t}$ . We now characterize the equilibrium tariff with given sizes of the two lobby groups:

**Lemma 2.1** *When the participation shares of the sectors are  $\Theta_1$  and  $\Theta_2$ , the equilibrium tariff is given by*

$$t^O = \underbrace{\left(\frac{-\Theta_1\alpha K_1 + \Theta_2 K}{a}\right)}_{\text{(Political Economy Effect)}} + \underbrace{\frac{m}{\lambda}}_{\text{(Terms of Trade Effect)}}. \quad (2.5)$$

There are two components of the above expression. The first term  $\left(\frac{-\Theta_1\alpha K_1 + \Theta_2 K}{a}\right)$  captures the political economy channel in the model. Note sector 2 is *anti-trade* since it produces the tradable good, and sector 1 is *pro-trade* because it uses the tradable good as an intermediate good. The equilibrium tariff increases in the size of the anti-trade lobby (represented by  $\Theta_2 K$ ) and decrease in the size of the pro-trade lobby represented by  $\Theta_1\alpha K_1$ ).

The second term captures the fact that the government manipulates the terms of trade to improve its welfare, as first prove by Johnson (1954). When the size of the country becomes smaller (as represented by a smaller  $\lambda$ ), the importance of this term diminishes.

Using equation (2.5), we can also solve for the closed form solution for  $t^O$ :

$$t^O(\Theta_1, \Theta_2) = \left( \frac{-\Theta_1 \alpha K_1 + \Theta_2 K}{a} \right) \left( \frac{1 + \lambda}{2 + \lambda} \right) + \frac{\alpha K_1}{2 + \lambda}. \quad (2.6)$$

#### ENDOGENOUS LOBBY PARTICIPATION

We now go back to the first stage of the tariff setting game  $\Gamma$  to pin down the sizes of the two lobbies. As is shown by Laussel and Le Breton (2001), in a common agency game with two competing groups of bidders, the equilibrium contribution of one group is the joint welfare loss of the principal (in this case, the government) and the other competing group due to the first group's existence. Given the sizes of groups  $\Theta_1$  and  $\Theta_2$ , the equilibrium contribution of group  $i$  is

$$C_i^* = \underbrace{\max_t \{aW(\cdot) + \Theta_j \Pi_j(\cdot)\}}_{\text{(Joint payoff of the government and lobby } j \text{ if lobby } i \text{ were absent.)}} - \underbrace{\left( aW(t^O(\Theta_1, \Theta_2)) + \Theta_j \Pi_j(t^O(\Theta_1, \Theta_2)) \right)}_{\text{(Joint payoff of the government and lobby } j \text{ in equilibrium.)}} \quad (i \neq j). \quad (2.7)$$

In general,  $C_i^*$  depends on both  $\Theta_1$  and  $\Theta_2$ , which renders the problem untractable. However, in this model with fixed production and linear demand,  $C_i^*$  is just a function of  $\Theta_i$ :

$$C_1^*(\Theta_1) = \frac{1}{2} \frac{(\alpha K_1)^2}{a} \left( \frac{\lambda}{2 + \lambda} \right) \Theta_1^2, \quad (2.8)$$

$$C_2^*(\Theta_2) = \frac{1}{2} \frac{K^2}{a} \left( \frac{\lambda}{2 + \lambda} \right) \Theta_2^2. \quad (2.9)$$

$C_i^*(\Theta_i)$  represents the cost lobby group  $i$  must bear when its size is  $\Theta_i$ . Next we look at how much a lobby can collect from its members. Define the *willingness to pay* of a firm as the increase in its profit due to the change in the tariff in the favorable direction resulting from this firm's participation:

$$w_1 = \frac{1}{N_1} \Pi_1(t^O(\Theta_1, \Theta_2)) - \frac{1}{N_1} \Pi_1\left(t^O\left(\Theta_1 - \frac{1}{N_1}, \Theta_2\right)\right),$$

$$w_2 = \frac{1}{N_2} \Pi_2(t^O(\Theta_1, \Theta_2)) - \frac{1}{N_2} \Pi_2(t^O(\Theta_1, \Theta_2 - \frac{1}{N_2})),$$

where  $w_i$  represents a firm's willingness to pay in sector  $i$ .

Then the willingness to pay of the lobby group  $i$  is

$$\mathcal{W}_i = n_i w_i,$$

where  $n_i$  is the number of firms in lobby  $i$ . In this stylized economic structure,  $\mathcal{W}_i$  turns out to be a function of  $\Theta_i$ , i.e., the maximum amount of political contribution a lobby group can collect from its members depend on the size of its size, but not on the size of the other competing lobby group.

$$\mathcal{W}_1(\Theta_1) = \frac{(\alpha K_1)^2}{a} \left( \frac{\lambda}{2 + \lambda} \right) \frac{\Theta_1}{N_1}, \quad (2.10)$$

$$\mathcal{W}_2(\Theta_2) = \frac{K^2}{a} \left( \frac{\lambda}{2 + \lambda} \right) \frac{\Theta_2}{N_2}. \quad (2.11)$$

The amount of contribution of lobby group  $i$  cannot exceed  $\mathcal{W}_i$ , otherwise at least one firm will contribute an amount that is greater than this firm's willingness to pay, and it will be better off with free-riding and not join the group in the first place. Also note that when  $N_i$  is smaller, i.e., the sector is more concentrated,  $\mathcal{W}_i$  will be greater. Thus when the sector is composed of a smaller number of larger firms, the lobby group, given its size, will be able to collect more contribution from its members.

The equilibrium participation share (or size) of lobby group  $i$   $\Theta_i^*$  is found by equating the equilibrium contribution with the group's willingness to pay:

$$\mathcal{W}_i(\Theta_i^*) = C_i^*(\Theta_i^*).$$

Note when solving for  $\Theta_i^*$ , we have allowed the measure of firms (and thus  $\Theta_i$ ) to be continuous. When  $\mathcal{W}_i(\Theta_i)$  is smaller than  $C_i^*(\Theta_i)$ , the size of the group cannot be maintained because some firm will choose to free-ride in stage 1 of game  $\Gamma$ ; when

$\mathcal{W}_i(\Theta_i)$  is greater than  $C_i^*(\Theta_i)$ , some firm outside the lobby group will be better off with participation, and therefore the size of the group tends to grow. We now state the following lemma:

**Lemma 2.2** *In the unilateral policy setting game, the equilibrium participation share of sector  $i$  ( $i = 1, 2$ ) is given by  $\Theta_i^* = \frac{2}{N_i}$ .*

Then using Lemma (2.1) and equation (2.6), the equilibrium unilateral tariff is

$$\begin{aligned}
 t^O &= \underbrace{\left( \frac{-\left(\frac{2}{N_1}\right)\alpha K_1 + \left(\frac{2}{N_2}\right)K}{a} \right)}_{\text{(Lobbying)}} + \underbrace{\frac{m}{\lambda}}_{\text{(Market Power)}} \\
 &= \left( \frac{-\left(\frac{2}{N_1}\right)\alpha K_1 + \left(\frac{2}{N_2}\right)K}{a} \right) \left( \frac{1+\lambda}{2+\lambda} \right) + \frac{\alpha K_1}{2+\lambda}. \tag{2.12}
 \end{aligned}$$

We have now fully pinned down the endogenous tariff  $t^O$  in the unilateral setting. It depends on the market power employed by the country, and the sizes of the pro-trade and anti-trade lobbies, which in turn hinge on the degree of concentration of the relevant sector (which is represented by the number of firms in the sector).

### 2.3 TRADE AGREEMENTS WITH ENDOGENOUS LOBBY PARTICIPATION

In this section we fully characterize the trade agreements between two countries. To sharply focus on the the relationship between the Home bound and applied tariffs, we abstract from the lobbying of Foreign firms. The full game  $\Gamma_0$  determining the trade agreement is defined as follows:

**Stage 1:** Each firm in Home voluntarily decides whether to participate in the next stage.

**Stage 2:** The two governments negotiates over a trade agreement that can set a tariff cap  $\bar{t} \in (-\infty, +\infty)$ . Those Home firms that have chosen to participate in sector

$i$  form an *ex ante* lobby group  $\mathcal{L}_{0i}$  ( $i = 1, 2$ ), which in turn offers the government a contribution scheduling  $Contr_{0,i}(\cdot)$ , with political contributions depending on the tariff cap specified in the trade agreement.

**Later stages:** The events of  $\Gamma$  unfold, which have been described in Section 2.2.2.

Note that we distinguish between *ex ante* lobbying, which takes place when the government is negotiating toward a trade agreement, and *ex post* lobbying, which occurs when the trade agreement leaves the government some discretion over the applied tariff, as in Maggi and Rodriguez-Clare (2007). The contribution of this analysis is that we explicitly take account of the fact that interest groups constantly need to overcome the free rider problem, and some groups can sometimes appear more active. This is again achieved by letting firms voluntarily participate in the tariff-ceiling-setting game  $\Gamma_0$ .

### 2.3.1 EXOGENOUS TARIFF CAP

We solve game  $\Gamma_0$  with backward induction. Suppose now Home has committed to set its tariff below a tariff cap  $\bar{t}$ . When  $\bar{t}$  is arbitrarily high, it certainly will not change any part of the equilibrium of Home in any way. However, we consider a case where  $\bar{t}$  is higher than the equilibrium applied tariff but satisfies

$$\bar{t} < t^O(0, \Theta_2^*). \quad (2.13)$$

$t^O(0, \Theta_2^*)$  is the off-the-equilibrium-path tariff if the pro-trade lobby group were absent, i.e.,  $\Theta_1 = 0$ . We now show when the condition in (2.13) is met,  $\bar{t}$  is low enough to affect the equilibrium applied tariff. The equilibrium contribution of the

pro-trade lobby representing sector 1 is

$$\begin{aligned} \tilde{C}_1(\Theta_1, \Theta_2; \bar{t}) &= \underbrace{(aW(\bar{t}) + \Theta_2\Pi_2(\bar{t}))}_{\text{(Joint payoff of the government and the anti-trade lobby without the pro-trade lobby.)}} \\ &- \underbrace{(aW(t^O(\Theta_1, \Theta_2)) + \Theta_2\Pi_2(t^O(\Theta_1, \Theta_2)))}_{\text{(Joint payoff of the government and the anti-trade lobby in equilibrium.)}}. \end{aligned} \quad (2.14)$$

The condition in equation (2.13) ensures that  $(aW(\bar{t}) + \Theta_2\Pi_2(\bar{t})) < \max_t\{aW(\cdot) + \Theta_2\Pi_2(\cdot)\}$ , which is the first term in the equilibrium contribution equation (2.7) when there is no external constraint faced by the country. Therefore, when participation shares are  $\Theta_1$  and  $\Theta_2$ , the equilibrium tariff is still given by  $t^O(\Theta_1, \Theta_2)$  if  $t^O(\Theta_1, \Theta_2) < \bar{t}$ , but the pro-trade contribution needed to incur the equilibrium tariff has decreased due to the imposition of  $\bar{t}$  (or a lower  $\bar{t}$ ).

Meanwhile, the contribution of the anti-trade lobby group is still represented by equation (2.7), since there is no lower bound on tariff, and if the anti-trade lobby were not present, the joint payoff of the government and the pro-trade lobby would not change due to the tariff ceiling. Therefore the result for the equilibrium participation share of the anti-trade sector 2 is the same as in Section 2.2.2: equating the equilibrium contribution  $C_2(\Theta_2)$  with this group's willingness to pay  $\mathcal{W}_2(\Theta_2)$ , we still find

$$\Theta_2^* = \frac{2}{N_2}.$$

In contrast, the tariff cap will affect the lobby participation decision of the pro-trade firms in the model and thus change the size of the pro-trade lobby group. Given  $\bar{t}$  that satisfies equation (2.13), the size of the pro-trade lobby is characterized by

$$\mathcal{W}_1(\Theta_1^*) = \tilde{C}_1(\Theta_1^*, \Theta_2^*; \bar{t}). \quad (2.15)$$

We now states the following lemma which characterizes how an exogenously given tariff cap affect the size of the pro-trade lobby and therefore the equilibrium applied tariff. The detail of the derivation is relegated to Appendix B.1.

**Lemma 2.3** *Given a tariff cap  $\bar{t} \in [t^O(1, \frac{2}{N_2}), t^O(0, \frac{2}{N_2})]$ , the applied tariff decreases with  $\bar{t}$  with positive binding overhang (i.e.,  $t^* < \bar{t}$ ). Specifically,*

$$t^* = t_{-1} - \left( \frac{\eta_1}{N_1} + \sqrt{\left( \frac{\eta_1}{N_1} \right)^2 + (t_{-1} - \bar{t})^2} \right) \quad (2.16)$$

and the participation share of the pro-trade group is

$$\Theta_1^* = \frac{1}{N_1} + \sqrt{\frac{1}{N_1^2} + \left( \frac{t_{-1} - \bar{t}}{\eta_1} \right)^2}, \quad (2.17)$$

where  $\eta_1 \equiv \left( \frac{\alpha K_1}{a} \right) \left( \frac{1+\lambda}{2+\lambda} \right)$ , and  $t_{-1} \equiv t^O(0, \frac{2}{N_2}) = \left( \frac{\frac{2}{N_2} K}{a} \right) \left( \frac{1+\lambda}{2+\lambda} \right) + \frac{\alpha K_1}{2+\lambda}$  is the out-of-equilibrium tariff if the pro-trade lobby were absent.

In this model, given the same tariff ceiling  $\bar{t}$  countries with a larger size (and thus market power) also exhibit a greater difference between the bound and applied tariff. To see this, let  $h = \bar{t} = t^*$  denotes the “binding overhang” and use (2.14) to write the pro-trade contribution in terms of  $h$ :

$$\hat{C}_1(h) = a \left( \frac{\lambda}{1+\lambda} \frac{2+\lambda}{1+\lambda} \right) \left( \frac{1}{2} h^2 + (t_{-1} - \bar{t}) h \right).$$

We then use (2.6) and (2.10) to write the willingness to pay of the pro-trade group in terms of  $h$ :

$$\hat{\mathcal{W}}_1(h) = \alpha K_1 \left( \frac{\lambda}{1+\lambda} \right) \frac{(t_{-1} - \bar{t}) + h}{N_1}.$$

The equilibrium binding overhang  $h^*$  can be found by equating  $\hat{C}_1(h^*)$  with  $\hat{\mathcal{W}}_1(h^*)$ , which leads to the following equation:

$$\frac{1}{2} h^2 + (t_{-1} - \bar{t}) h = \frac{\eta_1 \left( (t_{-1} - \bar{t}) + h \right)}{N_1}. \quad (2.18)$$

When  $\lambda$  is smaller (i.e., the country has larger market power), the out-of-equilibrium tariff  $t_{-1}$  will be greater, simply because the country will raise the tariff to gain from a better terms of trade. Therefore, as is reflected in the formula of  $\hat{C}_1(h)$  as well as

the LHS of (2.18), the pro-trade lobby needs to incur a larger amount of political contribution to bring the tariff down from  $\bar{t}$  and induce a binding overhang  $h$ . As a result, it achieves a smaller amount of  $h$  (and a higher applied tariff) in equilibrium given the same tariff ceiling  $\bar{t}$ , which is implied by equation (2.18).

As is in practice, we can also define the binding overhang in the ad valorem form:  $\frac{h}{p^*}$ . The next lemma shows the above property of the model that larger countries have a smaller binding overhang also extends to this definition. Appendix B.2 provides its proof.

**Lemma 2.4** *Given a tariff cap  $\bar{t} \in [t^O(1, \frac{2}{N_2}), t^O(0, \frac{2}{N_2})]$ , the “water” in the tariff ceiling  $\left(\frac{h}{p^*}\right)$  increases in  $\lambda$ , namely, the measure of tariff binding overhang decreases in the size of the country.*

### 2.3.2 ENDOGENOUS TRADE AGREEMENTS

#### THE TRADE AGREEMENT WITH EXOGENOUS EX ANTE LOBBIES

Let the number of participating firms in sector  $i$  in stage 2 of the tariff-ceiling-setting game  $\Gamma_0$  be  $n_{0i}$  ( $i = 1, 2$ ). The ex ante participation share of sector  $i$  is

$$\Theta_{0i} = \frac{n_{0i}}{N_i}.$$

As in Maggi and Rodriguez-Clare (2007), the trade agreement maximizes the joint surplus of the two governments and the lobby groups<sup>5</sup>. Since we abstract from lobbying in Foreign, the ex ante payoff of the Foreign government  $G_\Gamma^*$  is just the social welfare level  $aW^*(t)$ <sup>6</sup> incurred by Home tariff  $t$ . The ex ante payoff of Home government  $G_\Gamma$  (i.e., the payoff of Home government in the tariff-setting game  $\Gamma$ ) takes account of the

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<sup>5</sup>International transfers must be allowed to justify this result.

<sup>6</sup>To simplify notations, we normalized  $G_\Gamma^*$  by multiplying it by the parameter  $a$ .

Home welfare level  $W(t)$  and the ex post political contribution from two competing lobbies  $\mathcal{W}_1$  and  $\mathcal{W}_2$ <sup>7</sup>:

$$G_\Gamma = aW + \mathcal{W}_1 + \mathcal{W}_2.$$

The ex ante payoff of sector  $i$  can be written as

$$V_i = \Pi - \mathcal{W}_i,$$

and the ex ante payoff of the lobby group  $\mathcal{L}_{0i}$  is  $\Theta_{0i}V_i$  when its size is  $\Theta_{0i}$ .

The trade agreement then maximizes the objective function  $\Psi$ :

$$\Psi = G_\Gamma^* + G_\Gamma + \Theta_{01}V_1 + \Theta_{02}V_2.$$

Rearranging terms yields

$$\Psi = (a(W + W^*) + \Theta_{01}\Pi_1 + \Theta_{02}\Pi_2) + ((1 - \Theta_{01})\mathcal{W}_1 + (1 - \Theta_{02})\mathcal{W}_2). \quad (2.19)$$

The first term  $(a(W + W^*) + \Theta_{01}\Pi_1 + \Theta_{02}\Pi_2)$  simply takes account of the social welfare of the two countries and the surpluses of the lobby groups. The second term  $((1 - \Theta_{01})\mathcal{W}_1 + (1 - \Theta_{02})\mathcal{W}_2)$  stems from free riding: the ex ante players in stage 2 of the tariff-ceiling-setting game  $\Gamma_0$  jointly benefit from more ex post political contribution in the tariff-setting game  $\Gamma$ , since the burden of the ex post contribution is share by some firms not in the ex ante lobby groups. Note when  $\Theta_{0i} = 1$  ( $i = 1, 2$ ), the players will not benefit from more contribution in game  $\Gamma$  and the second term disappears, since all of the firms participate in ex ante lobbying and there is not any firm outside the ex ante lobby groups that can share the contribution ex post. Thus the model has the feature that *the ex ante players are prone to inducing more intense ex post political competition when the trade agreement leaves the government some discretion over the policy choice.*

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<sup>7</sup>To simplify notations, here we use the willingness to pay to denote contributions.

We solve the game when  $\bar{t} \in [t^O(1, \frac{2}{N_2}), t^O(0, \frac{2}{N_2})]$ . As Lemma 2.4, when the tariff ceiling is in this range, the bound and applied tariff co-moves and there is a one-to-one mapping from one to another. Therefore, in solving the model we think of governments as choosing the applied tariff  $t$  although they are really choosing  $\bar{t}$ . In other words, the tariff ceiling is treated as instrumental in achieving a certain level of applied tariff.

We now look at how marginal change in  $t$  affect the terms in the objective function (2.19). The marginal change in Home and Foreign welfare with respect to  $t$  can be expressed as  $\frac{\partial W}{\partial t} = \frac{\partial}{\partial t}(\Pi_1 + \Pi_2 + CS + TR) = -m \frac{\partial p^*}{\partial t} + t \frac{\partial m}{\partial t}$  and  $\frac{\partial W^*}{\partial t} = \frac{\partial}{\partial t}(CS^* + \Pi_2^*) = m \frac{\partial p^*}{\partial t}$ . Therefore, the tariff affects the world welfare ( $W + W^*$ ) in the following way:

$$\frac{\partial(W + W^*)}{\partial t} = t \frac{\partial m}{\partial t} = - \left( \frac{\lambda}{1 + \lambda} \right) t. \quad (2.20)$$

A higher tariff unambiguously reduces world welfare ( $W + W^*$ ), as now the terms of trade externality has been internalized in the calculation. When  $\lambda \rightarrow \infty$ , i.e., the country is very small, then effect of its policy on world welfare can be ignored.

Next we look at how tariff ceiling (represented by the corresponding applied tariff  $t$ ) affects the ex post contribution  $\mathcal{W}_1$  and  $\mathcal{W}_2$  in the objective function (2.19). As we have shown, the tariff ceiling will not affect the equilibrium participation share  $\Theta_2^*$  for the anti-trade group, since it is an upper bound on tariff. According to equation (2.11), the political contribution remains fixed at  $\mathcal{W}_2(\Theta_2^*)$ . On the other hand, as Lemma 2.3 shows, the equilibrium size of the pro-trade lobby  $\Theta_1^*$  increases when  $\bar{t}$  is lower. Then according to (2.10), the pro-trade contribution of the pro-trade lobby group  $\mathcal{W}_1(\Theta_1^*)$  will also increase. Given the fixed size of the anti-trade lobby  $\Theta_2^*$ , equation (2.6) gives us the relationship between equilibrium applied tariff  $t$  and the size of the pro-trade lobby  $\Theta_1$ :

$$t = \left( \frac{-\Theta_1 \alpha K_1 + \Theta_2^* K}{a} \right) \left( \frac{1 + \lambda}{2 + \lambda} \right) + \frac{\alpha K_1}{2 + \lambda}, \quad (2.21)$$

where  $\Theta_2^* = \left(\frac{2}{N_2}\right)$ . Equation (2.21) implicitly leads to a function  $\tilde{\Theta}_1(t)$ . The willingness to pay of the pro-trade lobby can then be written as a function of equilibrium tariff  $t$ :

$$\tilde{\mathcal{W}}_1(t) = \mathcal{W}_1(\tilde{\Theta}_1(t)).$$

The the marginal change of the ex post pro-trade contribution with respect to the equilibrium tariff  $t$  (which is induced by a certain  $\bar{t}$ ) is

$$\frac{\partial \mathcal{W}_1}{\partial t} = -\frac{1}{N_1} \alpha K_1 \left( \frac{\lambda}{1 + \lambda} \right). \quad (2.22)$$

A lower tariff cap  $\bar{t}$  (inducing a lower applied tariff  $t$ ) will increase the size and therefore the contribution of the pro-trade lobby group, as in the above expression of  $\frac{\partial \mathcal{W}_1}{\partial t}$ . When the number of pro-trade firms  $N_1$  gets greater, this effect becomes weaker (as exemplified by a smaller  $\frac{\partial \mathcal{W}_1}{\partial t}$ ) due to the fact that firms free-ride more on each other and it is more difficult to use the tariff cap as an instrument to make the sector more organized.

We are now ready to characterize the equilibrium when the sizes of the ex ante lobbies are exogenously given. Using the first order condition  $\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial t} (a(W + W^*)) + \frac{\partial}{\partial t} (\Theta_{01}\Pi_1 + \Theta_{02}\Pi_2) + \frac{\partial \tilde{\mathcal{W}}_1}{\partial t} = 0$ , we arrive at the following lemma.

**Lemma 2.5** *Given the sizes of the ex ante lobbies  $\Theta_{0i}$  ( $i = 1, 2$ ) at stage 2 of the tariff-ceiling-setting game  $\Gamma_0$ , the trade agreement will specify a tariff ceiling that results in an applied tariff*

$$t^*(\Theta_{01}, \Theta_{02}) = \left( \frac{-\Theta_{01}\alpha K_1 + \Theta_{02}K}{a} \right) - \frac{\alpha K_1}{aN_1} (1 - \Theta_{01}), \quad (2.23)$$

and the tariff ceiling  $\bar{t}$  is implicitly given by

$$t^* = t_{-1} - \left( \frac{\eta_1}{N_1} + \sqrt{\left( \frac{\eta_1}{N_1} \right)^2 + (t_{-1} - \bar{t})^2} \right),$$

where  $\eta_1 \equiv \left(\frac{\alpha K_1}{a}\right) \left(\frac{1+\lambda}{2+\lambda}\right)$ , and  $t_{-1} \equiv t^O(0, \frac{2}{N_2}) = \left(\frac{\frac{2}{N_2}K}{a}\right) \left(\frac{1+\lambda}{2+\lambda}\right) + \frac{\alpha K_1}{2+\lambda}$  is the out-of-equilibrium tariff if the pro-trade lobby were absent.

The first term  $\left(\frac{-\Theta_{01}\alpha K_1 + \Theta_{02}K}{a}\right)$  in (2.23) simply captures the effects of tariff on the producer surpluses of the competing lobby groups. The tariff does not reflect Home's market power since the terms-of-trade externality has been internalized with the trade agreement, as opposed to (2.5) in the unilateral setting. Moreover, since a lower tariff ceiling will lead to more ex post political contribution from the pro-trade lobby group, the ex ante players will agree to a even lower ex post tariff, as is reflected by the second term  $-\frac{\alpha K_1}{aN_1}(1 - \Theta_{01})$  in (2.23).

#### ENDOGENOUS EX ANTE LOBBY PARTICIPATION

Now we go back to stage 1 of the tariff-ceiling-setting game  $\Gamma_0$  and pin down the sizes of the ex ante lobbies. Recalling that in stage 2 of  $\Gamma_0$ , by choosing a tariff ceiling  $\bar{t}$ , the governments are as if choosing a future applied tariff  $t^*$ . In this setting, the principals of the common agency game are the ex ante lobbies, and the agent is a player that has a payoff function representing the payoff of the two governments:  $a(G_\Gamma + G_\Gamma^*)(\cdot)$ . The logic in Section 2.2.2 applies, that the contribution of one principal depends on what the agent and the other competing principal can jointly achieve absent the first principal. Therefore, the ex ante contribution of group  $i$  can be written as

$$C_{0i}^* = \underbrace{\max_{t^*} \{a(G_\Gamma + G_\Gamma^*)(\cdot) + \Theta_{0j}V_j(\cdot)\}}_{\text{(Joint payoff of the governments and ex ante lobby } j \text{ absent lobby } i.)} - \underbrace{\left(a(G_\Gamma + G_\Gamma^*)(t^O(\Theta_{01}, \Theta_{02})) + \Theta_{0j}V_j(t^O(\Theta_{01}, \Theta_{02}))\right)}_{\text{(Joint payoff of the governments and ex ante lobby } j \text{ in equilibrium.)}} \quad (i \neq j). \quad (2.24)$$

If the ex ante pro-trade lobby group were absent, the governments would have set a higher tariff cap. We assume this tariff cap will still put downward pressure on the future tariff. Let

$$t_{-01} = t^*(0, \frac{2}{N_2}) = \frac{\frac{2}{N_2}K}{a} - \frac{\alpha K_1}{aN_1},$$

which denotes the out-of-equilibrium future tariff absent the ex ante pro-trade lobby, and

$$t^O(\frac{2}{N_1}, \frac{2}{N_2}) = \left( \frac{-\frac{2}{N_1}\alpha K_1 + \frac{2}{N_2}K}{a} \right) \left( \frac{1+\lambda}{2+\lambda} \right) + \frac{\alpha K_1}{2+\lambda},$$

which is the future tariff is the tariff ceiling is arbitrarily high. We solve the model in the case in which  $t_{-01} \leq t^O(\frac{2}{N_1}, \frac{2}{N_2})$ .

**Assumption 2.1** *Assume  $\lambda$  is small enough (or the country is large enough) such that  $\frac{\alpha K_1}{aN_1} - \frac{\alpha K_1 - \left( \frac{-\frac{2}{N_1}\alpha K_1 + \frac{2}{N_2}K}{a} \right)}{2+\lambda} \leq 0$ .*

According to the above assumption, even without the ex ante pro-trade lobby group, the countries will still sign a trade agreement that lowers the future applied tariff with a bound one, since the the size of the home country (represented by  $\frac{1}{\lambda}$ ) and thus is market power are large enough. The focus in this paper differs from that in Qu (2014), because here we model the interplay between a country's market power and the organization of the competing lobby groups. Qu (2014) considers a small country with no market power, and shows that at the stage where the home government is negotiating toward a trade agreement that may stipulate a tariff ceiling, the pro-trade group becomes more "organized" while the anti-trade remains the same. This is due to the fact that for a small country, if the pro-trade lobby were absent in the ex ante stage, the tariff ceiling would have been arbitrarily high and have had no effect in the future. In that model, the stronger pro-trade ex ante lobby group pushes the government to enter the trade agreement that will reduce the future applied tariff

with a tariff ceiling. In contrast, in the model in this paper, as we will see, the ex ante pro-trade lobby group will not become more organized when they know about the opportunity for the governments to sign a trade agreement. The trade agreement and the resulting tariff cut are for the internalization of the terms-of-trade effect, and when the bound and applied tariffs do not coincide, the trade agreement still achieves its purpose since the ex ante players have anticipated that the tariff ceiling can indirectly reduce the applied tariff by making the ex post pro-trade group more organized.

Under Assumption 2.1, the ex ante contribution can be written as a function of the size of the corresponding lobby group:

$$C_{01}^*(\Theta_{01}) = \frac{1}{2} \left(1 - \frac{1}{N_1}\right)^2 \frac{(\alpha K_1)^2}{a} \left(\frac{\lambda}{1+\lambda}\right) \Theta_{01}^2, \quad (2.25)$$

$$C_{02}^*(\Theta_{02}) = \frac{1}{2} \frac{K^2}{a} \left(\frac{\lambda}{1+\lambda}\right) \Theta_{02}^2. \quad (2.26)$$

The formulas for the ex ante willingness to pay can be derived with the same logic as in Section 2.2.2. The marginal changes of the ex ante payoffs of the two sectors with respect to the applied tariff (induced by a corresponding tariff cap according to (2.16)) are given by

$$\begin{aligned} \frac{\partial V_1}{\partial t} &= \frac{\partial \Pi_1}{\partial t} - \frac{\partial \mathcal{W}_1}{\partial t} = \left(\frac{\lambda}{1+\lambda}\right) \left(1 + \frac{1}{N_1}\right) (-\alpha K_1), \\ \frac{\partial V_2}{\partial t} &= \frac{\partial \Pi_2}{\partial t} = \left(\frac{\lambda}{1+\lambda}\right) K. \end{aligned}$$

In the tariff-ceiling-setting game  $\Gamma_0$ , denote the willingness to pay of a firm in sector  $i$  ( $i = 1, 2$ ) with  $w_{0i}$ . We have

$$\begin{aligned} w_{01} &= \frac{1}{N_1} V_1(t^*(\Theta_{01}, \Theta_{02})) - \frac{1}{N_1} V_1\left(t^*\left(\Theta_{01} - \frac{1}{N_1}, \Theta_{02}\right)\right), \\ w_{02} &= \frac{1}{N_2} V_2(t^*(\Theta_{01}, \Theta_{02})) - \frac{1}{N_2} V_2\left(t^*\left(\Theta_{01}, \Theta_{02} - \frac{1}{N_2}\right)\right). \end{aligned}$$

We then define the willingness to pay of a *group* of sector  $i$ 's firms

$$\mathcal{W}_{0i} = n_i w_{0i},$$

where  $n_i$  denotes the number of firms the group consists of.

The willingness to pay of an ex ante lobby group can therefore be written as a function of the size of this group:

$$\mathcal{W}_{01}(\Theta_{01}) = \frac{1}{N_1} \left(1 - \frac{1}{N_1}\right)^2 \frac{(\alpha K_1)^2}{a} \left(\frac{\lambda}{1+\lambda}\right) \Theta_{01}, \quad (2.27)$$

$$\mathcal{W}_{02}(\Theta_{02}) = \frac{1}{N_2} \frac{K^2}{a} \left(\frac{\lambda}{1+\lambda}\right) \Theta_{02}. \quad (2.28)$$

The equilibrium ex ante participation share  $\Theta_{0i}^*$  can be found with the equation

$$C_{01}^*(\Theta_{0i}^*) = \mathcal{W}_{0i}(\Theta_{0i}^*) \quad (i = 1, 2).$$

The above equation immediately leads to the result that  $\Theta_{0i}^* = \frac{2}{N_i}$  ( $i = 1, 2$ ). The participation shares of the tariff-ceiling setting game  $\Gamma_0$  are exactly the same as those in the tariff-setting game  $\Gamma$  if there is no tariff ceiling imposed on the latter game. We are now ready to fully characterize the trade agreement that endogenously emerges from  $\Gamma_0$ .

**Proposition 2.1** *Under Assumption 2.1, the two countries will sign a trade agreement that specifies a tariff ceiling  $\bar{t}$ . The resulting applied tariff is*

$$t^* = \left( \frac{-\frac{2}{N_1} \alpha K_1 + \frac{2}{N_2} K}{a} \right) - \frac{\alpha K_1}{aN_1} \left(1 - \frac{2}{N_1}\right), \quad (2.29)$$

and the tariff ceiling  $\bar{t}$  is implicitly given by

$$t^* = t_{-1} - \left( \frac{\eta_1}{N_1} + \sqrt{\left(\frac{\eta_1}{N_1}\right)^2 + (t_{-1} - \bar{t})^2} \right), \quad (2.30)$$

where  $\eta_1 \equiv \left(\frac{\alpha K_1}{a}\right) \left(\frac{1+\lambda}{2+\lambda}\right)$ , and  $t_{-1} \equiv t^O(0, \frac{2}{N_2}) = \left(\frac{\frac{2}{N_2} K}{a}\right) \left(\frac{1+\lambda}{2+\lambda}\right) + \frac{\alpha K_1}{2+\lambda}$  is the out-of-equilibrium tariff if the pro-trade lobby were absent.

A notable feature of the equilibrium applied tariff  $t^*$  that emerges from the tariff-ceiling-setting game is that it does not depend on the measure of the country's size or market power,  $\frac{1}{\lambda}$ . Comparing (2.29) with the equilibrium tariff formula (2.12), we see that the term reflecting Home market power disappears. The trade agreement will specify a tariff ceiling as in (2.30) which is instrumental in achieving the desired level of applied tariff. As opposed to the models with unverifiable contingencies or contracting costs, in this model countries are able to eliminate the terms-of-trade externalities when the applied tariff is still below the bound one. Moreover, there is a further reduction in the future tariff due to the design of the trade agreement: the ex ante players would agree to have an additional tariff cut  $\frac{\alpha K_1}{aN_1}(1 - \frac{2}{N_1})$ , as in (2.29), since a lower tariff ceiling induces greater lobby competition in the ex post tariff-setting game.

Although the trade agreement fully eliminates the terms-of-trade externality and the resulting applied tariff is irrelevant of the market power of the country, the model generates the result that larger countries have a lower “binding overhang”, as is observed empirically.

**Proposition 2.2** *When  $\lambda$  is smaller (i.e., the size of Home is larger), the binding overhang, either defined as  $h = \bar{t} - t^*$  or  $\hat{h} = \frac{h}{p^*}$ , becomes smaller.*

**Proof.** With a smaller  $\lambda$ , Lemma 2.4 shows if  $\bar{t}$  is the same, the binding overhang will be smaller. However, since the equilibrium applied tariff keeps the same, as is shown in Proposition 2.1, the trade agreement now must specify a lower tariff ceiling, will lead to an even smaller binding overhang. ■

The intuition behind the above result is simple. The trade agreement reduces the tariff of the home country to a level that does not reflect its market power. It achieves this goal through a tariff ceiling that improves the off-the-equilibrium-path outcome

for the pro-trade group. Specifically, in the model the pro-trade lobby bring the tariff down from the tariff ceiling  $\bar{t}$  to the equilibrium level. For a larger country, lowering tariff will incur greater loss in social welfare as the government forgoes its power to manipulate the terms of trade, and therefore to lower the tariff marginally, the pro-trade lobby will need a larger amount of political contribution. To compensate this, the trade agreement will stipulate a better off-the-equilibrium-path outcome for the pro-trade lobby group, i.e., a lower  $\bar{t}$ , which leaves a smaller binding overhang.

#### WHY DO COUNTRIES NEGOTIATE TARIFF CEILINGS?

Why do countries negotiate over tariff ceilings, rather than precise applied tariffs at GATT/WTO? Our model also sheds light on this problem. If the policy instrument considered in stage 2 of  $\Gamma_0$  is an applied tariff  $t$  rather than a tariff ceiling  $\bar{t}$ , the negotiation will lead to an applied tariff  $\left(\frac{-\frac{2}{N_1}\alpha K_1 + \frac{2}{N_2}K}{a}\right)$ , which is the first term in (2.29), and higher than the equilibrium applied tariff if countries were to negotiate over a tariff ceiling. This is because when countries negotiate over a precise applied tariff, the ex post game  $\Gamma$  disappears, and it is as if we are in a tariff-setting game described in Section 2.2.2 where the objective function of the government represents world welfare. In such a setting, in each sector, the measure of participating firms is 2 and the equilibrium tariff depends on the sizes of the two competing lobbies but not Home's market power. In contrast, when countries negotiates over tariff ceilings, there is a tariff cut in addition to the elimination of the terms-of-trade manipulation. It is represented by the term  $-\frac{\alpha K_1}{aN_1}(1 - \frac{2}{N_1})$  in (2.29). The reason for this additional tariff reduction is as follows. The tariff ceiling keeps the lobbying game active as opposed to a precise tariff. Furthermore, as we assume the tariff ceiling is not so low as to induce full participation of the pro-trade group, lowering the tariff ceiling marginally will mobilize more pro-trade firms to participate ex post. Since some of these firms

are not the participants at the stage leading to the trade agreement (i.e., ex ante participants), the ex ante players jointly benefit from the participation of and thus contribution from such firms, which result from a lower tariff ceiling. In this way, our model offers a potential explanation for the norm regulating the GATT/WTO negotiations that countries should negotiate over upper bounds on tariffs as opposed to precise tariff levels: it simply helps countries to achieve freer trade<sup>8</sup>.

## 2.4 CONCLUSION

This chapter proposes a model in which countries negotiate over a tariff ceiling that will indirectly lead to a certain level of applied tariff with a positive binding overhang, and thereby remedy the terms-of-trade externality. The tariff ceiling improves the off-the-equilibrium-path outcome for the pro-trade lobby group if this group were not present, and therefore reduces the costs of lobby participation for the pro-trade firms and strengthens the organization of the pro-trade group. Furthermore, we show that the binding overhang decreases in the size of the tariff-setting country.

The model helps reconcile the discrepancy between the argument that terms-of-trade motives should have played a central role in GATT/WTO trade agreements and the implication of existing theoretical models that countries set unilateral optimal tariffs if the applied tariffs fall below the bound tariffs. It generates the stark benchmark result that under a certain range of parameters, the applied tariff resulting from the trade agreement is unrelated to the market power of the country setting the tariff. Future research may generalize the model to a setting with uncertainties to obtain richer and more realistic predictions.

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<sup>8</sup>Admittedly, in this model a lower tariff does not necessarily improve welfare.

## CHAPTER 3

### PROTECTION (OR LIBERALIZATION) FOR SALE WITH FIRM SELECTION

#### 3.1 INTRODUCTION

The “protection for sale” paradigm, first proposed by Grossman and Helpman (1994, 1995), have studied how lobbying shapes the cross-industry variation in protection levels. In models with perfect competition and industrial specific factors, the endogenous tariff imposed on a sector is elegantly related to three variables: the import demand elasticity, the import penetration ratio and whether the sector is “organized”. Many subsequent empirical studies, the first among which being Goldberg and Maggi (1999) and Gawande and Bandyopadhyay (2000), have confirmed the prediction of the protection-for-sale model. Chang (2005) extends this model to a setting of monopolistic competition, where the both the domestic and imported varieties that are available in a market are fixed, and consumers exogenously spends a fixed amount of expenditure on any sector’s differentiated goods.

In reviewing this line of research, one could not help realizing a gap between the simple economic structures assumed in models of endogenous trade policy and the recent trade models that emphasize firm heterogeneity, fixed production and entry costs and (thus) firm selection into a particular market, starting with Melitz (2003). There are (at least) two reasons why incorporating the latter more complicated trade structure into the analysis of the political economy of trade policy can add value. First, recent research on firm-level trade, both theoretical and empirical, has established

that the extensive margin of trade responds strongly to trade barriers. Arkolakis et al. (2008) use Costa Rican data from 1986 to 1992 to show that products with a larger tariff reduction see greater increases in imported varieties. They also show that the “curvature” of the firm productivity distribution matters for the welfare gains from trade. Kehoe and Ruhl (2013) found that the growth in trade of the “least traded goods” is important surrounding trade liberalization episodes. In contrast, the adjustment in the extensive margin of trade is relatively small between the US and its trade partners without any major policy or structural change. Meanwhile, a lot of studies have documented that trade liberalizations tend to weed out the less efficient domestic firms in a country. To the extent that the government trades off political contributions against the loss of social welfare and in most industries relatively larger firms participate in lobbying (as is documented in Bombardini (2008)), it is of interest to know what the protection structure will look like when the response of the set of surviving domestic and imported varieties in a market to the changes in its trade barriers enters the calculation of the relevant players of the political game. Second, trade liberalizations (especially multilateral ones) may create both losers and winners *within the same industry*, because while all domestic firms are faced with greater foreign competition, only the more productive ones can afford to enter the foreign market (and thus benefit from lower trade barriers). It is also of interest to derive a sector’s protection level when such intra-industry conflicts of interests arise.

This chapter proposes a model in which heterogeneous firms lobby for trade policies. There are a fixed measure of domestic firms as well as foreign firms, yet only a subset of them are going to survive in a particular market. When trade policies are set unilaterally, all the domestic firms presents the government with a contribution schedule that is a function of home tariffs. Then the government chooses the tariffs, maximizing a weighted sum of social welfare and political contributions. After that,

the domestic firms decide whether to incur the fixed production costs, and the foreign ones choose whether to pay the fixed exporting costs. Only the firms that have incurred these fixed costs are able to be active in the domestic market, so the varieties available are endogenously determined. Those domestic firms that will be driven out of market in equilibrium must pay zero political contributions on the equilibrium path. We show in this setting, the government will essentially maximize a weighted sum of social welfare and the profits (gross of the fixed production costs) of a set of more productive and (therefore) larger domestic firms. Furthermore, this set of larger firms that dominate the political game coincides with the set of firms that survive in equilibrium. Besides knowing that a higher tariff will ease the market condition by directly raising the prices of imported varieties, these firms face a critical trade-off when lobbying: greater protection will drive out some weaker foreign firms, but it also has the side effect of allowing some less efficient domestic competitors to survive. This trade-off appears in the government's consideration about how protection affects social welfare as well.

We are able to derive explicit formulas for the structure of cross-industry variation in tariff levels. The protection level of a sector depends on its import penetration ratio and the elasticity of the demand function for the sector's CES composite good, in a fashion similar to that of an "organized" sector in the original protection-for-sale model. More interestingly, we also link a sector's endogenous tariff to the "curvatures" of the productivity distributions of both the domestic and foreign firms. The endogenous tariff of a sector increases in the heterogeneity of the productivity distribution of foreign firms, because, roughly speaking, when the curvature is higher (i.e. firms are more heterogeneous), the marginal foreign varieties driven out by protection are of lower value to domestic consumers. The relation between a sector's endogenous tariff and the curvature of domestic firm distribution turns out to be more subtle: when the

political economy channel is dominant in a sector, the endogenous tariff increases in the heterogeneity of domestic firms; when the political economy channel is dominated, the above relationship is reversed. Furthermore, we show that the political economy channel is more important in a sector when its varieties are more substitutable, consumers are less likely to shift expenditures out of this sector, the inverse import penetration ratio is higher, and the government attaches a greater weight on political contributions. The above pattern emerges because the marginal firms that survive due to protection affect social welfare (which the government cares about) and the larger firms' profits differently: these marginal firms alleviate the distortionary effects of the tariff, but erode the profits of the larger and more productive firms that dominate the political game.

We then extend the model to study multilateral trade liberalization. We assume tariff is the only available policy instrument. Firms condition political contributions to their own government on domestic tariffs as well as foreign ones. Then the two governments, both maximizing a weighted sum of political contributions and social welfare, bargain efficiently. We also derive explicit expressions for this setting. Naturally, the cooperative tariffs in both countries are lower compared with those of the unilateral setting. Thus multilateral trade liberalization creates both winners (i.e., the most efficient firms whose gains from exporting outweigh the losses due to greater competition in the domestic market) and losers (i.e., the less efficient firms) within the same industry. We show the relationship between a sector's protection level and its foreign firm heterogeneity in the unilateral setting is now reversed: greater foreign firm heterogeneity in a sector leads to a lower cooperative domestic tariff. This result is due to the lobbying of foreign firms. As is detailed in the main text, when the foreign firms become less heterogeneous, more of them will concentrate toward the exporting cut-off, and the fixed exporting costs will occupy a greater fraction of the

firms' revenue, making the firms' profits a smaller fraction of the corresponding trade volume which directly impacts the domestic market. This drives the above result.

We also show that our results are consistent with some of the theoretical findings in the previous literature. In the unilateral setting, we show some special cases of our results collapse to the ones in Demidova and Rodriguez-Clare (2009) and Chang (2005). In the cooperative setting, we present conditions under which free trade will emerge in equilibrium.

The chapter is organized as follows. Section 3.2 lays out the basic model and derives protection structure when the trade policies are set unilaterally. Section 3.3 studies the protection structure in the cooperative setting. Section 3.4 concludes.

## 3.2 THE BASIC MODEL AND UNILATERAL TRADE POLICY

### 3.2.1 THE ECONOMIC FRAMEWORK

There are two countries: Home and Foreign. We will lay down the variables relating to the economic structure of Home, and it is understood that the structure of the Foreign economy is the same and the counterparts of the Home variables are denoted with “\*”s.

#### DEMAND

Home is populated by individuals with identical quasi-linear utility functions:

$$U = Q_0 + \sum_{i=1}^n U_i(Q_i), \quad (3.1)$$

where  $Q_0$  is the consumption of the homogeneous good, and  $Q_i$  ( $i = 1, \dots, n$ ) is the index of consumption that takes the Dixit-Stiglitz form:

$$Q_i = \left[ \int_{\omega \in \Omega_i} q(\omega)^{\rho_i} d\omega \right]^{\frac{1}{\rho_i}} \quad (0 < \rho_i < 1), \quad (3.2)$$

where  $\Omega_i$  denotes the mass of available varieties in industry  $i$  and  $\omega$  represents a single variety. The goods in industry  $i$  are substitutes, with elasticity of substitution  $\sigma_i > 1$ . We know  $\rho_i = \frac{\sigma_i - 1}{\sigma_i}$ .

The price index associated with the varieties consumed in industry  $i$  is:

$$P_i = \left[ \int_{\omega \in \Omega_i} p(\omega)^{1-\sigma_i} d\omega \right]^{\frac{1}{1-\sigma_i}}, \quad (3.3)$$

where  $p(\omega)$  is the consumer price of variety  $\omega$ . Then total spending of home on industry  $i$ 's differentiated goods is then  $E_i = P_i Q_i$ . Let  $\Lambda_i = Q_i P_i^\sigma = E_i P_i^{\sigma-1}$ , it is well known that the demand for a particular variety  $\omega$  in industry  $i$  is

$$q(\omega) = \Lambda_i p(\omega)^{-\sigma}. \quad (3.4)$$

Given the sub-utility function  $U_i(Q_i)$  defined over industry  $i$ 's composite good  $Q_i$  which is in turn defined in (3.2), and with the quasi-linear utility we assumed, one can derive the demand function for the  $Q_i$ :

$$Q_i = D_i(P_i). \quad (3.5)$$

The consumer surplus derived from industry  $i$ 's goods is then  $S_i(P_i) = U_i(Q_i) - P_i Q_i$ .

## PRODUCTION

Home is endowed with  $L$  units of labor, which is the only factor of production. The homogeneous good is the numeraire. It is freely traded, each produced with one unit of labor, fixing the wage rate at one.

Industry  $i$  has a continuum of entrants with a measure of  $M_{e,i}$ , which is exogenously given<sup>1</sup>. Each of these firms may produce a particular variety. Let  $\varphi > 0$  represents a Home firm's productivity. Production of the firm entails a fixed cost  $f_i$  and

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<sup>1</sup>We abstract from free entry condition in order to focus on the political economy mechanism.

marginal cost  $\frac{1}{\varphi}$ , so to produce  $q > 0$  units of the good the required labor input is  $l(q) = f_i + \frac{q}{\varphi}$ . Faced with a residual demand with constant elasticity  $\sigma_i$  as in (3.4), the profit-maximizing firm charges a markup equal to  $\frac{1}{\rho}$ . Thus the price for the variety is

$$p_i(\varphi) = \frac{1}{\rho_i \varphi}. \quad (3.6)$$

The firm's domestic revenue and domestic profit are respectively (if it were able to operate in equilibrium):

$$r_{d,i}(\varphi) = \Lambda_i p_i(\varphi)^{1-\sigma_i}, \quad (3.7)$$

$$\pi_{d,i}(\varphi) = \max\left\{\frac{r_{d,i}(\varphi)}{\sigma_i} - f_i, 0\right\}. \quad (3.8)$$

As in a typical Melitz-type model, a marginal firm is indifferent between whether to produce since its domestic profit is just enough to cover the fixed cost of production. Let  $\frac{r_{d,i}(\varphi_d)}{\sigma_i} - f_i = 0$  where  $\varphi_d$  denotes the minimum level of productivity required to be active in the domestic market. Then we have the Home Zero Cutoff Profit (ZCP) condition:

$$\frac{\Lambda_i \left(\frac{1}{\rho_i \varphi_d}\right)^{1-\sigma_i}}{\sigma_i} = f_i. \quad (3.9)$$

## TARIFF

Home and Foreign may choose to impose import tariffs, and let  $\tau_i$  and  $\tau_i^*$  denote the corresponding one plus ad valorem tariff rates for the goods of industry  $i$ . If a Home (Foreign) firm in industry  $i$  wants to export its products to the other country, it must incur a fixed cost  $f_x$  ( $f_{x,i}^*$ ). Then the profit-maximizing Home firm with productivity  $\varphi$  charges an offshore price  $p_i(\varphi) = \frac{1}{\rho_i \varphi}$  to Foreign consumers, leading to the consumer price  $p_i(\varphi)\tau_i^* = \frac{\tau_i^*}{\rho_i \varphi}$  in Foreign. Then the firm's exporting revenue and profit are respectively (if it decides to enter the Foreign market):

$$r_{x,i}(\varphi) = p_i(\varphi) \Lambda_i^* [p_i(\varphi)\tau_i^*]^{-\sigma_i}, \quad (3.10)$$

$$\pi_{x,i}(\varphi) = \max\left\{\frac{r_{x,i}(\varphi)}{\sigma_i} - f_{x,i}, 0\right\}, \quad (3.11)$$

where  $\Lambda_i^* = Q_i^* P_i^{*\sigma} = E_i^* P_i^{*\sigma-1}$  denotes the Foreign overall demand condition for industry  $i$ 's differentiated goods.

A Home firm with productivity level  $\varphi_x$  breaks even by choosing to export. All firms with productivities above  $\varphi_x$  will export in equilibrium while those with productivities below this value will not. The corresponding Home exporting cutoff condition is given by

$$\frac{r_{x,i}(\varphi_x)}{\sigma_i} - f_{x,i} = 0 \Leftrightarrow \frac{\left(\frac{1}{\rho_i \varphi_x}\right) \Lambda_i^* \left(\frac{\tau_i^*}{\rho_i \varphi_x}\right)^{-\sigma_i}}{\sigma_i} = f_{x,i}. \quad (3.12)$$

The counterparts of equation (3.10)(3.14)(3.12) for a Foreign foreign firm with productivity  $\varphi^*$  are respectively

$$r_{x,i}^*(\varphi^*) = p_i(\varphi^*) \Lambda_i [p_i(\varphi^*) \tau_i]^{-\sigma_i}, \quad (3.13)$$

$$\pi_{x,i}^*(\varphi^*) = \max\left\{\frac{r_{x,i}^*(\varphi^*)}{\sigma_i} - f_{x,i}^*, 0\right\}, \quad (3.14)$$

and

$$\frac{r_{x,i}^*(\varphi_x^*)}{\sigma_i} - f_{x,i}^* = 0 \Leftrightarrow \frac{\left(\frac{1}{\rho_i \varphi_x^*}\right) \Lambda_i \left(\frac{\tau_i}{\rho_i \varphi_x^*}\right)^{-\sigma_i}}{\sigma_i} = f_{x,i}^*, \quad (3.15)$$

where  $\varphi_x^*$  denotes the productivity level of the least productive Foreign firm that will choose to enter the Home market.

## WELFARE

The aggregate social welfare of Home can be written as

$$W = L + \sum_{i=1}^n (\Pi_{d,i} + \Pi_{x,i} + TR_i + S_i), \quad (3.16)$$

where  $\Pi_{d,i}$  and  $\Pi_{x,i}$  denotes total domestic and foreign profits of the entrants in industry  $i$ ,  $TR_i$  denotes the tariff revenue generated from industry  $i$ 's imports, and  $S_i$  is the consumer surplus associated with industry  $i$ 's varieties defined in section

3.2.1. We will pin down the relationship between  $\Pi_d$ ,  $\Pi_x$  and  $TR_i$  and other variables below.

Suppose the productivity levels of the  $M_{e,i}$  Home entrants are characterized by a distribution with cumulative distribution function  $G_i(\varphi)$  and probability density function  $g_i(\varphi)$ . All firms with productivities above  $\varphi_{d,i}$  will cater to the domestic Home market and earn positive profits, while those with productivities below  $\varphi_{d,i}$  find it optimal to quit and thus earn zero profits. So the total domestic profit of the  $M_{e,i}$  entrants of industry  $i$  is

$$\Pi_{d,i} = M_{e,i} \int_{\varphi_{d,i}}^{\infty} \pi_{d,i}(\varphi) g_i(\varphi) d\varphi = M_{e,i} \int_{\varphi_{d,i}}^{\infty} \left[ \frac{\Lambda_i \left( \frac{1}{\rho_i \varphi} \right)^{1-\sigma_i}}{\sigma_i} - f_i \right] g_i(\varphi) d\varphi. \quad (3.17)$$

Equations (3.8)(3.7) are used in deriving the above expression.

For firms that produce for the domestic markets, only those with productivities above  $\varphi_{x,i}$  find it profitable to enter the Foreign market. Therefore the total exporting profit of the  $M_{e,i}$  entrants of industry  $i$  is

$$\Pi_{x,i} = M_{e,i} \int_{\varphi_{x,i}}^{\infty} \pi_{x,i}(\varphi) g_i(\varphi) d\varphi = M_{e,i} \int_{\varphi_{x,i}}^{\infty} \left[ \frac{\left( \frac{1}{\rho_i \varphi} \right) \Lambda_i^* \left( \frac{\tau_i^*}{\rho_i \varphi} \right)^{-\sigma_i}}{\sigma_i} - f_{x,i} \right] g_i(\varphi) d\varphi. \quad (3.18)$$

We can decompose the domestic price index in the following way:

$$P_i^{1-\sigma_i} = P_{d,i}^{1-\sigma_i} + P_{m,i}^{1-\sigma_i}, \quad (3.19)$$

where  $P_{d,i}$  and  $P_{m,i}$  are price indices for domestic and imported varieties, defined as follows:

$$P_{d,i}^{1-\sigma_i} = M_{e,i} \int_{\varphi_{d,i}}^{\infty} \left( \frac{1}{\rho_i \varphi} \right)^{1-\sigma_i} g_i(\varphi) d\varphi, \quad (3.20)$$

$$P_{m,i}^{1-\sigma_i} = M_{e,i}^* \int_{\varphi_{x,i}^*}^{\infty} \left( \frac{\tau_i}{\rho_i \varphi^*} \right)^{1-\sigma_i} g_i^*(\varphi^*) d\varphi^*, \quad (3.21)$$

where  $g_i^*(\varphi^*)$  is the density of the distribution of Foreign firms' productivities in industry  $i$ .

Domestic consumers' total spending on imported varieties of industry  $i$  is

$$E_{m,i} = M_{e,i}^* \int_{\varphi_{x,i}^*}^{\infty} \Lambda_i \left( \frac{\tau_i}{\rho_i \varphi^*} \right)^{1-\sigma_i} g_i^*(\varphi^*) d\varphi^* = \Lambda_i P_{m,i}^{1-\sigma_i}, \quad (3.22)$$

and the corresponding Foreign entrants' total export revenue (net of tariff payments) is

$$R_{x,i}^* = \frac{E_{m,i}}{\tau_i} = \frac{\Lambda_i P_{m,i}^{1-\sigma_i}}{\tau_i}. \quad (3.23)$$

The tariff revenue from the imported differentiated goods of industry  $i$  is then

$$TR_i = M_{e,i}^* \int_{\varphi_{x,i}^*}^{\infty} \left( \frac{\tau_i - 1}{\rho_i \varphi^*} \right) \Lambda_i \left( \frac{\tau_i}{\rho_i \varphi^*} \right)^{-\sigma_i} g_i^*(\varphi^*) d\varphi^* = (\tau_i - 1) R_{x,i}^* = \left( \frac{\tau_i - 1}{\tau_i} \right) E_{m,i}. \quad (3.24)$$

### 3.2.2 LOBBYING

Now suppose that each government sets tariffs unilaterally. Similar to Bombardini (2008), we assume *firms* are principals that offer the government political contribution schedules in exchange for favorable policies, in a common agency game first proposed by Bernheim and Whinston (1986) and applied in the work of Grossman and Helpman (1994) (where sectors may get organized) which then has become the workhorse model in the literature of the political economy of trade policy. Unlike Bombardini (2008), we abstract from fixed costs of political participation and assume in any industry  $i$  all the  $M_{e,i}$  ( $M_{e,i}^*$ ) entrants are players of the game and incentivize the Home (Foreign) government to heed their needs. Specifically, we assume a Home firm in sector  $i$  with productivity  $\varphi$  presents to its government a contribution schedule  $C_{\varphi,i}(\tau, \cdot)$  before it incurs any fixed cost of production. The government then chooses a tariff vector, weighting political contributions relative to social welfare. After the tariffs are set, a firm must pay the contribution it has promised, and decides whether to pay the fixed cost  $f_i$  and stay in the market, or to exit. We assume the contribution schedule is “truthful”:

$$C_{\varphi,i}(\tau, \cdot) = \max\{0, \pi_i(\varphi; \tau, \cdot) - B_i(\varphi)\}, \quad (3.25)$$

where  $\pi_i(\varphi; \tau, \cdot)$  denotes the firm's total profit<sup>2</sup> given Home tariff  $\tau$ , and  $B_i(\varphi)$  is a constant. Furthermore, we assume  $B_i(\varphi)$  is the firm's *equilibrium* profit<sup>3</sup>. Also note the contribution schedule  $C_{\varphi,i}(\tau, \cdot)$  should be seen as a function of Home tariff  $\tau$ , because in the unilateral tariff setting game, players in Home treat the Foreign tariff as given; it is represented by “.” and kept in the background.

As in the “protection for sale” paradigm, the government maximizes a weighted sum of social welfare and political contributions. Note with a perfectly competitive numeraire sector and separable sub-utility functions for each manufacturing sector, we need only consider the effect of tariff  $\tau_i$  on variables relating to sector  $i$  ( $i = 1, \dots, n$ ). Moreover, Home firms' total exporting profit  $\Pi_{x,i}$  and exporting cutoff  $\varphi_{x,i}$  only depend the Foreign tariff  $\tau_i^*$  and will not be affected by Home tariffs. Thus the equilibrium Home tariff is

$$\tau_i^O = \arg \max_{\tau_i} \{b\Pi_{d,i}(\tau_i) + TR_i(\tau_i) + S_i(P_i(\tau_i))\} \quad (i = 1, \dots, n), \quad (3.26)$$

where  $b \geq 1$  is the weight attached to producer profits. When  $b = 1$ , the government has no affinity for political contribution and will choose a tariff that maximizes the social welfare. Generally  $b > 1$ , and politics becomes more important as  $b$  increases. Also note we have written industry  $i$ 's aggregate domestic profit  $\Pi_{d,i}$ , tariff revenue

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<sup>2</sup>We assumes owners of each firm represents a negligible share of the population and thus only care about the firm's profits. This is only for expositional purposes. One can immediately extend the main results of this paper to the case where lobbies also consider their consumer surplus and income from tariff revenues.

<sup>3</sup>This clarification is important. A common agency game typically features multiple equilibria which differ in their efficiencies. Hence the literature has focused on “truthful equilibria”, which has the property that the outcome maximizes the joint payoffs of the players. If one only assumes each principal offers a “truthful” contribution schedule and the game ends up in a Nash Equilibrium, it is *not* guaranteed that we have a “truthful equilibrium”. One can construct examples where the above two assumptions are satisfied yet the outcome of the common agency game is not Pareto efficient. To arrive at a truthful equilibrium, we need to further assume the constant  $B_i(\varphi)$  is the principal's equilibrium payoff. This point is not made explicit in most of the literature.

$TR_i$  and price index  $P_i$  as functions of  $\tau_i$ . Furthermore, using the expression in (3.17), we have

$$\begin{aligned}
\frac{d\Pi_{d,i}(\tau_i)}{d\tau_i} &= M_{e,i} \frac{d(\int_{\varphi_{d,i}(\tau_i)}^{\infty} [\frac{\Lambda_i(\tau_i)(\frac{1}{\rho_i\varphi})^{1-\sigma_i}}{\sigma_i} - f_i]g_i(\varphi)d\varphi)}{d\tau_i} \\
&= \underbrace{-M_{e,i}[\frac{\Lambda_i(\tau_i)(\frac{1}{\rho_i\varphi_{d,i}})^{1-\sigma_i}}{\sigma_i} - f_i]g_i(\varphi_{d,i})\frac{\varphi_{d,i}}{d\tau_i}}_{\text{Equal to 0 due to the Home ZCP condition (3.9)}} + \int_{\varphi_{d,i}(\tau_i)}^{\infty} \frac{d}{d\tau_i} [\frac{\Lambda_i(\tau_i)(\frac{1}{\rho_i\varphi})^{1-\sigma_i}}{\sigma_i} - f_i]g_i(\varphi)d\varphi \\
&= \int_{\varphi_{d,i}(\tau_i)}^{\infty} \frac{d}{d\tau_i} [\frac{r_{d,i}(\varphi; \tau_i)}{\sigma_i}]g_i(\varphi)d\varphi, \tag{3.27}
\end{aligned}$$

where  $r_{d,i}(\varphi; \tau_i) = \Lambda_i(\tau_i)(\frac{1}{\rho_i\varphi})^{1-\sigma_i}$  is a (surviving) firm's domestic revenue as defined in (3.7) and we now write it as a function of sector  $i$ 's Home tariff  $\tau_i$ .

Let  $\varphi_{d,i}^O = \varphi_{d,i}(\tau_i^O)$  denote the domestic cutoff that results from equilibrium tariff  $\tau_i^O$ . Then with the result in (3.27), the first order condition of the equilibrium tariff equation (3.26) can be written as

$$\frac{d}{d\tau_i} [b \int_{\varphi_{d,i}^O}^{\infty} \frac{r_{d,i}(\varphi; \tau_i)}{\sigma_i} g_i(\varphi)d\varphi] + \frac{d}{d\tau_i} [TR_i(\tau_i) + S_i(P_i(\tau_i))] = 0. \tag{3.28}$$

This has led to the following lemma:

**Lemma 3.1** *(Only larger firms matter in the lobbying game.)*

*In the unilateral tariff setting game, let us assume in each sector  $i$  ( $i = 1, \dots, n$ ) all the  $M_{e,i}$  entrants lobby the government in a common agency game. Let  $\varphi_{d,i}^O$  denote the productivity of the least productive firm that will produce under the equilibrium tariff  $\tau_i^O$ .*

*In equilibrium, the government chooses the tariff as if (with respect to the first order condition) it is maximizing a weighted sum of social welfare and the domestic profits (gross of fixed costs of production) of sector  $i$ 's firms with productivity levels higher than  $\varphi_{d,i}^O$ . In particular, this object function can be written as*

$$G_i(\tau) = b \int_{\varphi_{d,i}^O}^{\infty} \frac{r_{d,i}(\varphi; \tau_i)}{\sigma_i} g_i(\varphi)d\varphi + TR_i(\tau_i) + S_i(P_i(\tau_i)), \tag{3.29}$$

where  $b \geq 1$  measures the government's affinity for political contributions,  $r_{d,i}(\varphi; \tau_i)$  is the firm  $\varphi$ 's domestic revenue (if it produces) under tariff  $\tau_i$ ,  $\frac{r_{d,i}(\varphi; \tau_i)}{\sigma_i}$  is the corresponding profit (gross of the fixed cost of production), and  $TR_i(\cdot)$  and  $S_i(\cdot)$  denote tariff revenue and consumer surplus generated from varieties in sector  $i$ .

Furthermore, in equilibrium, all the entrants with productivity levels below  $\varphi_{d,i}^O$  must exit and make zero political contributions.

**Proof.** One can immediately verify that the first order condition  $\frac{dG_i(\tau)}{d\tau} = 0$  coincides with equation (3.28), which we have shown is the first order condition of the game where all firms are principals of the common agency game. ■

According to Lemma 3.1, this model generates the result that the government acts *as if* it considers social welfare and the profits (gross of fixed costs of production) of only the relatively more productive (and thus larger) firms, in spite of the assumption that all of the firms are involved in the political game. Moreover, this subset of larger firms coincides with the set of surviving firms in equilibrium. Under the equilibrium trade policy, the relatively less productive firms are driven out of the market and they find it optimal to make zero contribution to the politician<sup>4</sup>. Intuitively, these firms would make less profits if they were to produce, and they require higher protection in order to survive. Therefore in equilibrium they decide to not incur the fixed cost of production and not influence the policy choice.

The above property of the model is desirable. Bombardini (2008) shows that in most 2-digit SIC sectors in US, relatively smaller firms do not participate (in terms of making positive PAC contributions) and among those firms that do participate, larger

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<sup>4</sup>These firms may make positive contributions out of equilibrium. Actually, according to the logic of the "truthful equilibrium", a firm in this group would submit all the profit to the government if the tariff were high enough such that it would be able to produce. As for the contribution of the surviving firms, there are multiple equilibrium. The possible payoff vectors of these firms are constrained in core of some properly defined cooperative game. See the contribution of Laussel and Le Breton (2001).

firms tend to contribute more. Note that we abstract from the fixed costs of political contribution assumed in Bombardini (2008), in order to make the model tractable. However, the fixed production costs naturally afford us the property that only larger firms will influence the policy outcome.

### 3.2.3 REWRITING THE MODEL IN TERMS OF MARGINAL CHANGES

We have outlined the economic framework and the lobbying game, and are going to derive the cross-industry variation in protection levels in section 3.2.4. To this end, we derive the marginal changes in the key variables in response to changes in tariffs in this section. We will extensively use the following algebra: let  $X$  be a variable, define

$$\hat{X} = \frac{dX}{X}, \quad (3.30)$$

which captures the percentage changes of  $X$ . This “hat algebra”<sup>5</sup> linearizes a non-linear system in terms of marginal changes and will greatly simplify our derivation of the equilibrium tariff structure. It also allows us to characterize the marginal effect of tariff changes on any key variable of the model in a much easier way. Some useful mathematical properties of the “hat algebra” will be used and they are reviewed in appendix C.1.

In our model, due to the existence of a constant-returns-to-scale numeraire sector and the quasi-linear utility form, a Home tariff  $\tau_i$  ( $i = 1, \dots, n$ ) affects the variables related sector  $i$  independently. Thus to save notations we now drop the industrial index  $i$ , without causing any confusion. Given any value of the tariff  $\tau > 0$ , we can view the outcomes in the corresponding sector as characterized by 6 variables: the price index  $P$ , the price index of domestic varieties  $P_d$ , the price index of imported

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<sup>5</sup>This sort of algebraic transformation is referred to as “Jone’s (1965) algebra” in the textbook on international trade of Feenstra (2003).

varieties  $P_m$ , the cutoff for domestic firms  $\varphi_d$ , the cutoff for Foreign exporting firms  $\varphi_x^*$ , and the overall demand condition

$$\Lambda = D(P)P^\sigma, \quad (3.31)$$

where  $D(P)$  is the demand for the industrial composite good. Given any value of  $\tau$ , we have 6 equations to (implicitly) pin down the above 6 unknown variables: the sectoral demand condition (3.31), the Home ZCP condition (3.9), the Foreign export cutoff condition (3.15), and the definitions of the price indices (3.19), (3.20) and (3.21). We now rewrite these equations with the “hat algebra”, revealing the relations between marginal changes in the relevant variables.

The sectoral overall demand condition can be written as

$$\widehat{\Lambda} = (\sigma - \varepsilon)\widehat{P}, \quad (3.32)$$

where  $\varepsilon = \frac{dD(P)}{dP} \frac{P}{D(P)}$  denotes the price elasticity of the demand for the sectoral composite good. Throughout the paper we assume  $\sigma > \varepsilon$ , which implies goods in the same sector are more substitutable than good among different sectors.  $\widehat{\Lambda}$  and  $\widehat{P}$  are of the same sign, meaning a higher price index will increase the demand for every variety in the sector. This effect is stronger when goods in this sector are more substitutable (i.e.  $\sigma$  is higher), but is weaker when consumers are more likely to shift their expenditures toward the numeraire sector (i.e.  $\varepsilon$  is higher).

Given the distribution of domestic firms  $g(\varphi)$  of the relevant sector, define function  $H(\psi) = \int_{\psi}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi$ . Let  $\varepsilon_{H(\psi),\psi} = -\frac{dH(\psi)}{d\psi} \frac{\psi}{H(\psi)} > 0$  denote the elasticity of  $H(\psi)$  in response to changes in  $\psi$ . If  $g(\varphi) = \beta\varphi_0^\beta \varphi^{\beta-1} (\varphi > \varphi_0)$  is the density of the Pareto distribution, one can show  $\varepsilon_{H(\psi),\psi} = \beta - (\sigma - 1)$  when  $\psi > \varphi_0$ . A larger  $\varepsilon_{H(\psi),\psi}$  (or a larger  $\beta$  with Pareto distribution) indicates firms are more homogeneous (with respect to their productivities). Intuitively, when the productivity distribution is more

concentrated toward the lower bound, a marginal change in the cutoff will induce a greater influx of firms, and the function  $H(\psi)$  will be more sensitive to changes in the cutoff  $\psi$ .

Let  $\gamma = \frac{\varepsilon_{H(\varphi_d), \varphi_d}}{\sigma-1}$ . For the above Pareto distribution, we have  $\gamma = \frac{\beta-(\sigma-1)}{\sigma-1}$ . This parameter  $\gamma$  measures the homogeneity of productivity levels of domestic firms. Then equation (3.20) implies that the percentage change of the price index for domestic varieties can be expressed as

$$\widehat{P}_d = \gamma \widehat{\varphi}_d. \quad (3.33)$$

Intuitively, when  $\gamma$  is higher, firms are more homogeneous, which implies: (1) marginal varieties will be more valuable (Arkolakis et al. (2008)); (2) a decrease in  $\varphi_d$  will induce a greater influx of domestic firms. Thus a marginal change in  $\varphi_d$  will have a bigger impact on the corresponding price index (in terms of marginal changes) when homogeneity is high.

For the productivity distribution of foreign firms, define the counterparts of the relevant domestic variables:  $H^*(\psi^*) = \int_{\psi^*}^{\infty} \varphi^{*\sigma-1} g^*(\varphi^*) d\varphi^*$ ,  $\varepsilon_{H^*(\psi^*), \psi^*} = -\frac{dH^*(\psi^*)}{\psi^*} \frac{\psi^*}{H(\psi^*)} > 0$  and  $\gamma^* = \frac{\varepsilon_{H^*(\varphi_x^*), \varphi_x^*}}{\sigma-1}$ . (For a Pareto distribution  $g^*(\varphi^*) = \beta^* \varphi_0^{*\beta^*} \varphi^{*(\beta^*-1)}$ ,  $\varepsilon_{H^*(\varphi_x^*), \varphi_x^*} = \beta^* - (\sigma - 1)$ , and  $\gamma^* = \frac{\beta^*-(\sigma-1)}{\sigma-1}$ .) Thus  $\gamma^*$  measures the homogeneity of Foreign productivity levels. The percentage change of the imported price index can then be written as (using the definition (3.21))

$$\widehat{P}_m = \widehat{\tau} + \gamma^* \widehat{\varphi}_x^*, \quad (3.34)$$

which shows that the Home tariff raises  $P_m$  for two reasons: (1) it directly raises the prices of imported goods, which is captured by the first term  $\widehat{\tau}$ , (2) it increases the cutoff for Foreign exporting firms, and import price index  $P_m$  rises due to lost Foreign varieties. Effect (2) is represented by the second term  $\gamma^* \widehat{\varphi}_x^*$ , and it is more important when the homogeneity measure  $\gamma^*$  is higher.

The percentage change of the sectoral price index can be decomposed as (using (3.19))

$$\widehat{P} = s_d \widehat{P}_d + s_m \widehat{P}_m, \quad (3.35)$$

where  $s_d = \frac{P_d^{1-\sigma}}{P^{1-\sigma}} = \frac{E_d}{E}$  denotes the share of consumers' spending on domestic varieties in total spending on the industry's differentiated goods, and  $s_m = \frac{P_m^{1-\sigma}}{P^{1-\sigma}} = \frac{E_m}{E}$  represents the corresponding share of consumers' spending on imported varieties.

The Home ZCP condition (3.9) can be written as

$$\widehat{\varphi}_d = -\frac{1}{\sigma-1} \widehat{\Lambda}, \quad (3.36)$$

which implies a higher  $\Lambda$  (which means a larger demand for each variety) will lead to a lower cutoff for domestic firms.

Finally, we also write the Foreign exporting cutoff condition (3.15) in terms of percentage changes:

$$\widehat{\varphi}_x^* = \frac{1}{\rho} \widehat{\tau} - \frac{1}{\sigma-1} \widehat{\Lambda}. \quad (3.37)$$

Equation (3.37) shows a tariff raises the Foreign exporting cutoff by directly raising the price of and thus reducing the demand for each imported variety (which is captured by the first term). However, the impact of this effect is dampened by a higher  $\Lambda$  (implying that doing business is now easier in the Home market).

We now view the marginal conditions (3.32)-(3.37) as a *linear* equation system with 6 unknowns:  $\widehat{\Lambda}$ ,  $\widehat{P}$ ,  $\widehat{P}_d$ ,  $\widehat{P}_m$ ,  $\widehat{\varphi}_d$ , and  $\widehat{\varphi}_x^*$ . Thus it is straightforward to solve for these unknown variables (which captures the percentage changes). We arrive at the following lemma:

**Lemma 3.2** *When Home changes its tariff  $\tau$  on the varieties of a particular sector, the marginal (percentage) changes of the relevant variables can be expressed as follows.*

Let  $\chi = \frac{(\sigma-\varepsilon)s_m}{1+(\frac{\gamma}{\rho})(\frac{\sigma-\varepsilon}{\sigma})s_d} \in (0, \sigma)$ , then

$$\widehat{P}_m = \frac{1 + \frac{\gamma^*}{\rho}}{1 + \frac{\gamma^*}{\sigma-1}\chi} \widehat{\tau}, \quad (3.38)$$

and

$$\widehat{\Lambda} = \chi \widehat{P}_m. \quad (3.39)$$

**Proof.** See appendix C.2. ■

Lemma 3.2 immediately leads to some standard results in a typical Melitz-type model, which are summarized in the following lemma.

**Lemma 3.3** (*Standard properties of Melitz-type models.*)

When  $\tau$  increases,

- (1)  $\Lambda$  and  $P$  increases;
- (2)  $\varphi_d$  decreases, and  $\varphi_x^*$  increases.

**Proof.** Using the results in lemma 3.2 we see when  $\widehat{\tau} > 0$ ,  $\widehat{P}_m > 0$  and  $\widehat{\Lambda} = \chi \widehat{P}_m > 0$ . This proves the results in part (1). The proof of part (2) entails some algebra, so it is relegated to appendix C.3. ■

Lemme 3.3 formally establishes that a higher tariff will increase the industry's overall price index and shift out the domestic demand curve for each domestic variety, and that it will drive out foreign firms and allow less efficient domestic ones to survive.

### 3.2.4 THE STRUCTURE OF PROTECTION

In this section we will derive the structure of cross-industry variation in tariffs. First we characterize how tariffs affect producers' profits, consumer surpluses and tariff revenues in lemmas 3.4, 3.5 and 3.6.

**Lemma 3.4** (*The impact of tariff on firms' total domestic profit in the relevant sector.*)

$$\left[\frac{d\Pi_d}{d\tau}\right] = \left[\frac{1}{\frac{\sigma}{(\sigma-\varepsilon)s_d} + \frac{\gamma}{\rho}}\right] (R_x^* \frac{\widehat{P}_m}{\widehat{\tau}}). \quad (3.40)$$

**Proof.** See appendix C.4. ■

Notably, keeping other variables fixed, when the homogeneity measure  $\gamma$  is higher,  $\left[\frac{d\Pi_d}{d\tau}\right]$  will be smaller, i.e., the effect of tariff on firms' domestic profits is smaller when the productivities are more homogeneous. The reason is as follows. As explained in section 3.2.2, when we look at the impact of a marginal change in a tariff on all the entrants' profits in the relevant industry, due to the Home ZCP condition, it is *as if* we are just considering how the tariff affects the profits (gross of fixed costs of production) of a fixed set of larger and more productive firms. (These firms turn out to be the ones that survive *in equilibrium*). Let us denote this set of firms  $\mathcal{S}$ . Intuitively, a higher tariff will raise the prices of imported varieties and drive out Foreign firms, but it will allow less efficient domestic firms to survive, eroding the profits of firms in  $\mathcal{S}$ . When the productivities of domestic entrants are more homogeneous, the emerging domestic firms due to protection will have a bigger impact on the market, making protection less desirable for firms in  $\mathcal{S}$ .

**Lemma 3.5** (*The impact of tariff on consumer surplus from the relevant sector.*)

$$\begin{aligned}
\left[\frac{dS(P(\tau))}{d\tau}\right] &= \underbrace{\left[\frac{\frac{\gamma}{\rho}}{(\sigma-\varepsilon)s_d + \frac{\gamma}{\rho}}\right]}_{\text{(The effect of gaining marginal domestic varieties due to protection.)}} \left(R_x^* \frac{\widehat{P}_m}{\widehat{\tau}}\right) \\
&\quad - \underbrace{R_x^* \frac{\widehat{P}_m - \widehat{\tau}}{\widehat{\tau}}}_{\text{(The effect of losing marginal imported varieties due to protection.)}} \\
&\quad - \underbrace{R_x^*}_{\text{(Direct effect of tariff on the prices of imported varieties.)}}. \tag{3.41}
\end{aligned}$$

**Proof.** See appendix C.5. ■

The above lemma says the tariff impacts consumer surplus in three ways: (1) introducing domestic varieties, (2) forcing out foreign varieties, and (3) directly raising the prices of imported goods. Keeping other variables fixed, effect (1) is more pronounced when  $\gamma$  is higher. This is because when firms are more homogeneous, the marginal domestic varieties induced by tariff are more important, as discussed previously.

**Lemma 3.6** *(The impact of tariff on tariff revenue from the relevant sector.)*

$$\left[\frac{dTR}{d\tau}\right] = R_x^* - \left(\frac{\tau-1}{\tau}\right) R_x^* \left[ (\sigma - \chi) \frac{\widehat{P}_m}{\widehat{\tau}} - \frac{\widehat{P}_m - \widehat{\tau}}{\widehat{\tau}} \right]. \tag{3.42}$$

**Proof.** Foreign exporting firms' total revenue (earned in Home) is  $R_x^* = \frac{P_m}{\tau} \Lambda P_m^{-\sigma}$ , so

$$\begin{aligned}
\widehat{R}_x^* &= \widehat{\Lambda} - \sigma \widehat{P}_m + (\widehat{P}_m - \widehat{\tau}) \\
&= -(\sigma - \chi) \widehat{P}_m + (\widehat{P}_m - \widehat{\tau}). \tag{3.43}
\end{aligned}$$

Since  $TR = (\tau - 1)R_x^*$  (from (3.24)),

$$\frac{dTR}{d\tau} = R_x^* + (\tau - 1) \frac{R_x^* \widehat{R}_x^*}{\widehat{\tau}}. \tag{3.44}$$

Combining the above equation with (3.43), we get (3.42). ■

Let us define

$$\xi = \frac{\widehat{P}_m - \widehat{\tau}}{\widehat{P}_m} = \frac{1 - (\frac{\chi}{\sigma})}{1 + (\frac{\rho}{\gamma^*})} > 0. \quad (3.45)$$

The tariff increases the imported price index  $P_m$  due to two effects: (1) it drives out Foreign varieties; (2) it directly raises the prices of the corresponding imported goods. Thus  $\xi$  measures the relative importance of effect (1). It increases in  $\gamma^*$ , the homogeneity measure of Foreign firms.

Now the results of lemmas 3.4, 3.5 and 3.6 allow us to characterize the marginal change of the government's objective function in response to tariff changes, as in the following lemma.

**Lemma 3.7** (*Decomposition of the Home government's objective function in the unilateral setting.*)

*When the government is lobbied by all the entrants in the industry, the derivative of its objective function  $G(\tau)$  (defined in lemma 3.1) can be written as*

$$\begin{aligned}
\left[\frac{dG(\tau)}{\tau}\right] &= \left[\frac{dTR}{d\tau}\right] + \left[\frac{dS(P(\tau))}{d\tau}\right] + b\left[\frac{d\Pi_d}{d\tau}\right] \\
&= \underbrace{-\left(\frac{\tau-1}{\tau}\right)R_x^*\left[(\sigma-\chi)\frac{\widehat{P}_m}{\widehat{\tau}} - \frac{\widehat{P}_m - \widehat{\tau}}{\widehat{\tau}}\right]}_{\text{(Tariff revenue loss from protection due to less spending on imported varieties.)}} \\
&\quad - \underbrace{R_x^*\frac{\widehat{P}_m - \widehat{\tau}}{\widehat{\tau}}}_{\text{(Consumer surplus loss from protection due to less imported varieties.)}} \\
&\quad + \underbrace{\left[\frac{\frac{\gamma}{\rho}}{\frac{\sigma}{(\sigma-\varepsilon)s_d} + \frac{\gamma}{\rho}}\right]\left(R_x^*\frac{\widehat{P}_m}{\widehat{\tau}}\right)}_{\text{(Consumer surplus gain from protection due to more domestic varieties.)}} \\
&\quad + \underbrace{\left[\frac{b}{\frac{\sigma}{(\sigma-\varepsilon)s_d} + \frac{\gamma}{\rho}}\right]\left(R_x^*\frac{\widehat{P}_m}{\widehat{\tau}}\right)}_{\text{(Increase in (relatively larger) firms' profits (gross of fixed costs of production) due to protection.)}} \\
&= \left(R_x^*\frac{\widehat{P}_m}{\widehat{\tau}}\right)\left[\frac{b + \frac{\gamma}{\rho}}{\frac{\sigma}{(\sigma-\varepsilon)s_d} + \frac{\gamma}{\rho}} - \xi - \left(\frac{\tau-1}{\tau}\right)((\sigma-\chi) - \xi)\right] \tag{3.46}
\end{aligned}$$

Let  $\left[\frac{dG(\tau^O)}{\tau^O}\right] = 0$ , we have  $\left(\frac{\tau^O-1}{\tau^O}\right) = \frac{\frac{b+\frac{\gamma}{\rho}}{\frac{\sigma}{(\sigma-\varepsilon)s_d} + \frac{\gamma}{\rho}} - \xi}{(\sigma-\chi) - \xi} = 1 - \frac{\sigma - \frac{b+\frac{\gamma}{\rho}}{\frac{\varepsilon}{(\sigma-\varepsilon)s_d} + 1 + \frac{\gamma}{\rho}}}{\sigma - \frac{1}{1+\frac{\rho}{\gamma^*}}}$  (with details of the derivation for the last “=” in appendix C.6). Thus  $\tau^O = \frac{\sigma - \frac{1}{1+\frac{\rho}{\gamma^*}}}{\sigma - \frac{b+\frac{\gamma}{\rho}}{\frac{\varepsilon}{(\sigma-\varepsilon)s_d} + 1 + \frac{\gamma}{\rho}}}$ . We state this result formally in the next proposition, retrieving the industry index  $i$ .

**Proposition 3.1** *(The structure of protection.)*

The equilibrium endogenous unilateral tariff imposed on imported varieties of industry  $i$  ( $i = 1, \dots, n$ ) is

$$\tau_i^O = \frac{\sigma_i - \frac{1}{1+\frac{\rho_i}{\gamma_i^*}}}{\sigma_i - \frac{b+\frac{\gamma_i}{\rho_i}}{\frac{\varepsilon_i}{(\sigma_i-\varepsilon_i)s_{d,i}} + 1 + \frac{\gamma_i}{\rho_i}}}, \tag{3.47}$$

where  $b \geq 1$  is the weight placed on producers' profits in the government's objective function,  $\gamma_i$  and  $\gamma_i^*$  respectively measure the homogeneity of the productivities of Home

and Foreign firms,  $s_{d,i} = \frac{E_{d,i}}{E_i}$  is the share of expenditure on domestic varieties in the total spending on industry  $i$ 's varieties,  $\varepsilon_i$  is the price elasticity of the demand  $D_i(P_i)$  for industry  $i$ 's composite good, and  $\sigma_i$  is the elasticity of substitution between the varieties of industry  $i$  with  $\rho_i = \frac{\sigma_i - 1}{\sigma_i}$ . (When Home (Foreign) productivities are subject to Pareto distribution of parameter  $\beta_i$  ( $\beta_i^*$ ), we have  $\gamma_i = \frac{\beta_i - (\sigma_i - 1)}{\sigma_i - 1}$  ( $\gamma_i^* = \frac{\beta_i^* - (\sigma_i - 1)}{\sigma_i - 1}$ )).

This had led to the following proposition regarding how the protection level of an industry is related to the relevant political and economic parameters.

**Proposition 3.2 (Comparative statics.)** For each industry  $i$  ( $i = 1, \dots, n$ ),

- (1) The endogenous unilateral tariff  $\tau_i^O$  increases in  $b$  and  $s_{d,i}$ , and decreases in  $\varepsilon_i$ .
- (2)  $\tau_i^O$  decreases in the Foreign homogeneity measure  $\gamma_i^*$ .
- (3) When  $b < \frac{\varepsilon_i}{(\sigma_i - \varepsilon_i)s_{d,i}} + 1$ ,  $\tau_i^O$  increases in the Home homogeneity measure  $\gamma$ ; when  $b > \frac{\varepsilon_i}{(\sigma_i - \varepsilon_i)s_{d,i}} + 1$ ,  $\tau_i^O$  decreases in the Home homogeneity measure  $\gamma$ .

Part (1) of proposition 3.2 is consistent with the prediction of the Grossman-Helpman theory for an “organized” industry. When the government has greater affinity for political contributions or the import penetration is lower, the domestic firms’ lobbying activity will have a greater influence on the policy outcome. On the other hand, when the consumers are more likely to shift their expenditures out of this sector (i.e., a higher  $\varepsilon_i$ ), the equilibrium endogenous tariff will be lower.

Part (2) of the proposition is due to the fact that tariffs have the effect of reducing imported varieties, in addition to raising the prices of imported goods. When Foreign firms are more homogeneous (i.e. higher  $\gamma^*$ ), the lost marginal imported varieties are more important for domestic consumers, so raising the tariff will incur a larger welfare loss. This leads to a lower equilibrium tariff in our political economy model.

Part (3) of the proposition says the protection level of an industry increases with domestic firm heterogeneity *only if* the political economy effect is dominant, namely,  $b > \frac{\varepsilon_i}{(\sigma_i - \varepsilon_i)s_{d,i}} + 1$ . On the other hand, when  $b < \frac{\varepsilon_i}{(\sigma_i - \varepsilon_i)s_{d,i}} + 1$ , the endogenous tariff level will decrease with the heterogeneity of domestic firms. This is due to the fact that the “curvature” of the domestic productivity distribution influences social welfare and the profits of larger firms in different ways. A lower “curvature” (i.e., less heterogeneity) alleviates the distortionary effect of the tariff, because the marginal domestic firms (or varieties) that survive due to this tariff will be more important. Precisely due to the same reason, these marginal firms have a bigger impact on the market (for example, as can be seen in equation (3.20)), so protection is less desirable for the larger and more productive firms in the relevant industry, and they are less willing to pay for the political contributions in exchange for protection that has the side-effect of sheltering smaller and weaker firms. For a particular sector, when the latter political economy effect dominates, greater domestic firm heterogeneity will lead to higher protection; when the political economy effect is dominated, protection level will appear inversely correlated with firm heterogeneity. Moreover, the political economy consideration is more important if the government has a greater affinity for political contributions (a larger  $b$ ), goods are more substitutable in the relevant industry (a higher  $\sigma_i$ ), consumers are less likely to shift spending out of this industry (a lower  $\varepsilon_i$ ), and the inverse import penetration is larger (a larger  $s_{d,i}$ ).

### 3.2.5 COMPARISON WITH RESULTS IN PREVIOUS LITERATURE

In this section we show some special cases of our results are consistent with those in two related papers: Demidova and Rodriguez-Clare (2009) and Chang (2005). We also gain further intuition into the mechanisms that drive our results.

**Corollary 3.1** (*Demidova and Rodriguez-Clare (2009).*)

For industry  $i$  ( $i = 1, \dots, n$ ), if  $b = 1$  (i.e., the government does not care about political contributions),  $\varepsilon_i = 0$  (i.e., consumers demand a fixed amount of the industrial composite good  $Q_i$ ), and the productivities of Foreign firms are subject to Pareto distribution with parameter  $\beta_i^*$ , then the endogenous tariff in our model is

$$\begin{aligned} \tau_{DR2009,i}^O &= \left( \frac{\beta_i^*}{\beta_i^* - \rho_i} \right) \\ &= \underbrace{\left( \frac{1}{\rho_i} \right)}_{\text{(Account for the domestic mark-up distortion.)}} \cdot \underbrace{\left( \frac{\beta_i^* \rho_i}{\beta_i^* - \rho_i} \right)}_{\text{(Account for the imported variety distortion.)}}. \end{aligned} \quad (3.48)$$

**Proof.** Given the Pareto distribution with parameter  $\beta_i^*$ , we know  $\gamma_i^* = \frac{\beta_i^* - (\sigma_i - 1)}{\sigma_i - 1}$ . Then using proposition 3.1 we get  $\tau_{DR2009,i}^O = \frac{\sigma_i - \frac{1}{1 + \frac{\rho_i}{\gamma_i^*}}}{\sigma_i - 1} = \left[ \sigma_i - \frac{1}{1 + \frac{\rho_i(\sigma_i - 1)}{\beta_i^* - (\sigma_i - 1)}} \right] \left( \frac{1}{\sigma_i - 1} \right) = \frac{\beta_i^*}{\rho_i - \beta_i^*}$ . ■

Corollary 3.1 says when the government is a social welfare maximizer and the demand for the industrial composite good is inelastic, the equilibrium tariff in our model collapse to that in Demidova and Rodriguez-Clare (2009)<sup>6</sup>. As illustrated in equation (3.48), the optimal tariff deals with two distortions: (1) the prices of the domestic varieties are above the opportunity costs, while the prices of the imported varieties are equal to the opportunity costs, so we need a tariff  $\left(\frac{1}{\rho_i}\right)$  to neutralize this distortion; (2) when making their decision, consumers do not take into account their welfare gain due to the entry of foreign firms, and a tariff  $\left(\frac{\beta_i^* \rho_i}{\beta_i^* - \rho_i}\right)$  can neutralize this distortion.

Next we analyze the unilateral tariff that maximizes social welfare in our full model.

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<sup>6</sup>In their model, they do not differentiate between domestic and foreign firm distributions. However, one can extend their result slightly and how that when the domestic firms and the foreign firms are distributed differently, the optimal tariff depends only on the distribution of foreign firms.

**Corollary 3.2** (*The unilateral optimal tariff.*)

For industry  $i$  ( $i = 1, \dots, n$ ), if  $b = 1$  (i.e., the government does not care about political contributions) and the productivities of Foreign firms are subject to Pareto distribution with parameter  $\beta_i^*$ , then the endogenous tariff in our model is

$$\tau_{optimal,i}^O = \frac{\sigma_i - \frac{1}{1+(\frac{\rho_i}{\gamma_i^*})}}{\sigma_i - \frac{\frac{\varepsilon_i}{(\sigma_i - \varepsilon_i)^{s_{d,i}}} + 1 + (\frac{\rho_i}{\gamma_i^*})}{\rho_i - \beta_i^*}} \leq \left( \frac{\beta_i^*}{\rho_i - \beta_i^*} \right) = \tau_{DR2009,i}^O. \quad (3.49)$$

Moreover, “=” holds in the above inequality only when  $\varepsilon_i = 0$  (i.e., consumers demand a fixed amount of the industrial composite good  $Q_i$ ).

According to corollary 3.2, the tariff that maximizes the social welfare are generally smaller than the one in corollary 3.1. This is because in addition to the two distortions described in and following corollary 3.1, we now have a third distortion: domestic producers in the manufacturing industry  $i$  charges a markup  $\frac{1}{\rho_i}$ , so consumers consume too little industrial composite good  $Q_i$  relative to numeraire good. A tariff will further shift expenditure toward the numeraire sector, exacerbating this third distortion. On the other hand, this deterioration will be alleviated if domestic firms are more homogeneous, because the marginal domestic varieties saved by the protection will be more important, as we discussed in section. So  $\tau_{optimal,i}^O$  will be closer to  $\tau_{DR2009,i}^O$  when  $\gamma_i$  is higher.

Next we link our results to Chang (2005), which explores the endogenous protection structure that emerges from a setting of monopolistic competition.

**Corollary 3.3** (*Chang (2005).*)

Assume  $\varepsilon_i = 1$  (i.e., fixed expenditure on industry  $i$ 's varieties). When  $\gamma_i \rightarrow 0$  and  $\gamma_i^* \rightarrow 0$  (or  $\beta \rightarrow \sigma_i - 1$  and  $\beta^* \rightarrow \sigma_i - 1$  in the case of Pareto distributions), the

endogenous tariff  $\tau_i^O$  will have the property

$$\frac{\tau_i^O}{\tau_i^O - 1} \rightarrow b \frac{\frac{\sigma_i - 1}{\sigma_i}}{\frac{E_{m,i}}{E_{d,i}} + \sigma_i}, \quad (3.50)$$

where  $E_{d,i}$  and  $E_{m,i}$  denote the Home expenditures on domestic and imported varieties of industry  $i$ .

**Proof.** Plugging the relevant values into (3.47) and rearranging terms will yield the result. ■

Corollary 3.3 says if we shut down the firm selection channels (for both Home and Foreign firms), the endogenous tariff we derived will be the same as the one in Chang (2005) for an industry that is “organized”<sup>7</sup>. Specifically, when  $\gamma_i$  and  $\gamma_i^*$  approach zero, the Home (Foreign) firms are so heterogeneous that the impact of the marginal firms selected into the market due to higher (lower) protection can almost be ignored. So asymptotically it is as if we have a *fixed* number of monopolistic competing firms and all the domestic ones lobby the government. This is the setting that leads to the same result as the one in Chang (2005) for an “organized sector”.

### 3.3 COOPERATIVE TRADE POLICY

We have studied the unilateral tariff-setting game and the resulting tariff structure. Especially, through the firm selection channel, we have linked the cross-industry variation of endogenous tariffs to the “curvatures” of a sector’s domestic and foreign firm distributions. In this section, we assume the Home and Foreign government are involved in a “trade talk” and examine the structure of protection that emerges from this setting.

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<sup>7</sup>Chang (2005) assumed the home expenditure on goods in any manufacturing industry is fixed.

Following Grossman and Helpman (1995), we proposed a two-stage game. In the first stage, in sector  $i$  ( $i = 1, \dots, n$ ), each Home (Foreign) firm with productivity  $\varphi$  offers its government a contribution schedule  $C_{\varphi,i}(\tau_i, \tau_i^*)$  ( $C_{\varphi,i}^*(\tau_i, \tau_i^*)$ ) that depends on both the Home tariff  $\tau_i$  and the Foreign tariff  $\tau_i^*$ . In the second stage, the two governments, both maximizing a weighted sum of social welfare and political contributions, bargain efficiently and agree on a tariff pair  $(\tau_{c,i}^O, \tau_{c,i}^{*O})$ . (In this process, international transfers may be necessitated, as explained in Grossman and Helpman (1995).) It is well-known that in such a setting, the resulting tariff pair maximizes the joint payoff of all the involved players (in our case, the two governments and all the surviving and non-surviving firms).

In the economics structure we assumed, the Home tariff affects welfare of Foreign only through the exporting profits of Foreign firms. Therefore when the Home government sets its tariffs cooperatively, it considers Foreign exporting profits, in addition to domestic firms' profits, tariff revenue and consumer surplus. (Also note the exporting profits of the Home firms only depend on Foreign tariffs.) The Home cooperative tariff is then

$$\tau_{c,i}^O = \arg \max_{\tau_i} \{b^* \Pi_{x,i}^*(\tau_i) + [b \Pi_{d,i}(\tau_i) + TR_i(\tau_i) + S_i(P_i(\tau_i))]\} \quad (i = 1, \dots, n), \quad (3.51)$$

where  $b^*$  is the weight the Foreign government places on its producers' profits. The cooperative Foreign tariff is defined analogously.

Comparing (3.51) with the Home unilateral tariff equation (3.26), one immediately sees that  $\tau_{c,i}^O < \tau_i^O$  ( $i = 1, \dots, n$ ), that is, the cooperative tariff is smaller than the unilateral tariff. Similar relationships hold for Foreign tariffs. Due to the lobbying of the Foreign (Home) exporting firms, in the cooperative setting the Home (Foreign) government reduces its tariffs relative to the unilateral level. This multilateral trade

liberalization leads to further firm selection (as implied by lemma 3.3): in both countries, the cut-off productivity levels for survival increases, so more of the relative less productive firms are driven out of the market; among the surviving firms, exporting to another country becomes more profitable, so the cut-off productivity levels for exporting decreases in both countries. Moreover, conflicts of interests arise in the *same* industry: when the two countries carry out the multilateral trade liberalization, the more productive (and thus larger) firms may gain because they are able to earn more profit from exporting, while the less productive (and thus smaller) firms must lose because they make zero or very little exporting profits, but are now faced with more competition from foreign firms in their domestic markets.

What will be the structure of protection in such a cooperative setting? In section 3.3.1, we will derive explicit expressions for the tariffs that emerge from this “trade talk”. In particular, we show that the relation between a sector’s protection level and “curvature” of foreign firms in the unilateral setting is now reversed: in any industry, a greater Foreign firm heterogeneity leads to a lower Home cooperative tariff.

### 3.3.1 THE STRUCTURE OF COOPERATIVE TARIFFS

Using the cut-off condition for Foreign exporting firms (3.15), we write the total exporting profit of the  $M_{e,i}^*$  Foreign firms in sector  $i$  as

$$\Pi_{x,i}^* = \int_{\varphi_{x,i}^*}^{\infty} [f_{x,i}^* \frac{\varphi^{\sigma_i-1}}{\varphi_{x,i}^{*\sigma_i-1}} - f_{x,i}^*] M_{e,i}^* g_i(\varphi) d\varphi \quad (3.52)$$

and their total exporting revenue (which is also defined in (3.23)) as

$$R_{x,i}^* = \int_{\varphi_{x,i}^*}^{\infty} \sigma_i f_{x,i}^* \frac{\varphi^{\sigma_i-1}}{\varphi_{x,i}^{*\sigma_i-1}} M_{e,i}^* g_i(\varphi) d\varphi, \quad (3.53)$$

where  $f_{x,i}^*$  denotes the fixed cost a Foreign firm must incur in order to export to Home.

**Lemma 3.8** (The impact of tariff on Foreign firms' total exporting profits in the relevant sector.)

$$\begin{aligned} \left[ \frac{d\Pi_{x,i}^*}{d\tau_i} \right] &= \frac{1}{\sigma_i(1+\gamma_i^*)} \left[ \frac{dR_{x,i}^*}{d\tau_i^*} \right] \\ &= -\frac{1}{\sigma_i(1+\gamma_i^*)} \left( \frac{1}{\tau_i} \right) R_{x,i}^* \left[ (\sigma_i - \chi_i) \frac{\widehat{P}_{m,i}}{\widehat{\tau}_i} - \frac{\widehat{P}_{m,i} - \widehat{\tau}_i}{\widehat{\tau}_i} \right] \end{aligned} \quad (3.54)$$

**Proof.** From (3.52) and (3.53) we know  $\frac{\left[ \frac{d\Pi_{x,i}^*}{d\tau_i} \right]}{\left[ \frac{dR_{x,i}^*}{d\tau_i^*} \right]} = \frac{\frac{d}{d\varphi_{x,i}^*} \left( \int_{\varphi_{x,i}^*}^{\infty} \left[ \frac{\varphi^{\sigma_i-1}}{\varphi_{x,i}^* \sigma_i^{\sigma_i-1}} - 1 \right] g_i(\varphi) d\varphi \right) \frac{d\varphi_{x,i}^*}{d\tau_i}}{\sigma_i \frac{d}{d\varphi_{x,i}^*} \left( \int_{\varphi_{x,i}^*}^{\infty} \frac{\varphi^{\sigma_i-1}}{\varphi_{x,i}^* \sigma_i^{\sigma_i-1}} g_i(\varphi) d\varphi \right) \frac{d\varphi_{x,i}^*}{d\tau_i}}$ , and after some straightforward algebra one sees that  $\frac{\frac{d}{d\varphi_{x,i}^*} \left( \int_{\varphi_{x,i}^*}^{\infty} \left[ \frac{\varphi^{\sigma_i-1}}{\varphi_{x,i}^* \sigma_i^{\sigma_i-1}} - 1 \right] g_i(\varphi) d\varphi \right)}{\sigma_i \frac{d}{d\varphi_{x,i}^*} \left( \int_{\varphi_{x,i}^*}^{\infty} \frac{\varphi^{\sigma_i-1}}{\varphi_{x,i}^* \sigma_i^{\sigma_i-1}} g_i(\varphi) d\varphi \right)} = \frac{1}{\sigma_i(1+\gamma_i^*)}$ .

Therefore the first “=” of equation (3.54) holds. In equation (3.43) we have derived that  $\widehat{R}_{x,i}^* = -(\sigma_i - \chi_i) \widehat{P}_{m,i} + (\widehat{P}_{m,i} - \widehat{\tau}_i)$ , so  $\left[ \frac{dR_{x,i}^*}{d\tau_i^*} \right] = \frac{R_{x,i}^* \widehat{R}_{x,i}^*}{\tau_i \widehat{\tau}_i} = -\left( \frac{1}{\tau_i} \right) R_{x,i}^* \left[ (\sigma_i - \chi_i) \frac{\widehat{P}_{m,i}}{\widehat{\tau}_i} - \frac{\widehat{P}_{m,i} - \widehat{\tau}_i}{\widehat{\tau}_i} \right]$ , which leads to the second “=” of equation (3.54). ■

Lemma 3.8 implies that when the Foreign firm homogeneity measure  $\gamma_i^*$  increases,  $\left[ \frac{d\Pi_{x,i}^*}{d\tau_i} \right]$  will be a smaller portion of  $\left[ \frac{dR_{x,i}^*}{d\tau_i^*} \right]$ . In other words, when Foreign firms are more homogeneous in terms of productivity, a change in tariff will induce a greater change in Foreign exporting revenue relative to the change in Foreign exporting profits. This is due to the fact that with a greater homogeneity of firms, when the tariff decreases, the marginal Foreign firms selected into the Home market are more important relative to the ones that are already exporting to Home<sup>8</sup>. These less productive marginal firms need to incur the fixed exporting costs to enter the market and they must use their exporting revenue to cover these costs, which makes their exporting profits a smaller fraction of their exporting revenue compared with the firms already exporting. (For

<sup>8</sup>Note we can use equation (3.53) to decompose the marginal percentage change of  $R_{x,i}^*$  (due to a change in tariff) as  $\widehat{R}_{x,i}^* = -(\sigma_i - 1) \gamma_i^* \widehat{\varphi}_{x,i}^* - (\sigma_i - 1) \widehat{\varphi}_{x,i}^*$ . The first term captures the effect of marginal Foreign firms selected into the Home market, while the second term captures the effect of the Foreign firms that are already exporting.

example, for a Foreign firm that is just able to enter the Home market, its profit is zero while its revenue (and the import volume of its product) is positive.) So when the impact of these marginal firms are more important (with a greater  $\gamma_i^*$ ), the change in total Foreign exporting profit  $\Pi_{x,i}^*$  will be smaller compared to the change in total Foreign exporting revenue  $R_{x,i}^*$ .

In this setting of multilateral trade liberalization, the Home government chooses the tariff  $\tau_i$  as if it has an objective function  $G_{c,i}(\tau_i) = b^*\Pi_{x,i}^*(\tau_i) + [b\Pi_{d,i}(\tau_i) + TR_i(\tau_i) + S_i(P_i(\tau_i))]$ , as can be seen in (3.51). Next we derive the marginal change of  $G_{c,i}(\tau_i)$  in response to changes in  $\tau_i$ .

**Lemma 3.9** (*Decomposition of the Home government's objective function in the cooperative setting.*)

*In the setting of "trade talks" between Home and Foreign, the Home government chooses the tariff  $\tau_i$  as if maximizing  $G_{c,i}(\tau_i)$ . The derivative of  $G_{c,i}(\tau_i)$  is*

$$\begin{aligned}
\left[\frac{dG_{c,i}(\tau_i)}{\tau_i}\right] &= b^* \left[\frac{d\Pi_{x,i}^*}{d\tau_i}\right] + \left[\frac{dT R_i}{d\tau_i}\right] + \left[\frac{dS_i(P_i(\tau_i))}{d\tau_i}\right] + b \left[\frac{d\Pi_{d,i}}{d\tau_i}\right] \\
&= \underbrace{-\left(\frac{b^*}{\sigma_i(1+\gamma_i^*)}\right) R_{x,i}^* [(\sigma_i - \chi_i) \frac{\widehat{P}_{m,i}}{\widehat{\tau}_i} - \frac{\widehat{P}_{m,i} - \widehat{\tau}_i}{\widehat{\tau}_i}]}_{\text{(Profit loss of Foreign exporting firms resulting from Home protection.)}} \\
&\quad \underbrace{-R_{x,i}^* \frac{\widehat{P}_{m,i} - \widehat{\tau}_i}{\widehat{\tau}_i}}_{\text{(Home consumer surplus loss from protection due to less imported varieties.)}} \\
&\quad \underbrace{-\left(\frac{\tau_i - 1}{\tau_i}\right) R_{x,i}^* [(\sigma_i - \chi_i) \frac{\widehat{P}_{m,i}}{\widehat{\tau}_i} - \frac{\widehat{P}_{m,i} - \widehat{\tau}_i}{\widehat{\tau}_i}]}_{\text{(Home tariff revenue loss from protection due to less spending on imported varieties.)}} \\
&\quad \underbrace{-R_{x,i}^* \frac{\widehat{P}_{m,i} - \widehat{\tau}_i}{\widehat{\tau}_i}}_{\text{(Home consumer surplus loss from protection due to less imported varieties.)}} \\
&\quad \underbrace{+\left[\frac{\frac{\gamma_i}{\rho_i}}{\frac{\sigma_i}{(\sigma_i - \varepsilon_i)s_{d,i}} + \frac{\gamma_i}{\rho_i}}\right] (R_{x,i}^* \frac{\widehat{P}_{m,i}}{\widehat{\tau}_i})}_{\text{(Home consumer surplus gain from protection due to more domestic varieties.)}} \\
&\quad \underbrace{+\left[\frac{b_i}{\frac{\sigma_i}{(\sigma_i - \varepsilon_i)s_{d,i}} + \frac{\gamma_i}{\rho_i}}\right] (R_{x,i}^* \frac{\widehat{P}_{m,i}}{\widehat{\tau}_i})}_{\text{(Increase in (relatively larger) Home firms' profits (gross of fixed costs of production) due to protection.)}} \\
&= \left(R_{x,i}^* \frac{\widehat{P}_{m,i}}{\widehat{\tau}_i}\right) \left[\frac{b_i + \frac{\gamma_i}{\rho_i}}{\frac{\sigma_i}{(\sigma_i - \varepsilon_i)s_{d,i}} + \frac{\gamma_i}{\rho_i}} - \xi_i - \left(\frac{\tau_i - 1 + \frac{b^*}{\sigma_i(1+\gamma_i^*)}}{\tau_i}\right) ((\sigma_i - \chi_i) - \xi_i)\right] \quad (3.55)
\end{aligned}$$

Lemma 3.9 is the counterpart of lemma 3.7. Both lemmas decompose the (marginal change of) the Home government's objective function. In contrast to the non-cooperative setting, in the cooperative one the Home government takes into account an additional term  $[\frac{d\Pi_{x,i}^*}{d\tau_i}]$ , which captures the impact of tariff on Foreign exporting firms' profits. As is in lemma 3.8, ceteris paribus, this term is smaller when the Foreign homogeneity measure  $\gamma_i^*$  is larger, because with a lower "curvature", the response of the *extensive margin* of import varieties to tariff changes is relatively more important compared with that of the *intensive margin*, and for the marginal

varieties (or firms) entering and exiting the market, the profits are small compared with the corresponding trade volume (or revenue).

We are now ready to characterize the cross-industry variation of protection levels.

**Proposition 3.3** (*The structure of protection in the cooperative setting.*)

When Home and Foreign engage in “trade talks”, the equilibrium endogenous Home tariff on imported varieties of sector  $i$  ( $i = 1, \dots, n$ ) is

$$\tau_{c,i}^O = \frac{(\sigma_i - 1) - \frac{(b^* - 1)}{(\frac{\gamma_i^*}{\rho_i}) + 1}}{\sigma_i - \frac{b + (\frac{\gamma_i}{\rho_i})}{\frac{\varepsilon_i}{(\sigma_i - \varepsilon_i)s_{d,i}} + 1 + (\frac{\gamma_i}{\rho_i})}}, \quad (3.56)$$

where  $b \geq 1$  ( $b^* \geq 1$ ) is the weight placed on producers’ profits in the Home (Foreign) government’s objective function,  $\gamma_i$  and  $\gamma_i^*$  respectively measure the homogeneity of the productivities of Home and Foreign firms,  $s_{d,i} = \frac{E_{d,i}}{E_i}$  is the share of expenditure on domestic varieties in the total spending on industry  $i$ ’s varieties,  $\varepsilon_i$  is the price elasticity of the demand  $D_i(P_i)$  for industry  $i$ ’s composite good, and  $\sigma_i$  is the elasticity of substitution between the varieties of industry  $i$  with  $\rho_i = \frac{\sigma_i - 1}{\sigma_i}$ . (When Home (Foreign) productivities are subject to Pareto distribution of parameter  $\beta_i$  ( $\beta_i^*$ ), we have  $\gamma_i = \frac{\beta_i - (\sigma_i - 1)}{\sigma_i - 1}$  ( $\gamma_i^* = \frac{\beta_i^* - (\sigma_i - 1)}{\sigma_i - 1}$ ).

The next proposition summarizes how the protection level of an industry varies with the relevant political and economic parameters in the cooperative setting.

**Proposition 3.4** (*Comparative statics.*) For each industry  $i$  ( $i = 1, \dots, n$ ),

(1) When  $b^* > 1$  (i.e., the Foreign government does care about political contributions),  $\tau_{c,i}^O$  increases in the Foreign homogeneity measure  $\gamma_i^*$ .

(2) The comparative statics relating to other parameters are identical to those in the unilateral setting (which are summarized in proposition 3.2).

Compared with the structure of protection in the unilateral setting, the cooperative tariff relates to the homogeneity (or heterogeneity) of Foreign firm productivities in a different way. When the Foreign government is not affected by firms' lobbying (i.e.,  $b^* = 1$ ), the cooperative tariff  $\tau_{c,i}^O$  does not vary with Foreign homogeneity measure  $\gamma^*$ : the unilateral incentives to subsidize imported varieties disappear in the cooperative setting. When the lobbying of Foreign firms matter, greater Foreign firm homogeneity is related to a higher Home tariff. This is due to that fact that when Foreign firms are more homogeneous, more of the adjustment in import volume (or the revenue of Foreign exporting firms) resulting from tariff changes comes from the adjustment in the extensive margin of imported varieties, which makes the change in the exporting profits of Foreign firms a smaller portion of their exporting revenue (or the corresponding Home import volume), as is shown in lemma 3.8. Thus with a higher Foreign homogeneity measure  $\gamma^*$ , the stake of Foreign firms is less compared with the impact of tariff on Home welfare, so the Foreign firms are less willingly to change the Home tariff through lobbying in the cooperative setting.

### 3.3.2 SPECIAL CASES OF THE COOPERATIVE TARIFF STRUCTURE

In this section we summarize two special cases of the endogenous cooperative tariff structure, where the governments are welfare maximizers.

The following corollary lays out the condition under which free trade will emerge from the cooperative setting.

**Corollary 3.4** *If  $b = b^* = 1$  and  $\varepsilon_i = 0$  ( $i = 1, \dots, n$ ), then  $\tau_{c,i}^O = 1$ , i.e. there will be no tariff imposed on varieties of industry  $i$ .*

Corollary 3.4 says when political contributions play no role in both countries and Home consumers demand a fixed amount  $Q_i$  of industry  $i$ 's composite good, the trade

talks between Home and Foreign lead to free trade in sector  $i$ . This is in contrast to corollary 3.1 that is for the unilateral setting and features the result in Demidova and Rodriguez-Clare (2009). Unilaterally, there are two distortions for Home as discussed in section 3.2.5: the imported varieties are sold at prices equal to the opportunity costs while the domestic varieties are not; Home consumers do not take into account their welfare gain through the entry of foreign firms. These two distortions are absent from the perspective of the cooperative setting, and therefore free trade results. This is also a case that features the optimality of market allocation in the CES economy shown in Dhingra and Morrow (2012).

**Corollary 3.5** *If  $b = b^* = 1$  and  $\varepsilon_i > 0$  ( $i = 1, \dots, n$ ), then  $\frac{1}{\rho_i} < \tau_{c,i}^O < 1$ .*

Corollary 3.5 shows when there is a substitution between the numeraire good and the varieties of industry  $i$  and the governments maximize their countries' social welfare, there will be import subsidies which neutralize the distortionary effects of the exporters' market powers. However, since there is no instrument to deal with the market power of the producers of domestic varieties, the import subsidy  $\tau_{c,i}^O$  will not reach  $\frac{1}{\rho_i}$ .

### 3.4 CONCLUDING REMARKS

Recent research on firm-level trade, both theoretical and empirical, has emphasized that trade liberalization influences heterogeneous firms in the same sector in different ways. This paper investigates the implication of the firm selection effects of international trade on the cross-industry variation in protection levels, and therefore broadens the protection for sale model of the political economy of trade policy. We posit a model where heterogeneous firms are “principals” of a lobbying game who offer the government implicit contribution schedules where the political contribution

is contingent on the domestic tariff in the unilateral setting as well as foreign tariff in the multilateral setting. A higher domestic tariff has the effect of weeding out less productive domestic firms, but also allows some marginal foreign firms to enter. In equilibrium, only the more productive firms contribute positive amounts, because the weaker firms will be driven out of the market and therefore find it optimal not to contribute.

We are able to derive novel predictions on how the “curvature” (i.e., level of heterogeneity) of firms’ productivity levels in a sector shapes the endogenous protection imposed on this sector. We show the influence of domestic firm heterogeneity in a sector on its protection level depends on the importance of the political economy channel: if the political economy effect is dominant, greater domestic firm heterogeneity leads to greater protection, and when the political economy effect is dominated, greater domestic firm heterogeneity results in lower protection. We lay out conditions characterizing the importance of the political economy channel. On the other hand, how foreign firm heterogeneity affects the protection level imposed on a sector by the home country hinges on whether trade policies are set unilaterally or multilaterally: in the unilateral setting, the home tariff increases in foreign firm heterogeneity; in the setting where “trade talks” takes place, the reverse is true. This research thus provides potentially novel explanations on whether some sectors enjoy greater protection than others by bridging the gap between the protection for sale paradigm and models on firm-level trade, and lays the groundwork for future empirical research.

## APPENDIX A

### A MODEL OF LOBBY PARTICIPATION AND TARIFF BINDING OVERHANG

#### A.1 PROOF FOR PROPOSITION 1.1

With equations (1.8)(1.9) on equilibrium contribution and equations (2.10)(2.10) on firms' willingness to pay, the conditions in (1.14) can be simplified to

$$\begin{aligned} \sum_{f \in \mathcal{L}_1} \left( \frac{\alpha^2 K_1^2}{ab_2} \right) \theta_f^2 &\geq \frac{1}{2} \left( \frac{\alpha^2 K_1^2}{ab_2} \right) \Theta_1^2, \\ \sum_{f \in \mathcal{L}_2} \left( \frac{K_2^2}{ab_2} \right) \theta_f^2 &\geq \frac{1}{2} \left( \frac{K_2^2}{ab_2} \right) \Theta_2^2. \end{aligned}$$

Rearranging terms, we get for sector  $i$  ( $i = 1, 2$ ),

$$\sum_{f \in \mathcal{L}_i} \left( \frac{\theta_f}{\Theta_i} \right)^2 \geq \frac{1}{2}. \tag{A.1}$$

This establishes part (1) for proposition 1.1.

The sharing rule that supports the equilibrium in part (2) can be constructed as follows. For each firm in  $\mathcal{L}_i$  ( $i = 1, 2$ ), the sharing rule stipulates that it should contribute an amount that is not greater than its willingness to pay. This is possible, since the Herfindahl index of  $\mathcal{L}_i$  is weakly above  $\frac{1}{2}$  and thus the group's total willingness to pay is weakly greater than its total contribution. If a firm  $f$  outside  $\mathcal{L}_i$  joins the lobby group, the sharing rule requires each firm in the new group contribute an amount greater than its willingness to pay. This is also possible, since the Herfindahl index of the new group must be smaller than  $\frac{1}{2}$ . Clearly, all sharing rules satisfying the above standards can support the equilibrium in part (2) of proposition 1.1.

## A.2 PROOF FOR PROPOSITION 1.5

With a little abuse of notation, still let  $\mathcal{L}_i$  ( $i = 1, 2$ ) be the set firms in sector  $i$  that have entered stage 2 of game  $\Gamma'$ . Let  $\Theta_1 = \frac{\sum_{f \in \mathcal{L}_1} k_f}{K_1}$  and  $\Theta_2 = \frac{\sum_{f \in \mathcal{L}_2} k_f}{K_2}$  denote the participation shares. Define cooperative games  $\Gamma'_1(\cdot)$  and  $\Gamma'_2(\cdot)$ , s.t.

$$\Gamma'_1(S) = \max_{t \leq \bar{t}} \{aW(t) + \Theta_2 \Pi_2(t) + \Theta_S \Pi_1(t)\} - \max_{t \leq \bar{t}} \{aW(t) + \Theta_2 \Pi_2(t)\}, \quad \text{for all } S \subseteq \mathcal{L}_1, \quad (\text{A.2})$$

and

$$\Gamma'_2(S) = \max_{t \leq \bar{t}} \{aW(t) + \Theta_1 \Pi_1(t) + \Theta_S \Pi_2(t)\} - \max_{t \leq \bar{t}} \{aW(t) + \Theta_1 \Pi_1(t)\}, \quad \text{for all } S \subseteq \mathcal{L}_2, \quad (\text{A.3})$$

where  $\Theta_S = \frac{\sum_{f \in S} k_f}{K_i}$  denotes the share of a certain group  $S$  in sector  $i$ .

The stages 2' and 3 compose a “two-sided” common agency game, as is defined in Laussel and Le Breton (2001), because  $\frac{\partial \Pi_1}{\partial t} = -\alpha K_1 < 0$  and  $\frac{\partial \Pi_2}{\partial t} = K_2 > 0$ . We have the following lemma which is a direct application of proposition 4.4 of the above paper.

**Lemma A.1** (*Laussel and Le Breton (2001)*)

*The cooperative games  $\Gamma'_1(S)$  and  $\Gamma'_2(S)$  are convex. And  $\{u^i \mid u^i$  is a vector of equilibrium payoffs of  $C(\Gamma'_i)$  ( $i = 1, 2$ ), where  $C(\Gamma'_i)$  denotes the core of  $\Gamma'_i$ .*

Note that each “side”  $\mathcal{L}_i$  act *as if* they have formed a group, and the joint payoff of  $\mathcal{L}_i$  is uniquely determined by (A.2) or (A.3):

$$\Theta_i \Pi_i(t^*) - C_i^* = \max_{t \leq \bar{t}} \{aW(t) + \Theta_j \Pi_j(t) + \Theta_i \Pi_i(t)\} - \max_{t \leq \bar{t}} \{aW(t) + \Theta_j \Pi_j(t)\}, \quad (j \neq i), \quad (\text{A.4})$$

where  $C_i^*$  denotes lobby  $i$ 's equilibrium contribution and  $t^*$  the equilibrium tariff.

Equation (A.4) gives us exactly the same level of group contributions as in (1.8)(1.15), because  $t^* = \arg \max_{t \leq \bar{t}} \{aW(t) + \Theta_1 \Pi_1(t) + \Theta_2 \Pi_2(t)\}$ . Generally there

are multiple equilibria due to different allocations of surplus in each “side”, and the cores of  $\Gamma_1(S)$  and  $\Gamma_2(S)$  put constraints on possible allocations. When we assume firms are symmetric and contribute the same amount in each lobby under assumption 1.1, we end up with a unique equilibrium identical to the one in section 1.2.8.

### A.3 PROOF FOR LEMMA 1.3

If in Stage 2 of game  $\Gamma_0$ , if the anti-trade lobby group  $\mathcal{L}_{02}$  were absent, the future applied tariff would be

$$t_{-02}^* = - \left( 1 - \frac{1}{N_1} \right) \frac{\alpha K_1}{ab_2} \Theta_{01}^* - \frac{1}{N_1} \frac{\alpha K_1}{ab_2}, \quad (\text{A.5})$$

while the equilibrium applied tariff  $t^*$  (induced by the corresponding tariff cap) was derived in (1.39). The downstream lobby  $\mathcal{L}_{02}$  must compensate the joint welfare loss of the government and the upstream lobby  $\mathcal{L}_{01}$  due to shift from  $t_{-02}^*$  to  $t^*$ . The equilibrium contribution of  $\mathcal{L}_{02}$  is

$$C_{0,2}(\Theta_{02}) = \int_{t^*}^{t_{-02}^*} \left( \frac{\partial G_\Gamma(t)}{\partial t} + \Theta_{01}^* \frac{\partial V_1(t)}{\partial t} \right) dt. \quad (\text{A.6})$$

Combined with equations (1.35)(1.37), we get

$$C_{0,2}(\Theta_{02}) = \int_{t^*}^{t_{-02}^*} \left( -ab_2 t - \frac{\alpha K_1}{N_1} - \Theta_{01}^* \alpha K_1 \left( 1 - \frac{1}{N_1} \right) \right) dt = \frac{1}{2} \frac{K_2^2}{ab_2} \Theta_{02}^2. \quad (\text{A.7})$$

Next we derive lobby group  $\mathcal{L}_{02}$ 's “total willingness to pay”, denoted with  $\mathcal{W}_{02}$ . Consider a single firm with sector-specific capital  $\frac{K_2}{N_2}$ . If it chooses not to participate, equation (1.39) shows that equilibrium  $t^*$  in game  $\Gamma$  will shift by the amount  $\frac{1}{N_2} \frac{K_2}{ab_2}$ , and from (1.36) we know this firm's payoff in  $\Gamma$  will decrease by  $w_f = \left( \frac{1}{N_2} \frac{K_2}{ab_2} \right) \cdot \frac{1}{N_2} \frac{\partial V_2}{\partial t} = \left( \frac{1}{N_2} \frac{K_2}{ab_2} \right) \cdot \frac{1}{N_2} K_2$ . This  $w_f$  is a firm's “willingness to pay” (in ex ante stages 1 and 2 of game  $\Gamma$ ). Then with  $n_{02}$  firms in group  $\mathcal{L}_{02}$ , the “total willingness to pay” can be

written as a function of participation share  $\Theta_{02}$ :

$$\mathcal{W}_{02}(\Theta_{02}) = n_{02} \cdot w_f = \Theta_{02} \frac{1}{N_2} \frac{K_2^2}{ab_2}. \quad (\text{A.8})$$

Let  $C_{0,2}(\Theta_{02}) = \mathcal{W}_{02}(\Theta_{02})$ , we have

$$\Theta_{02}^* = \frac{2}{N_2}. \quad (\text{A.9})$$

## APPENDIX B

### MARKET POWER, LOBBY PARTICIPATION AND TARIFF BINDING OVERHANG

#### B.1 DERIVATIONS FOR (2.16) AND (2.17)

Using (2.14),

$$\tilde{C}_1(\Theta_1, \Theta_2; \bar{t}) = \frac{1}{2} \frac{(\alpha K_1)^2}{a} \left( \frac{\lambda}{2 + \lambda} \right) \Theta_1^2 - \frac{a}{2} \left( \frac{\lambda}{1 + \lambda} \frac{2 + \lambda}{1 + \lambda} \right) (t_{-1} - \bar{t})^2.$$

The closed form expression for  $\mathcal{W}_1(\Theta_1)$  is given by (2.11). Solving the equation  $\tilde{C}_1(\Theta_1, \frac{2}{N_2}; \bar{t}) = \mathcal{W}_1(\Theta_1)$ , we get the solution for  $\Theta_1^*$  in (2.17). Once participation shares  $\Theta_1^*$  and  $\Theta_2^*$  are known, the equilibrium applied tariff  $t^*$  is given by (2.6).

#### B.2 PROOF FOR LEMMA 2.4

Using (2.2), we can write the equilibrium world price with respect to the equilibrium binding overhang  $h^*$ :

$$p^* = \frac{\alpha K_1 - t^*}{1 + \lambda} + (1 - K) = \frac{\alpha K_1 + h^* - \bar{t}}{1 + \lambda} + (1 - K).$$

Then

$$\frac{h^*}{p^*} = \frac{h^*}{\frac{\alpha K_1 + h^* - \bar{t}}{1 + \lambda} + (1 - K)}. \tag{B.1}$$

As is showed in the main text, when  $\lambda$  is larger,  $h^*$  will be larger. One can show in (B.1) that when  $\lambda$  and  $h^*$  both increase,  $\frac{h^*}{p^*}$  will also increase.

## APPENDIX C

### PROTECTION (OR LIBERALIZATION) FOR SALE WITH FIRM SELECTION

#### C.1 SOME USEFUL PROPERTIES OF THE “HAT ALGEBRA”

For any variable  $X$ , let  $\widehat{X} = \frac{dX}{X}$ . Note then for any two variables  $x$  and  $y$ ,  $\frac{\widehat{y}}{\widehat{x}} = \frac{dy}{dx} \frac{x}{y} = \varepsilon_{y,x}$  which is the elasticity of  $y$  in response to changes in  $x$ . Also we can write the derivative of  $y$  with respect to  $x$  as  $\frac{dy}{dx} = \frac{\widehat{y}}{\widehat{x}} \frac{y}{x}$ .

We state the following lemma:

**Lemma C.1** (*Properties of the “hat algebra”.*)

(1) If  $A = B_1 B_2 \cdots B_N$ , then  $\widehat{A} = \widehat{B}_1 + \widehat{B}_2 + \cdots + \widehat{B}_N$ .

(2) If  $a$  is a constant, then  $\widehat{(aA)} = \widehat{A}$ .

(3) If  $A = B_1 + B_2 + \cdots + B_N$ , then  $\widehat{A} = [\frac{B_1}{A}] \widehat{B}_1 + [\frac{B_2}{A}] \widehat{B}_2 + \cdots + [\frac{B_N}{A}] \widehat{B}_N$ .

(4) Consider a function  $g(x)$ , we have  $\widehat{g} = \varepsilon_{g,x} \widehat{x}$ , where  $\varepsilon_{g,x} = \frac{dg}{dx} \frac{x}{g}$  is the elasticity of  $g$  in response to changes in  $x$ .

#### C.2 PROOF FOR LEMMA 3.2

Equations (3.33)(3.36)  $\Rightarrow$

$$\widehat{P}_d = -\frac{\gamma}{\sigma - 1} \widehat{\Lambda} = -\frac{\gamma}{\sigma - 1} (\sigma - \varepsilon) \widehat{P}. \quad (\text{C.1})$$

Combine the above equation with (3.35), we have

$$\widehat{P} = s_d \left[ -\frac{\gamma}{\sigma - 1} (\sigma - \varepsilon) \widehat{P} \right] + s_m \widehat{P}_m,$$

$$\Leftrightarrow \widehat{P} = \frac{s_m}{1 + \left(\frac{\gamma}{\rho}\right)\left(\frac{\sigma-\varepsilon}{\sigma}\right)s_d} \widehat{P}_m. \quad (\text{C.2})$$

Thus

$$\widehat{\Lambda} = (\sigma - \varepsilon)\widehat{P} = \frac{(\sigma - \varepsilon)s_m}{1 + \left(\frac{\gamma}{\rho}\right)\left(\frac{\sigma-\varepsilon}{\sigma}\right)s_d} \widehat{P}_m = \chi \widehat{P}_m, \quad (\text{C.3})$$

which is the result in lemma 3.2.

Equations (3.34)(3.37)  $\Rightarrow$

$$\widehat{P}_m = \left(1 + \frac{\gamma^*}{\rho}\right)\widehat{\tau} - \frac{\gamma^*}{\sigma - 1}\widehat{\Lambda}. \quad (\text{C.4})$$

Together with (C.3) we have

$$\widehat{P}_m = \left(1 + \frac{\gamma^*}{\rho}\right)\widehat{\tau} - \frac{\gamma^*}{\sigma - 1}\chi \widehat{P}_m, \quad (\text{C.5})$$

which is equivalent to (3.38) in lemma 3.2.

### C.3 PROOF FOR LEMMA 3.3

We have shown in the main text that when  $\widehat{\tau} > 0$ ,  $\widehat{\Lambda} > 0$ . Then the result that  $\widehat{\varphi}_d < 0$  just follows from the Home ZCP condition (3.36).

Rearranging terms in (3.34) and using the expression for  $\widehat{P}_m$  in lemma 3.3, we have

$$\gamma^* \widehat{\varphi}_x^* = \widehat{P}_m - \widehat{\tau} = \frac{\frac{\gamma^*}{\rho}(1 - \frac{\chi}{\sigma})}{1 + \frac{\gamma^*}{\sigma-1}\chi} \widehat{\tau} = \frac{\frac{\gamma^*}{\rho}\left(1 - \frac{\frac{\sigma-\varepsilon}{\sigma}s_m}{1 + \left(\frac{\gamma}{\rho}\right)\left(\frac{\sigma-\varepsilon}{\sigma}\right)s_d}\right)}{1 + \frac{\gamma^*}{\sigma-1}\chi} \widehat{\tau}, \quad (\text{C.6})$$

which implies that  $\widehat{\varphi}_x^*$  and  $\widehat{\tau}$  are of the same sign and thus  $\varphi_x^*$  increases in  $\tau$ .

#### C.4 PROOF FOR LEMMA 3.4

Equation (3.27) implies  $\frac{d\Pi_d}{d\tau} = \frac{1}{\sigma}P_d^{1-\sigma}\frac{d\Lambda}{d\tau}$ , then

$$\begin{aligned}
\frac{d\Pi_d}{d\tau} &= \frac{1}{\sigma}P_d^{1-\sigma}\frac{\Lambda}{\tau}\frac{\widehat{\Lambda}}{\widehat{\tau}} \\
&= \frac{1}{\sigma}\frac{(P_d^{1-\sigma}\Lambda)}{\tau}\chi\frac{\widehat{P}_m}{\widehat{\tau}} \\
&= \frac{1}{\sigma}\frac{E_d}{\tau}\frac{(\sigma-\varepsilon)s_m}{1+(\frac{\gamma}{\rho})(\frac{\sigma-\varepsilon}{\sigma})s_d}\frac{\widehat{P}_m}{\widehat{\tau}} \\
&= \frac{1}{\sigma}\frac{\frac{E_d}{E}(\sigma-\varepsilon)}{1+(\frac{\gamma}{\rho})(\frac{\sigma-\varepsilon}{\sigma})s_d}\frac{E s_m \widehat{P}_m}{\tau \widehat{\tau}} \\
&= \frac{1}{\sigma}\frac{s_d(\sigma-\varepsilon)}{1+(\frac{\gamma}{\rho})(\frac{\sigma-\varepsilon}{\sigma})s_d}(R_x^*\frac{\widehat{P}_m}{\widehat{\tau}}), \tag{C.7}
\end{aligned}$$

where  $E_d$  and  $E_m$  denote consumers' spending on domestic and imported varieties of the sector. The last line of this equation directly leads to lemma 3.4.

#### C.5 PROOF FOR LEMMA 3.5

From (C.1) we know  $\widehat{P}_d = -\frac{\gamma}{\sigma-1}\widehat{\Lambda}$ , so

$$\begin{aligned}
[\frac{dP}{d\tau}] &= \frac{P}{\tau}\frac{\widehat{P}}{\widehat{\tau}} \\
&= \frac{P}{\tau}\left(\frac{s_d\widehat{P}_d + s_m\widehat{P}_m}{\widehat{\tau}}\right) \\
&= -\frac{P}{\tau}s_d\frac{\gamma}{\sigma-1}\frac{\widehat{\Lambda}}{\widehat{\tau}} + \frac{P}{\tau}s_m\frac{\widehat{P}_m - \widehat{\tau}}{\widehat{\tau}} + \frac{P}{\tau}s_m. \tag{C.8}
\end{aligned}$$

Then

$$\begin{aligned}
[\frac{dS(P(\tau))}{d\tau}] &= -D(P)\frac{dP}{d\tau} \\
&= -D(P)\left(-\frac{P}{\tau}s_d\frac{\gamma}{\sigma-1}\frac{\widehat{\Lambda}}{\widehat{\tau}} + \frac{P}{\tau}s_m\frac{\widehat{P}_m - \widehat{\tau}}{\widehat{\tau}} + \frac{P}{\tau}s_m\right) \\
&= \left[\frac{\frac{\gamma}{\rho}}{(\frac{\sigma-\varepsilon}{\sigma})s_d + \frac{\gamma}{\rho}}\right](R_x^*\frac{\widehat{P}_m}{\widehat{\tau}}) - R_x^*\frac{\widehat{P}_m - \widehat{\tau}}{\widehat{\tau}} - R_x^*. \tag{C.9}
\end{aligned}$$

The last “=” holds because  $-D(P)(-\frac{P}{\tau}s_d\frac{\gamma}{\sigma-1}\widehat{\Lambda}) = \frac{E}{\tau}s_d\frac{\gamma}{\sigma-1}\widehat{\Lambda} = \frac{E}{\tau}s_d\frac{\gamma}{\sigma-1}\chi\widehat{\frac{P_m}{\tau}} = \frac{E}{\tau}s_d\frac{\gamma}{\sigma-1}\frac{(\sigma-\varepsilon)s_m}{1+(\frac{\gamma}{\rho})(\frac{\sigma-\varepsilon}{\sigma})s_d}\widehat{\frac{P_m}{\tau}} = s_d\frac{\gamma}{\sigma-1}\frac{(\sigma-\varepsilon)}{1+(\frac{\gamma}{\rho})(\frac{\sigma-\varepsilon}{\sigma})s_d}(\frac{s_mE}{\tau}\widehat{\frac{P_m}{\tau}}) = [\frac{\frac{\gamma}{\rho}}{(\sigma-\varepsilon)s_d+\frac{\gamma}{\rho}}](R_x^*\widehat{\frac{P_m}{\tau}})$ .

## C.6 DERIVATION FOR THE ENDOGENOUS UNILATERAL TARIFF

Note  $\frac{\sigma s_d}{\chi} = \frac{\sigma}{(\sigma-\varepsilon)s_d} + \frac{\gamma}{\rho}$ , then

$$\begin{aligned}
\left(\frac{\sigma}{(\sigma-\varepsilon)s_d} + \frac{\gamma}{\rho}\right)(1 - \frac{\chi}{\sigma}) &= \frac{\sigma s_d}{\chi}(1 - \frac{\chi}{\sigma}) \\
&= \frac{1}{s_d}\left(\frac{\sigma s_m}{\chi} - (1 - s_d)\right) \\
&= \frac{1}{s_d}\left(\frac{\sigma}{\sigma-\varepsilon} + \frac{\gamma}{\rho}s_d - 1 + s_d\right) \\
&= \frac{1}{s_d}\left(\frac{\varepsilon}{\sigma-\varepsilon} + \left(1 + \frac{\gamma}{\rho}\right)s_d\right) \\
&= \frac{\varepsilon}{(\sigma-\varepsilon)s_d} + \left(1 + \frac{\gamma}{\rho}\right). \tag{C.10}
\end{aligned}$$

We have

$$\begin{aligned}
\left(\frac{\tau^O - 1}{\tau^O}\right) &= \frac{\frac{b+\frac{\gamma}{\rho}}{(\sigma-\varepsilon)s_d+\frac{\gamma}{\rho}} - \xi}{(\sigma-\chi) - \xi} \\
&= 1 - \frac{(\sigma-\chi) - \frac{b+\frac{\gamma}{\rho}}{(\sigma-\varepsilon)s_d+\frac{\gamma}{\rho}}}{(\sigma-\chi) - \xi} \\
&= 1 - \frac{\sigma\left(1 - \frac{\chi}{\sigma}\right) - \frac{b+\frac{\gamma}{\rho}}{(\sigma-\varepsilon)s_d+\frac{\gamma}{\rho}}}{\sigma\left(1 - \frac{\chi}{\sigma}\right) - \frac{1-\frac{\chi}{\sigma}}{1+\frac{\rho}{\gamma^*}}} \\
&= 1 - \frac{\sigma - \frac{b+\frac{\gamma}{\rho}}{(\frac{\sigma}{(\sigma-\varepsilon)s_d+\frac{\gamma}{\rho}})(1-\frac{\chi}{\sigma})}}{\sigma - \frac{1}{1+\frac{\rho}{\gamma^*}}} \\
&= 1 - \frac{\sigma - \frac{b+\frac{\gamma}{\rho}}{(\frac{\varepsilon}{(\sigma-\varepsilon)s_d}+1+\frac{\gamma}{\rho})}}{\sigma - \frac{1}{1+\frac{\rho}{\gamma^*}}}, \tag{C.11}
\end{aligned}$$

where (C.10) is used in the derivation for the last “=”.

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